Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/36-1.2.1.5-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [123]. This is test number [36].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (123)	0.00 (0)
Mathematica	100.00 (123)	0.00 (0)
Maple	98.37 (121)	1.63 (2)
Fricas	90.24 (111)	9.76 (12)
Giac	73.98 (91)	26.02 (32)
Maxima	54.47 (67)	45.53 (56)
Mupad	43.09 (53)	56.91 (70)
Sympy	34.96 (43)	65.04 (80)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

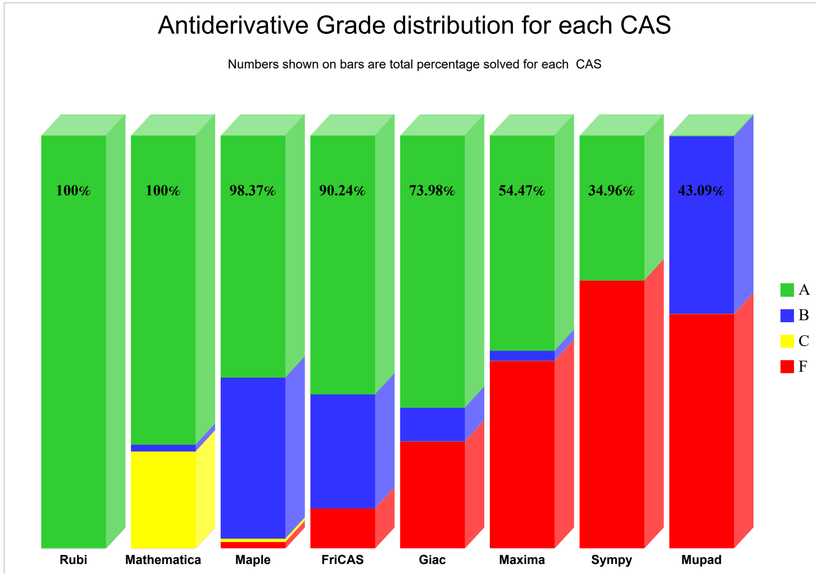
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

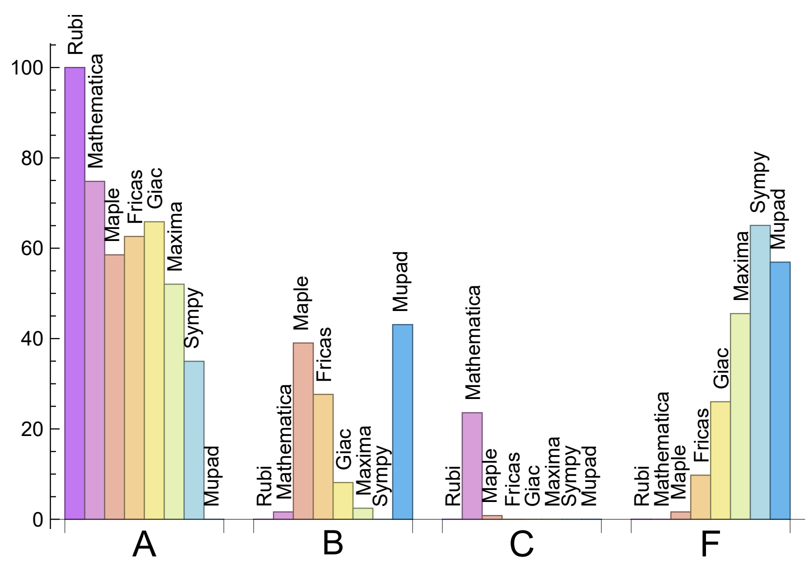
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	74.80	1.63	23.58	0.00
Giac	65.85	8.13	0.00	26.02
Fricas	62.60	27.64	0.00	9.76
Maple	58.54	39.02	0.81	1.63
Maxima	52.03	2.44	0.00	45.53
Sympy	34.96	0.00	0.00	65.04
Mupad	N/A	43.09	0.00	56.91

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Fricas	12	41.67 %	58.33 %	0.00 %
Giac	32	15.62 %	15.62 %	68.75 %
Maxima	56	64.29 %	0.00 %	35.71 %
Sympy	80	83.75 %	16.25 %	0.00 %
Mupad	70	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

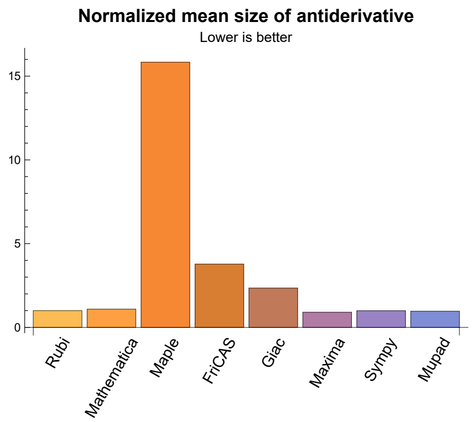
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

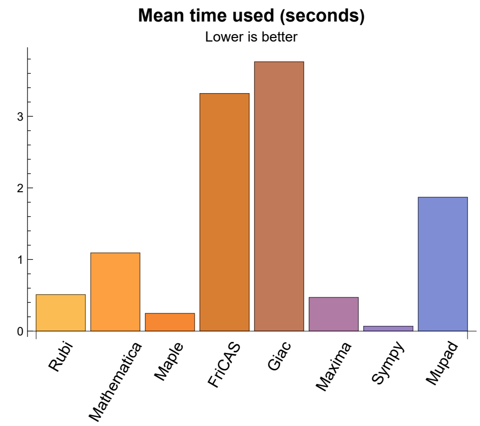
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.51	205.62	1.00	128.00	1.00
Mathematica	1.09	243.73	1.09	95.00	1.00
Maple	0.25	3970.25	15.83	144.00	0.83
Maxima	0.47	90.12	0.90	72.00	0.83
Fricas	3.32	825.00	3.78	112.00	1.24
Sympy	0.07	76.86	0.99	73.00	0.98
Giac	3.76	533.68	2.35	73.00	0.79
Mupad	1.87	105.40	0.96	64.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



Normalized mean size of antiderivative
Lower is better
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad



Mean time used (seconds)
Lower is better
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{13, 106, 107, 108, 113, 122\}$

Mathematica $\{108, 122, 123\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

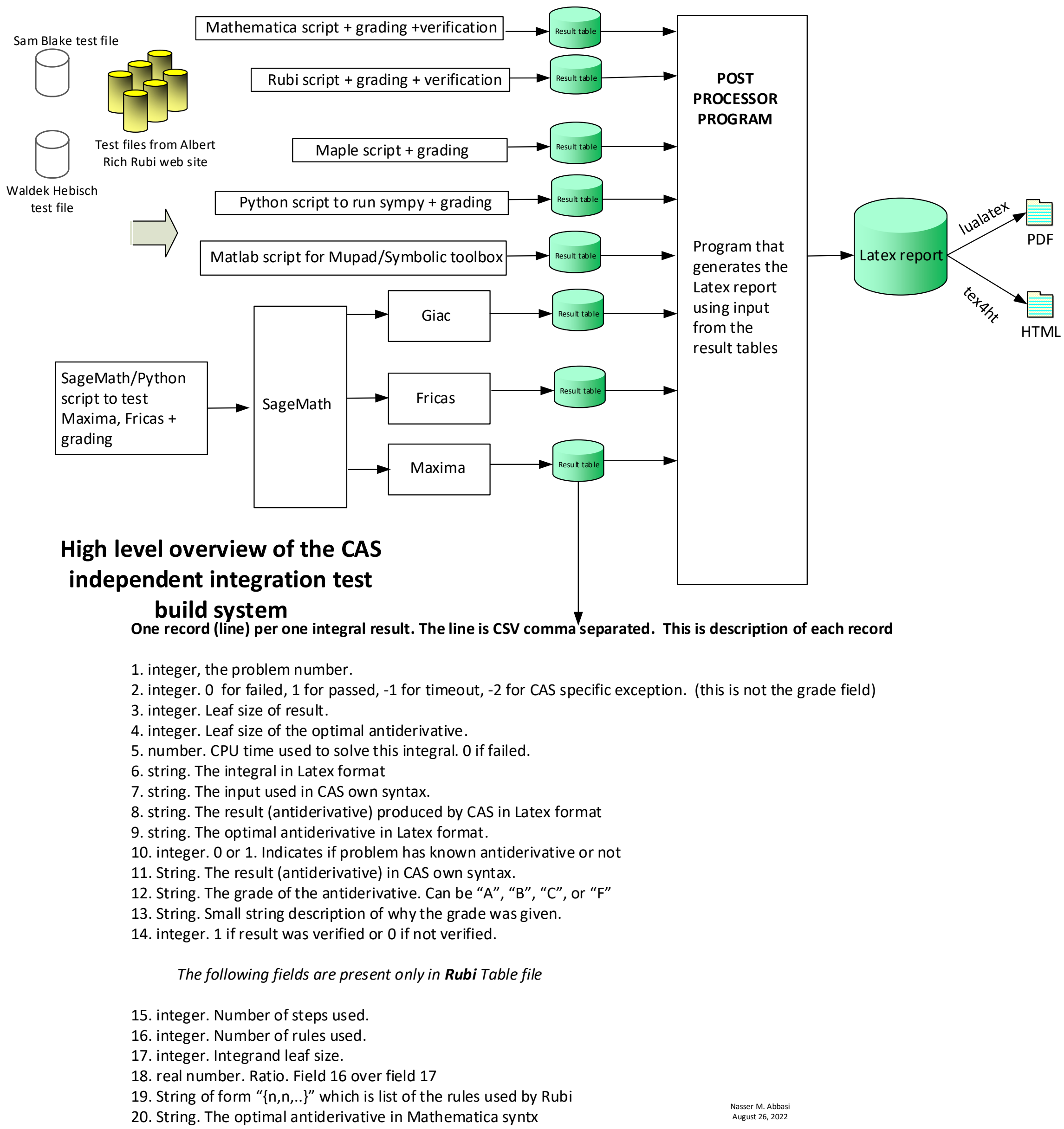
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 6, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 104, 105, 109, 110, 111, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123 }

B grade: { 5, 108 }

C grade: { 2, 3, 4, 7, 13, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 102, 103, 106, 107, 112, 117 }

F grade: { }

2.1.3 Maple

A grade: { 8, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 96, 111, 116, 122, 123 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 117, 118, 119, 120, 121 }

C grade: { 11 }

F grade: { 9, 10 }

2.1.4 Maxima

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 96 }

B grade: { 93, 94, 95 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

2.1.5 FriCAS

A grade: { 1, 8, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 93, 94, 96, 100, 101, 105, 109, 110, 111, 119 }

B grade: { 2, 3, 4, 5, 6, 7, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 89, 90, 91, 92, 95, 97, 98, 99, 104, 112, 114, 115, 116, 118, 120, 121 }

C grade: { }

F grade: { 9, 10, 13, 102, 103, 106, 107, 108, 113, 117, 122, 123 }

2.1.6 Sympy

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

2.1.7 Giac

A grade: { 1, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 105, 109, 110, 111, 114, 115, 116, 118, 119, 120 }

B grade: { 2, 3, 4, 5, 6, 8, 12, 14, 104, 121 }

C grade: { }

F grade: { 7, 9, 10, 13, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 102, 103, 106, 107, 108, 112, 113, 117, 122, 123 }

2.1.8 Mupad

A grade: { }

B grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 89, 96, 100, 101, 116, 120 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 121, 122, 123 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	87	195	0	208	0	84	-1
N.S.	1	1.00	0.85	1.91	0.00	2.04	0.00	0.82	-0.01
time (sec)	N/A	0.063	0.457	0.141	0.000	1.288	0.000	4.737	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	162	491	0	1062	0	847	-1
N.S.	1	1.00	1.98	5.99	0.00	12.95	0.00	10.33	-0.01
time (sec)	N/A	0.073	0.429	0.273	0.000	2.427	0.000	6.486	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	146	307	0	813	0	703	-1
N.S.	1	1.00	2.21	4.65	0.00	12.32	0.00	10.65	-0.02
time (sec)	N/A	0.051	0.390	0.207	0.000	2.463	0.000	3.447	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	215	827	0	1544	0	1166	-1
N.S.	1	1.00	1.67	6.41	0.00	11.97	0.00	9.04	-0.01
time (sec)	N/A	0.112	0.688	0.168	0.000	3.594	0.000	5.006	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	486	1868	0	3818	0	2986	-1
N.S.	1	1.00	2.17	8.34	0.00	17.04	0.00	13.33	-0.00
time (sec)	N/A	0.272	10.441	0.149	0.000	6.548	0.000	4.939	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	604	3649	0	8134	0	30280	-1
N.S.	1	1.00	1.84	11.12	0.00	24.80	0.00	92.32	-0.00
time (sec)	N/A	0.596	11.824	0.172	0.000	28.658	0.000	10.031	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	241	1352	0	1919	0	0	-1
N.S.	1	1.00	1.49	8.35	0.00	11.85	0.00	0.00	-0.01
time (sec)	N/A	0.196	1.011	0.151	0.000	4.544	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	39	27	0	38	0	52	-1
N.S.	1	1.00	1.39	0.96	0.00	1.36	0.00	1.86	-0.04
time (sec)	N/A	0.012	0.152	0.276	0.000	1.558	0.000	6.764	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	142	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.144	0.104	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	172	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.091	0.284	0.126	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	16	0	22	0	49	-1
N.S.	1	1.00	0.77	0.33	0.00	0.46	0.00	1.02	-0.02
time (sec)	N/A	0.010	0.070	0.089	0.000	1.190	0.000	2.257	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	84	0	82	0	143	-1
N.S.	1	1.00	0.89	1.20	0.00	1.17	0.00	2.04	-0.01
time (sec)	N/A	0.030	0.186	0.196	0.000	1.665	0.000	3.642	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1077	1077	600	714	0	0	0	0	-1
N.S.	1	1.00	0.56	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.042	3.132	0.268	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	75	341	0	161	0	171	-1
N.S.	1	1.00	0.77	3.48	0.00	1.64	0.00	1.74	-0.01
time (sec)	N/A	0.131	0.161	0.444	0.000	1.269	0.000	3.771	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	65	54	54
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79
time (sec)	N/A	0.028	0.003	0.099	0.286	5.340	0.015	3.547	0.062

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	44	44	53	44	44
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79
time (sec)	N/A	0.026	0.002	0.095	0.279	4.216	0.013	2.938	0.034

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.017	0.001	0.085	0.301	5.536	0.011	2.197	0.025

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.010	0.001	0.033	0.288	3.266	0.006	3.452	0.023

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	49	33	35
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.17	0.79	0.83
time (sec)	N/A	0.027	0.012	0.224	0.520	4.105	0.043	3.979	3.393

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	35
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.81
time (sec)	N/A	0.017	0.012	0.115	0.498	3.033	0.051	5.821	0.038

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	63	46	55
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86
time (sec)	N/A	0.022	0.019	0.105	0.509	1.814	0.066	3.610	0.049

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	64	64	76	64	64
N.S.	1	1.00	1.00	0.81	0.80	0.80	0.95	0.80	0.80
time (sec)	N/A	0.036	0.003	0.124	0.281	1.702	0.019	5.545	0.085

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	55	54	54	63	54	54
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.029	0.002	0.106	0.297	2.490	0.016	3.990	0.054

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	44	44
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.81	0.81
time (sec)	N/A	0.027	0.002	0.133	0.284	1.757	0.012	4.340	0.034

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	34	34	41	34	34
N.S.	1	1.00	1.00	0.76	0.74	0.74	0.89	0.74	0.74
time (sec)	N/A	0.016	0.001	0.096	0.272	2.549	0.009	3.315	0.025

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	44	43	43	63	43	45
N.S.	1	1.00	0.95	0.79	0.77	0.77	1.12	0.77	0.80
time (sec)	N/A	0.031	0.014	0.117	0.488	2.379	0.049	2.868	3.449

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	51	52	78	65	52	51
N.S.	1	1.00	0.94	0.81	0.83	1.24	1.03	0.83	0.81
time (sec)	N/A	0.041	0.024	0.138	0.492	2.639	0.067	3.020	0.050

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	63	46	55
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86
time (sec)	N/A	0.036	0.018	0.112	0.497	2.324	0.071	3.326	3.442

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	63	57	76	105	83	56	75
N.S.	1	1.00	0.74	0.67	0.89	1.24	0.98	0.66	0.88
time (sec)	N/A	0.039	0.032	0.122	0.509	1.883	0.085	3.648	3.468

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	75	74	74	92	74	74
N.S.	1	1.00	1.00	0.78	0.77	0.77	0.96	0.77	0.77
time (sec)	N/A	0.043	0.004	0.099	0.271	2.051	0.021	3.759	0.121

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	65	64	64	78	64	64
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.033	0.002	0.108	0.278	2.245	0.017	4.365	0.081

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	65	54	54
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79
time (sec)	N/A	0.029	0.002	0.114	0.278	3.040	0.014	3.632	0.054

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	44	44	53	44	44
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79
time (sec)	N/A	0.019	0.002	0.102	0.282	3.053	0.011	5.849	0.033

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	76	53	55
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.09	0.76	0.79
time (sec)	N/A	0.032	0.017	0.121	0.496	2.362	0.052	5.398	0.043

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	61	62	88	78	62	61
N.S.	1	1.00	1.00	0.79	0.81	1.14	1.01	0.81	0.79
time (sec)	N/A	0.048	0.021	0.125	0.511	2.296	0.070	2.326	3.431

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	63	72	118	85	62	71
N.S.	1	1.00	0.93	0.75	0.86	1.40	1.01	0.74	0.85
time (sec)	N/A	0.058	0.027	0.128	0.502	3.408	0.089	5.611	3.426

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	64	63	63	87	63	65
N.S.	1	1.00	0.86	0.76	0.75	0.75	1.04	0.75	0.77
time (sec)	N/A	0.041	0.021	0.203	0.499	1.721	0.058	5.122	3.441

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	73	53	55
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.04	0.76	0.79
time (sec)	N/A	0.032	0.016	0.129	0.503	2.616	0.051	4.698	0.042

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	44	43	43	60	43	45
N.S.	1	1.00	0.93	0.79	0.77	0.77	1.07	0.77	0.80
time (sec)	N/A	0.030	0.013	0.135	0.524	1.955	0.049	2.131	3.445

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	46	33	35
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.10	0.79	0.83
time (sec)	N/A	0.024	0.008	0.113	0.523	2.171	0.044	3.806	0.039

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	59	59	83	59	79
N.S.	1	1.00	1.00	0.82	0.81	0.81	1.14	0.81	1.08
time (sec)	N/A	0.035	0.023	0.140	0.535	3.069	0.098	2.466	0.187

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	77	78	117	102	78	95
N.S.	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.01
time (sec)	N/A	0.055	0.060	0.144	0.586	2.015	0.130	3.376	3.567

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	104	89	98	177	119	88	115
N.S.	1	1.00	0.90	0.77	0.85	1.54	1.03	0.77	1.00
time (sec)	N/A	0.078	0.114	0.139	0.513	2.312	0.153	2.249	0.179

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	71	72	98	90	72	72
N.S.	1	1.00	1.00	0.78	0.79	1.08	0.99	0.79	0.79
time (sec)	N/A	0.056	0.036	0.109	0.500	1.482	0.073	3.438	3.462

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	61	62	88	75	62	61
N.S.	1	1.00	1.00	0.79	0.81	1.14	0.97	0.81	0.79
time (sec)	N/A	0.047	0.021	0.138	0.657	2.257	0.071	2.987	3.424

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	51	52	78	61	52	52
N.S.	1	1.00	1.00	0.81	0.83	1.24	0.97	0.83	0.83
time (sec)	N/A	0.040	0.022	0.150	0.512	0.938	0.069	1.550	3.404

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	36
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.84
time (sec)	N/A	0.017	0.011	0.110	0.511	1.924	0.050	1.917	0.040

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	77	78	117	102	78	96
N.S.	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.02
time (sec)	N/A	0.055	0.045	0.142	0.510	1.648	0.127	1.413	3.578

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	106	94	96	167	122	96	115
N.S.	1	1.00	0.83	0.74	0.76	1.31	0.96	0.76	0.91
time (sec)	N/A	0.077	0.041	0.141	0.514	1.454	0.151	1.646	0.177

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	136	106	118	237	143	110	135
N.S.	1	1.00	0.92	0.72	0.80	1.60	0.97	0.74	0.91
time (sec)	N/A	0.104	0.055	0.158	0.500	2.305	0.174	1.052	3.586

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	73	82	128	95	72	81
N.S.	1	1.00	1.00	0.74	0.84	1.31	0.97	0.73	0.83
time (sec)	N/A	0.070	0.027	0.112	0.495	2.172	0.092	1.338	0.045

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	63	72	118	82	62	72
N.S.	1	1.00	1.00	0.75	0.86	1.40	0.98	0.74	0.86
time (sec)	N/A	0.060	0.026	0.123	0.509	1.909	0.087	2.002	0.049

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	47	56	75	63	46	56
N.S.	1	1.00	0.80	0.73	0.88	1.17	0.98	0.72	0.88
time (sec)	N/A	0.033	0.021	0.128	0.503	1.759	0.071	0.954	3.474

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	47	56	75	61	46	55
N.S.	1	1.00	0.80	0.73	0.88	1.17	0.95	0.72	0.86
time (sec)	N/A	0.021	0.021	0.109	0.507	1.565	0.065	1.427	0.044

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	99	89	98	177	122	88	116
N.S.	1	1.00	0.86	0.77	0.85	1.54	1.06	0.77	1.01
time (sec)	N/A	0.081	0.116	0.155	0.517	2.028	0.152	1.970	3.580

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	136	106	116	227	143	110	136
N.S.	1	1.00	0.85	0.66	0.72	1.42	0.89	0.69	0.85
time (sec)	N/A	0.102	0.088	0.163	0.518	2.388	0.174	1.027	3.597

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	151	118	138	297	163	116	155
N.S.	1	1.00	0.83	0.65	0.76	1.64	0.90	0.64	0.86
time (sec)	N/A	0.129	0.069	0.136	0.524	2.095	0.195	0.772	3.588

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	95	166	177	98	0	93	221
N.S.	1	1.00	0.46	0.80	0.85	0.47	0.00	0.45	1.06
time (sec)	N/A	0.192	0.808	0.145	0.519	1.797	0.000	1.444	5.028

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	85	132	143	88	0	83	187
N.S.	1	1.00	0.51	0.80	0.86	0.53	0.00	0.50	1.13
time (sec)	N/A	0.116	0.581	0.128	0.517	1.850	0.000	0.961	4.692

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	75	98	109	78	0	73	153
N.S.	1	1.00	0.60	0.79	0.88	0.63	0.00	0.59	1.23
time (sec)	N/A	0.060	0.363	0.123	0.505	2.191	0.000	1.673	4.185

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	64	75	68	0	63	119
N.S.	1	1.00	0.79	0.78	0.91	0.83	0.00	0.77	1.45
time (sec)	N/A	0.024	0.242	0.106	0.525	2.461	0.000	1.529	3.835

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	206	2065	0	2016	0	0	-1
N.S.	1	1.00	1.18	11.87	0.00	11.59	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.203	0.688	0.000	3.967	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	419	16357	0	2102	0	0	-1
N.S.	1	1.00	2.23	87.01	0.00	11.18	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.475	0.950	0.000	5.981	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	392	44343	0	2182	0	0	-1
N.S.	1	1.00	1.76	198.85	0.00	9.78	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.588	1.003	0.000	5.790	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	105	185	206	108	0	103	-1
N.S.	1	1.00	0.45	0.80	0.89	0.47	0.00	0.45	-0.00
time (sec)	N/A	0.211	1.016	0.143	0.514	5.622	0.000	5.187	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	95	151	172	98	0	93	-1
N.S.	1	1.00	0.50	0.80	0.91	0.52	0.00	0.49	-0.01
time (sec)	N/A	0.121	0.782	0.144	0.512	4.443	0.000	4.644	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	85	117	138	88	0	83	-1
N.S.	1	1.00	0.58	0.80	0.94	0.60	0.00	0.56	-0.01
time (sec)	N/A	0.073	0.566	0.125	0.512	4.104	0.000	2.980	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	75	83	104	78	0	73	-1
N.S.	1	1.00	0.71	0.79	0.99	0.74	0.00	0.70	-0.01
time (sec)	N/A	0.031	0.374	0.104	0.542	2.177	0.000	3.893	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	228	3460	0	2027	0	0	-1
N.S.	1	1.00	1.16	17.56	0.00	10.29	0.00	0.00	-0.01
time (sec)	N/A	0.314	0.343	0.833	0.000	4.390	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	416	28185	0	2150	0	0	-1
N.S.	1	1.00	1.79	121.49	0.00	9.27	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.520	0.982	0.000	2.793	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	572	81552	0	2183	0	0	-1
N.S.	1	1.00	2.57	365.70	0.00	9.79	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.788	0.970	0.000	3.274	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	115	204	235	118	0	113	-1
N.S.	1	1.00	0.45	0.80	0.93	0.46	0.00	0.44	-0.00
time (sec)	N/A	0.237	1.282	0.142	0.514	2.907	0.000	4.177	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	105	170	201	108	0	103	-1
N.S.	1	1.00	0.50	0.80	0.95	0.51	0.00	0.49	-0.00
time (sec)	N/A	0.140	1.019	0.118	0.504	3.744	0.000	5.250	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	95	136	167	98	0	93	-1
N.S.	1	1.00	0.56	0.80	0.98	0.58	0.00	0.55	-0.01
time (sec)	N/A	0.080	0.775	0.121	0.545	2.637	0.000	4.179	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	85	102	133	88	0	83	-1
N.S.	1	1.00	0.66	0.80	1.04	0.69	0.00	0.65	-0.01
time (sec)	N/A	0.037	0.561	0.098	0.499	3.214	0.000	3.195	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	238	4860	0	2010	0	0	-1
N.S.	1	1.00	1.07	21.89	0.00	9.05	0.00	0.00	-0.00
time (sec)	N/A	0.353	0.550	0.814	0.000	1.837	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	433	40028	0	2161	0	0	-1
N.S.	1	1.00	1.70	156.97	0.00	8.47	0.00	0.00	-0.00
time (sec)	N/A	0.425	0.718	0.957	0.000	3.764	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	616	119458	0	2240	0	0	-1
N.S.	1	1.00	2.19	425.12	0.00	7.97	0.00	0.00	-0.00
time (sec)	N/A	0.422	0.859	1.090	0.000	3.508	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	85	147	148	88	0	83	-1
N.S.	1	1.00	0.46	0.79	0.80	0.48	0.00	0.45	-0.01
time (sec)	N/A	0.197	0.636	0.133	0.517	3.331	0.000	4.180	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	75	113	114	78	0	73	-1
N.S.	1	1.00	0.52	0.79	0.80	0.55	0.00	0.51	-0.01
time (sec)	N/A	0.114	0.455	0.141	0.510	1.503	0.000	2.610	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	65	79	80	68	0	63	-1
N.S.	1	1.00	0.64	0.78	0.79	0.67	0.00	0.62	-0.01
time (sec)	N/A	0.057	0.297	0.119	0.495	2.098	0.000	4.843	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	45	46	58	0	53	-1
N.S.	1	1.00	0.93	0.76	0.78	0.98	0.00	0.90	-0.02
time (sec)	N/A	0.021	0.156	0.096	0.505	1.775	0.000	4.927	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	135	684	0	2002	0	0	-1
N.S.	1	1.00	0.91	4.62	0.00	13.53	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.188	0.569	0.000	2.374	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	230	5225	0	2102	0	0	-1
N.S.	1	1.00	1.22	27.79	0.00	11.18	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.365	0.632	0.000	3.568	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	396	13040	0	2183	0	0	-1
N.S.	1	1.00	1.78	58.48	0.00	9.79	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.634	0.646	0.000	2.860	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	85	166	148	112	0	82	-1
N.S.	1	1.00	0.51	1.00	0.89	0.67	0.00	0.49	-0.01
time (sec)	N/A	0.131	0.905	0.133	0.528	2.105	0.000	2.702	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	75	132	114	102	0	72	-1
N.S.	1	1.00	0.60	1.06	0.92	0.82	0.00	0.58	-0.01
time (sec)	N/A	0.079	0.638	0.132	0.514	1.782	0.000	4.178	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	98	80	92	0	62	-1
N.S.	1	1.00	0.79	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.045	0.487	0.126	0.504	1.753	0.000	2.506	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	55	64	46	82	0	53	87
N.S.	1	1.00	1.22	1.42	1.02	1.82	0.00	1.18	1.93
time (sec)	N/A	0.019	0.263	0.102	0.501	1.681	0.000	3.506	0.227

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	199	718	0	2083	0	0	-1
N.S.	1	1.00	1.13	4.08	0.00	11.84	0.00	0.00	-0.01
time (sec)	N/A	0.267	0.346	0.642	0.000	2.800	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	414	5942	0	2173	0	0	-1
N.S.	1	1.00	1.96	28.16	0.00	10.30	0.00	0.00	-0.00
time (sec)	N/A	0.312	0.673	0.697	0.000	2.806	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	607	18981	0	2263	0	0	-1
N.S.	1	1.00	2.47	77.16	0.00	9.20	0.00	0.00	-0.00
time (sec)	N/A	0.337	1.118	0.862	0.000	2.739	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	85	214	253	132	0	81	-1
N.S.	1	1.00	0.58	1.46	1.72	0.90	0.00	0.55	-0.01
time (sec)	N/A	0.105	1.056	0.138	0.515	1.818	0.000	5.787	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	75	180	219	122	0	72	-1
N.S.	1	1.00	0.71	1.71	2.09	1.16	0.00	0.69	-0.01
time (sec)	N/A	0.066	0.790	0.133	0.519	1.757	0.000	4.290	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	146	185	112	0	61	-1
N.S.	1	1.00	0.96	2.15	2.72	1.65	0.00	0.90	-0.01
time (sec)	N/A	0.040	0.582	0.121	0.516	1.558	0.000	3.967	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	69	59	51	0	29	29
N.S.	1	1.00	0.70	1.47	1.26	1.09	0.00	0.62	0.62
time (sec)	N/A	0.014	0.346	0.117	0.285	2.075	0.000	4.185	0.090

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	209	751	0	2133	0	0	-1
N.S.	1	1.00	1.05	3.77	0.00	10.72	0.00	0.00	-0.01
time (sec)	N/A	0.303	0.635	0.632	0.000	2.390	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	416	5975	0	2253	0	0	-1
N.S.	1	1.00	1.78	25.53	0.00	9.63	0.00	0.00	-0.00
time (sec)	N/A	0.348	1.046	0.769	0.000	3.361	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	605	19014	0	2343	0	0	-1
N.S.	1	1.00	2.25	70.68	0.00	8.71	0.00	0.00	-0.00
time (sec)	N/A	0.382	1.201	0.819	0.000	6.358	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	456	1186	0	1267	0	638	1299
N.S.	1	1.00	1.05	2.72	0.00	2.91	0.00	1.46	2.98
time (sec)	N/A	0.496	1.979	0.201	0.000	3.812	0.000	3.190	5.312

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	171	343	0	469	0	212	320
N.S.	1	1.00	0.98	1.96	0.00	2.68	0.00	1.21	1.83
time (sec)	N/A	0.103	0.629	0.134	0.000	4.324	0.000	5.102	3.906

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	396	1547	0	0	0	0	-1
N.S.	1	1.00	0.92	3.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.706	0.499	0.222	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	972	4691	0	0	0	0	-1
N.S.	1	1.00	1.99	9.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.892	1.662	0.179	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	766	1654	0	2175	0	1150	-1
N.S.	1	1.00	1.36	2.93	0.00	3.86	0.00	2.04	-0.00
time (sec)	N/A	0.606	5.080	0.149	0.000	4.584	0.000	6.533	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	290	499	0	849	0	417	-1
N.S.	1	1.00	1.23	2.11	0.00	3.60	0.00	1.77	-0.00
time (sec)	N/A	0.144	1.216	0.136	0.000	5.340	0.000	7.859	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	678	1150	2860	0	0	0	0	-1
N.S.	1	1.00	1.69	4.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.726	1.261	0.210	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	704	704	2416	7856	0	0	0	0	-1
N.S.	1	1.00	3.43	11.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.553	3.794	0.155	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	671	669	1621	16309	0	0	0	0	-1
N.S.	1	1.00	2.42	24.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.091	16.304	0.161	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	717	615	2207	0	1569	0	824	-1
N.S.	1	1.00	0.86	3.08	0.00	2.19	0.00	1.15	-0.00
time (sec)	N/A	1.712	2.787	0.148	0.000	3.698	0.000	3.672	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	251	714	0	629	0	304	-1
N.S.	1	1.00	0.79	2.26	0.00	1.99	0.00	0.96	-0.00
time (sec)	N/A	0.400	0.856	0.148	0.000	4.092	0.000	6.713	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	99	188	0	231	0	98	-1
N.S.	1	1.00	0.85	1.62	0.00	1.99	0.00	0.84	-0.01
time (sec)	N/A	0.061	0.441	0.131	0.000	3.586	0.000	5.546	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	211	761	0	11127	0	0	-1
N.S.	1	1.00	0.56	2.03	0.00	29.75	0.00	0.00	-0.00
time (sec)	N/A	0.362	0.393	0.156	0.000	15.622	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	789	787	1369	2108	0	0	0	0	-1
N.S.	1	1.00	1.74	2.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.679	12.951	0.161	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	768	2299	0	3109	0	1099	-1
N.S.	1	1.00	1.18	3.54	0.00	4.79	0.00	1.69	-0.00
time (sec)	N/A	1.274	4.238	0.160	0.000	4.188	0.000	3.938	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	291	749	0	1277	0	407	-1
N.S.	1	1.00	0.94	2.42	0.00	4.13	0.00	1.32	-0.00
time (sec)	N/A	0.270	1.503	0.160	0.000	3.188	0.000	6.399	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	114	201	0	435	0	122	143
N.S.	1	1.00	1.03	1.81	0.00	3.92	0.00	1.10	1.29
time (sec)	N/A	0.051	0.551	0.131	0.000	3.008	0.000	6.441	3.734

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	692	1906	0	0	0	0	-1
N.S.	1	1.00	1.04	2.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.147	1.057	0.164	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	891	891	872	3251	0	4005	0	1401	-1
N.S.	1	1.00	0.98	3.65	0.00	4.49	0.00	1.57	-0.00
time (sec)	N/A	1.103	10.899	0.204	0.000	9.649	0.000	1.971	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	392	1129	0	1595	0	587	-1
N.S.	1	1.00	0.88	2.54	0.00	3.59	0.00	1.32	-0.00
time (sec)	N/A	0.276	2.361	0.142	0.000	7.990	0.000	4.523	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	147	351	0	291	0	240	175
N.S.	1	1.00	1.12	2.68	0.00	2.22	0.00	1.83	1.34
time (sec)	N/A	0.055	0.882	0.134	0.000	6.657	0.000	3.737	3.708

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	144	0	154	0	205	-1
N.S.	1	1.00	1.04	2.82	0.00	3.02	0.00	4.02	-0.02
time (sec)	N/A	0.040	0.158	0.386	0.000	3.339	0.000	4.957	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1432	1432	670	906	0	0	0	0	-1
N.S.	1	1.00	0.47	0.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.900	3.920	0.292	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	390	420	0	0	0	0	-1
N.S.	1	1.00	0.60	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	2.246	0.393	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [10] had the largest ratio of [34]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	29	0.138
2	A	2	2	1.00	31	0.065
3	A	2	2	1.00	27	0.074
4	A	4	4	1.00	27	0.148
5	A	5	5	1.00	27	0.185
6	A	6	5	1.00	27	0.185
7	A	4	4	1.00	31	0.129
8	A	2	2	1.00	23	0.087
9	A	2	2	1.00	31	0.065
10	A	2	2	1.00	34	0.059
11	A	3	3	1.00	22	0.136
12	A	6	5	1.00	18	0.278
13	A	3	3	1.00	26	0.115
14	A	16	12	1.00	27	0.444
15	A	2	1	1.00	23	0.043
16	A	2	1	1.00	23	0.043
17	A	2	1	1.00	23	0.043
18	A	2	1	1.00	21	0.048
19	A	6	5	1.00	23	0.217
20	A	4	4	1.00	23	0.174
21	A	5	5	1.00	23	0.217
22	A	2	1	1.00	25	0.040
23	A	2	1	1.00	25	0.040
24	A	2	1	1.00	25	0.040
25	A	2	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	5	1.00	25	0.200
27	A	7	6	1.00	25	0.240
28	A	5	4	1.00	25	0.160
29	A	6	5	1.00	25	0.200
30	A	2	1	1.00	25	0.040
31	A	2	1	1.00	25	0.040
32	A	2	1	1.00	25	0.040
33	A	2	1	1.00	23	0.043
34	A	6	5	1.00	25	0.200
35	A	7	6	1.00	25	0.240
36	A	8	6	1.00	25	0.240
37	A	6	5	1.00	25	0.200
38	A	6	5	1.00	25	0.200
39	A	6	5	1.00	25	0.200
40	A	6	5	1.00	23	0.217
41	A	9	5	1.00	25	0.200
42	A	10	6	1.00	25	0.240
43	A	11	7	1.00	25	0.280
44	A	7	6	1.00	25	0.240
45	A	7	6	1.00	25	0.240
46	A	7	6	1.00	25	0.240
47	A	4	4	1.00	23	0.174
48	A	10	6	1.00	25	0.240
49	A	11	7	1.00	25	0.280
50	A	12	7	1.00	25	0.280
51	A	8	6	1.00	25	0.240
52	A	8	6	1.00	25	0.240
53	A	5	4	1.00	25	0.160
54	A	5	5	1.00	23	0.217
55	A	11	7	1.00	25	0.280
56	A	12	7	1.00	25	0.280
57	A	13	7	1.00	25	0.280
58	A	11	5	1.00	27	0.185
59	A	9	5	1.00	27	0.185
60	A	7	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	25	0.200
62	A	8	7	1.00	27	0.259
63	A	6	5	1.00	27	0.185
64	A	7	6	1.00	27	0.222
65	A	12	5	1.00	27	0.185
66	A	10	5	1.00	27	0.185
67	A	8	5	1.00	27	0.185
68	A	6	5	1.00	25	0.200
69	A	9	8	1.00	27	0.296
70	A	10	9	1.00	27	0.333
71	A	7	6	1.00	27	0.222
72	A	13	5	1.00	27	0.185
73	A	11	5	1.00	27	0.185
74	A	9	5	1.00	27	0.185
75	A	7	5	1.00	25	0.200
76	A	10	9	1.00	27	0.333
77	A	11	9	1.00	27	0.333
78	A	11	10	1.00	27	0.370
79	A	10	4	1.00	27	0.148
80	A	8	4	1.00	27	0.148
81	A	6	4	1.00	27	0.148
82	A	4	4	1.00	25	0.160
83	A	5	4	1.00	27	0.148
84	A	6	5	1.00	27	0.185
85	A	7	6	1.00	27	0.222
86	A	9	5	1.00	27	0.185
87	A	7	5	1.00	27	0.185
88	A	5	5	1.00	27	0.185
89	A	4	4	1.00	25	0.160
90	A	6	5	1.00	27	0.185
91	A	7	6	1.00	27	0.222
92	A	8	6	1.00	27	0.222
93	A	8	5	1.00	27	0.185
94	A	6	5	1.00	27	0.185
95	A	5	4	1.00	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	25	0.120
97	A	7	6	1.00	27	0.222
98	A	8	6	1.00	27	0.222
99	A	9	6	1.00	27	0.222
100	A	7	5	1.00	27	0.185
101	A	5	5	1.00	25	0.200
102	A	8	5	1.00	27	0.185
103	A	6	4	1.00	27	0.148
104	A	8	5	1.00	27	0.185
105	A	6	5	1.00	25	0.200
106	A	9	6	1.00	27	0.222
107	A	10	7	1.00	27	0.259
108	A	7	5	1.00	27	0.185
109	A	8	4	1.00	27	0.148
110	A	6	4	1.00	27	0.148
111	A	4	4	1.00	25	0.160
112	A	5	3	1.00	27	0.111
113	A	6	4	1.00	27	0.148
114	A	7	5	1.00	27	0.185
115	A	5	5	1.00	27	0.185
116	A	4	4	1.00	25	0.160
117	A	6	4	1.00	27	0.148
118	A	6	5	1.00	27	0.185
119	A	5	4	1.00	27	0.148
120	A	3	3	1.00	25	0.120
121	A	5	4	1.00	27	0.148
122	A	3	3	1.00	29	0.103
123	A	3	3	1.00	29	0.103

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(8af-b\left(e+\frac{4df}{e}\right)\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\,\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}$$

```
[Out] 1/8*(8*a*f-b*(e+4*d*f/e))*arctanh(1/2*(2*f*x+e)/f^(1/2)/(f*x^2+e*x+d)^(1/2)
)/f^(3/2)+1/4*b*(f*x^2+e*x+d)^(1/2)/f+1/2*b*x*(f*x^2+e*x+d)^(1/2)/e
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1675, 654, 635, 212}

$$\frac{\left(8af-b\left(\frac{4df}{e}+e\right)\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\,\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{b\sqrt{d+ex+fx^2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (b*Sqrt[d + e*x + f*x^2])/(4*f) + (b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((8*a
*f - b*(e + (4*d*f)/e))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2
])])/(8*f^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx = \frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\int \frac{\left(2a-\frac{bd}{e}\right)f+\frac{bfx}{2}}{\sqrt{d+ex+fx^2}}\,dx}{2f}$$

$$= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(-be+8af-\frac{4bdf}{e}\right)\int \frac{1}{\sqrt{d+ex+fx^2}}}{8f}$$

$$= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(-be+8af-\frac{4bdf}{e}\right)\operatorname{Subst}\left(\int \frac{1}{4f-x^2}\,dx\right.}{4f}$$

$$= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}-\frac{\left(be-8af+\frac{4bdf}{e}\right)\tanh^{-1}\left(\frac{e+}{2\sqrt{f}\sqrt{d}}\right.}{8f^{3/2}}$$

Mathematica [A]
time = 0.46, size = 87, normalized size = 0.85

$$\frac{2b\sqrt{f}(e+2fx)\sqrt{d+x(e+fx)}+\left(-8aef+b(e^2+4df)\right)\log\left(ef\left(e+2fx-2\sqrt{f}\sqrt{d+x(e+fx)}\right)\right)}{8ef^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (2*b*Sqrt[f]*(e + 2*f*x)*Sqrt[d + x*(e + f*x)] + (-8*a*e*f + b*(e^2 + 4*d*f
))*Log[e*f*(e + 2*f*x - 2*Sqrt[f]*Sqrt[d + x*(e + f*x)])])/(8*e*f^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(84) = 168$.
time = 0.14, size = 195, normalized size = 1.91

method	result
risch	$\frac{b(2fx+e)\sqrt{f\,x^2+ex+d}}{4fe}+\frac{\frac{ae\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{\sqrt{f}}-\frac{\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)bd}{2\sqrt{f}}-\frac{\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{j\right.}}{}}{e}$
default	$\frac{bf\left(\frac{x\sqrt{f\,x^2+ex+d}}{2f}-\frac{3e\left(\frac{\sqrt{f\,x^2+ex+d}}{f}-\frac{e\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{2f^{\frac{3}{2}}}\right)}{4f}-\frac{d\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{f\,x^2+ex+\right.}}{2f^{\frac{3}{2}}}\right.}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(b*f*(1/2*x/f*(f*x^2+e*x+d)^(1/2)-3/4*e/f*(1/f*(f*x^2+e*x+d)^(1/2)-1/2*
e/f^(3/2)*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2)))-1/2*d/f^(3/2)*ln((1/
2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2)))+e*b*(1/f*(f*x^2+e*x+d)^(1/2)-1/2*e/f
^(3/2)*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2)))+a*e*ln((1/2*e+f*x)/f^(1
/2)+(f*x^2+e*x+d)^(1/2))/f^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-%e^2>0)', see `assume?` for m
ore det
```

Fricas [A]
time = 1.29, size = 208, normalized size = 2.04

$$\left[\frac{\left((4\,bdf-8\,afe+be^2)\sqrt{f}\,\log\left(8\,f^2x^2+8\,fxe-4\,\sqrt{fx^2+xe+d}\,(2\,fx+e)\sqrt{f}+4\,df+e^2\right)+4\,(2\,bf^2x+bfe)\sqrt{fx^2+xe+d}\right)e^{(-1)}}{16\,f^2},\ \frac{\left((4\,bdf-8\,afe+be^2)\sqrt{-f}\,\arctan\left(\frac{\sqrt{fx^2+xe+d}\,(2\,fx+e)\sqrt{-f}}{2\,(f^2x^2+fxe+df)}\right)+2\,(2\,bf^2x+bfe)\sqrt{fx^2+xe+d}\right)e^{(-1)}}{8\,f^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((4*b*d*f - 8*a*f*e + b*e^2)*sqrt(f)*log(8*f^2*x^2 + 8*f*x*e - 4*sqrt
(f*x^2 + x*e + d)*(2*f*x + e)*sqrt(f) + 4*d*f + e^2) + 4*(2*b*f^2*x + b*f*e
)*sqrt(f*x^2 + x*e + d))*e^(-1)/f^2, 1/8*((4*b*d*f - 8*a*f*e + b*e^2)*sqrt(
-f)*arctan(1/2*sqrt(f*x^2 + x*e + d)*(2*f*x + e)*sqrt(-f)/(f^2*x^2 + f*x*e
+ d*f)) + 2*(2*b*f^2*x + b*f*e)*sqrt(f*x^2 + x*e + d))*e^(-1)/f^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ae}{\sqrt{d+ex+fx^2}}\,dx+\int \frac{bex}{\sqrt{d+ex+fx^2}}\,dx+\int \frac{bfx^2}{\sqrt{d+ex+fx^2}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] (Integral(a*e/sqrt(d + e*x + f*x**2), x) + Integral(b*e*x/sqrt(d + e*x + f*
x**2), x) + Integral(b*f*x**2/sqrt(d + e*x + f*x**2), x))/e
```

Giac [A]
time = 4.74, size = 84, normalized size = 0.82

$$\frac{1}{4}\sqrt{fx^2+xe+d}\left(2\,bxe^{(-1)}+\frac{b}{f}\right)+\frac{(4\,bdf-8\,afe+be^2)e^{(-1)}\log\left(\left|-2\left(\sqrt{f}\,x-\sqrt{fx^2+xe+d}\right)\sqrt{f}-e\right|\right)}{8\,f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(f*x^2 + x*e + d)*(2*b*x*e^(-1) + b/f) + 1/8*(4*b*d*f - 8*a*f*e + b
*e^2)*e^(-1)*log(abs(-2*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*sqrt(f) - e))/f
^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x+\frac{b\,f\,x^2}{e}}{\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2),x)
```

```
[Out] int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2), x)
```

3.2 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{bd-ae}\,(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

```
[Out] -2*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)/e^(1/2)/(-4*a*f+b*e)^(1/2)/(f*x^2+e*x
+d)^(1/2))*e^(1/2)/(-a*e+b*d)^(1/2)/(-4*a*f+b*e)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {996, 214}

$$-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] (-2*Sqrt[e]*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f
]*Sqrt[d + e*x + f*x^2])])/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = -\left((2e)\text{Subst}\left(\int \frac{1}{e(be-4af)-(bd-ae)x^2}\,dx, x, \frac{e+2fx}{\sqrt{d+ex+j}}\right.\right.$$

$$= -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{bd-ae}\,(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.43, size = 162, normalized size = 1.98

$$e\text{RootSum}\left[-bde^2+ae^3+bd^2f+2bde\sqrt{f}\#1-4ae^2\sqrt{f}\#1+be^2\#1^2-2bdf\#1^2+4aef\#1^2-2be\sqrt{f}\#1^3+bf\#1^4\&, \frac{\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right)}{bd\sqrt{f}-2ae\sqrt{f}+be\#1-b\sqrt{f}\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] e*RootSum[-(b*d*e^2) + a*e^3 + b*d^2*f + 2*b*d*e*Sqrt[f]*#1 - 4*a*e^2*Sqrt[
f]*#1 + b*e^2*#1^2 - 2*b*d*f*#1^2 + 4*a*e*f*#1^2 - 2*b*e*Sqrt[f]*#1^3 + b*f
*#1^4 & , Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]/(b*d*Sqrt[f] - 2*a
*e*Sqrt[f] + b*e*#1 - b*Sqrt[f]*#1^2) & ]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(68) = 136$.
time = 0.27, size = 491, normalized size = 5.99

method	result
default	$e\left(\ln\left(\frac{-\frac{2(ae-bd)}{b}-\frac{\sqrt{-eb(4fa-eb)}\left(x+\frac{eb+\sqrt{-eb(4fa-eb)}}{2bf}\right)}{b}+2\sqrt{-\frac{ae-bd}{b}}\sqrt{\left(x+\frac{eb+\sqrt{-eb(4fa}}{2bf}\right.}}{x+\frac{eb+\sqrt{-eb(4fa-eb}}{2bf}}\right)\right.\Big/ \sqrt{-eb(4fa-eb)}\sqrt{-\frac{ae}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/(-e*b*(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-e
*b*(4*a*f-b*e))^(1/2)/b*(x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*
(a*e-b*d))^(1/2)*((x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-e*b*(4*a
*f-b*e))^(1/2)/b*(x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^
(1/2))/(x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f))-1/(-e*b*(4*a*f-b*e))^(1/
2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-e*b*(4*a*f-b*e))^(1/2)/b*(x-
1/2*(-e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(
-e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-e*b*(4*a*f-b*e))^(1/2)/b*(x-1/2*(
-e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-e*b+(-e*
b*(4*a*f-b*e))^(1/2))/b/f)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

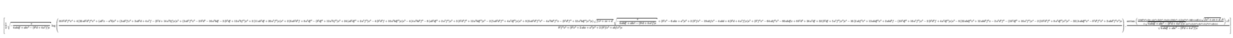
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*f-%e*b>0)', see `assume?` for m
ore det
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(72) = 144$.
time = 2.43, size = 1062, normalized size = 12.95



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-e/(4*a*b*d*f + a*b*e^2 - (b^2*d + 4*a^2*f)*e))*log((16*b^2*d^2*f
^4*x^4 + 4*(32*a*b^2*d^2*f^4*x^3 + (a*b^2*x - a^2*b)*e^6 + (3*a*b^2*f*x^2 +
 3*a*b^2*d + 4*a^3*f - (b^3*d + 14*a^2*b*f)*x)*e^5 + (2*a*b^2*f^2*x^3 - 2*b
^3*d^2 - 16*a^2*b*d*f - 3*(b^3*d*f + 12*a^2*b*f^2)*x^2 + 2*(11*a*b^2*d*f +
20*a^3*f^2)*x)*e^4 + 2*(6*a*b^2*d^2*f + 8*a^3*d*f^2 - (b^3*d*f^2 + 12*a^2*b
*f^3)*x^3 + 24*(a*b^2*d*f^2 + 2*a^3*f^3)*x^2 - 4*(b^3*d^2*f + 10*a^2*b*d*f^
2)*x)*e^3 - 4*(4*a^2*b*d^2*f^2 - 8*(a*b^2*d*f^3 + 2*a^3*f^4)*x^3 + 3*(b^3*d
```

```
^2*f^2 + 12*a^2*b*d*f^3)*x^2 - 2*(5*a*b^2*d^2*f^2 + 4*a^3*d*f^3)*x)*e^2 + 8
*(6*a*b^2*d^2*f^3*x^2 - 4*a^2*b*d^2*f^3*x - (b^3*d^2*f^3 + 12*a^2*b*d*f^4)*
x^3)*e)*sqrt(f*x^2 + x*e + d)*sqrt(-e/(4*a*b*d*f + a*b*e^2 - (b^2*d + 4*a^2
*f)*e)) + (b^2*x^2 - 6*a*b*x + a^2)*e^6 + 2*(b^2*f*x^3 - 19*a*b*f*x^2 - 4*a
*b*d + 4*(b^2*d + 4*a^2*f)*x)*e^5 + (b^2*f^2*x^4 - 64*a*b*f^2*x^3 - 80*a*b*
d*f*x + 8*b^2*d^2 + 24*a^2*d*f + 32*(b^2*d*f + 5*a^2*f^2)*x^2)*e^4 - 16*(2*
a*b*f^3*x^4 + 13*a*b*d*f^2*x^2 + 2*a*b*d^2*f - (3*b^2*d*f^2 + 16*a^2*f^3)*x
^3 - 2*(b^2*d^2*f + 4*a^2*d*f^2)*x)*e^3 - 8*(32*a*b*d*f^3*x^3 + 12*a*b*d^2*
f^2*x - 2*a^2*d^2*f^2 - (3*b^2*d*f^3 + 16*a^2*f^4)*x^4 - 2*(3*b^2*d^2*f^2 +
 8*a^2*d*f^3)*x^2)*e^2 - 32*(4*a*b*d*f^4*x^4 - b^2*d^2*f^3*x^3 + 3*a*b*d^2*
f^3*x^2)*e)/(b^2*f^2*x^4 + (b^2*x^2 + 2*a*b*x + a^2)*e^2 + 2*(b^2*f*x^3 + a
*b*f*x^2)*e)), arctan(1/2*(4*b*d*f^2*x^2 + (b*x - a)*e^3 + (b*f*x^2 - 8*a*f
*x + 2*b*d)*e^2 - 4*(2*a*f^2*x^2 - b*d*f*x + a*d*f)*e)*sqrt(f*x^2 + x*e + d
)*e^(1/2)/(sqrt(4*a*b*d*f + a*b*e^2 - (b^2*d + 4*a^2*f)*e)*(x*e^3 + (3*f*x^
2 + d)*e^2 + 2*(f^2*x^3 + d*f*x)*e)))*e^(1/2)/sqrt(4*a*b*d*f + a*b*e^2 - (b
^2*d + 4*a^2*f)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\int \frac{1}{ae\sqrt{d+ex+fx^2}+bex\sqrt{d+ex+fx^2}+bfx^2\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] e*Integral(1/(a*e*sqrt(d + e*x + f*x**2) + b*e*x*sqrt(d + e*x + f*x**2) + b
*f*x**2*sqrt(d + e*x + f*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(72) = 144.
time = 6.49, size = 847, normalized size = 10.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*log(abs(-4*(sqrt(f)*
x - sqrt(f*x^2 + x*e + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d)
)^2*a*f^2*e - 4*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*b*d*f^(3/2)*e + 4*b*d^2
*f^2 - (sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*b*f*e^2 + 8*(sqrt(f)*x - sqrt(
f*x^2 + x*e + d))*a*f^(3/2)*e^2 + 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f
*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*f^(3/2) - 3*b*d*f*e^2
 - (sqrt(f)*x - sqrt(f*x^2 + x*e + d))*b*sqrt(f)*e^3 + 4*sqrt(-4*a*b*d*f*e
+ b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*f*
```

```
e + 2*a*f*e^3 + sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*sqrt
(f)*e^2))/(4*a*b*d*f - b^2*d*e - 4*a^2*f*e + a*b*e^2) - sqrt(-4*a*b*d*f*e +
 b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*log(abs(-4*(sqrt(f)*x - sqrt(f*x^2 + x*
e + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*a*f^2*e - 4*(sq
rt(f)*x - sqrt(f*x^2 + x*e + d))*b*d*f^(3/2)*e + 4*b*d^2*f^2 - (sqrt(f)*x -
 sqrt(f*x^2 + x*e + d))^2*b*f*e^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*a
*f^(3/2)*e^2 - 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*(sq
rt(f)*x - sqrt(f*x^2 + x*e + d))^2*f^(3/2) - 3*b*d*f*e^2 - (sqrt(f)*x - sqr
t(f*x^2 + x*e + d))*b*sqrt(f)*e^3 - 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2
*f*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*f*e + 2*a*f*e^3 - sqr
t(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*sqrt(f)*e^2))/(4*a*b*d*
f - b^2*d*e - 4*a^2*f*e + a*b*e^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f\,x^2+e\,x+d}\,\left(a+b\,x+\frac{b\,f\,x^2}{e}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)),x)
[Out] int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)), x)
```

3.3 $\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx$

Optimal. Leaf size=66

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

```
[Out] -2*arctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-
d)^(1/2)/(b^2-4*c*d)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {996, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx = -\left((2b)\text{Subst}\left(\int \frac{1}{b\,(b^2-4cd)-(ab-bd)x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)\right)$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.39, size = 146, normalized size = 2.21

$$\text{RootSum}\left[-ab^2+a^2c+b^2d+2ab\sqrt{c}\,\#1-4b\sqrt{c}\,d\#1+b^2\#1^2-2ac\#1^2+4cd\#1^2-2b\sqrt{c}\,\#1^3+c\#1^4\&, \frac{\log\left(-\sqrt{c}\,x+\sqrt{a+bx+cx^2}-\#1\right)}{a\sqrt{c}-2\sqrt{c}\,d+b\#1-\sqrt{c}\,\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

```
[Out] RootSum[-(a*b^2) + a^2*c + b^2*d + 2*a*b*Sqrt[c]*#1 - 4*b*Sqrt[c]*d*#1 + b^
2*#1^2 - 2*a*c*#1^2 + 4*c*d*#1^2 - 2*b*Sqrt[c]*#1^3 + c*#1^4 & , Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]/(a*Sqrt[c] - 2*Sqrt[c]*d + b*#1 - Sqrt
[c]*#1^2) & ]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(56) = 112.
time = 0.21, size = 307, normalized size = 4.65

method	result
default	$\frac{\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\,\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\,\sqrt{\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)^2 c-\sqrt{b^2-4cd}\,\left(x+\frac{b\cdots}{}\right.}}{x+\frac{b+\sqrt{b^2-4cd}}{2c}}\right)}{\sqrt{b^2-4cd}\,\sqrt{a-d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^
2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-
4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c
*d)^(1/2))/c))-1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2
)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1
```

```
/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/
(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c*d-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(56) = 112.
time = 2.46, size = 813, normalized size = 12.32

$$\left[\frac{\log\left(\frac{8a^2b^4+\left(b^4c^2+24ab^2c^3+16a^2c^4+128c^4d^2-32\left(b^2c^3+4ac^4\right)d\right)x^4+2\left(b^5c+24ab^3c^2+16a^2bc^3+128bc^3d^2-32\left(b^3c^2+4abc^3\right)d\right)x^3+\left(b^4+24ab^2c+16a^2c^2\right)d^2+\left(b^6+32ab^4c+48a^2b^2c^2+32\left(5b^2c^2+4ac^3\right)d^2-2\left(19b^4c+104ab^2c^2+48a^2c^3\right)d\right)x^2-4\left(2ab^3+2\left(b^2c^2+4ac^3-8c^3d\right)x^3+3\left(b^3c+4abc^2-8bc^2d\right)x^2-\left(b^3+4abc\right)d+\left(b^4+8ab^2c-2\left(5b^2c+4ac^2\right)d\right)x\right)\sqrt{ab^2+4cd^2-\left(b^2+4ac\right)d}\sqrt{cx^2+bx+a}-8\left(ab^4+4a^2b^2c\right)d+2\left(4ab^5+16a^2b^3c+16\left(b^3c+4abc^2\right)d^2-\left(3b^5+40ab^3c+48a^2bc^2\right)d\right)x}{c^2x^4+2bcx^3+2bdx+\left(b^2+2cd\right)x^2+d^2}\right)}{2\sqrt{ab^2+4cd^2-\left(b^2+4ac\right)d}},\ -\frac{\sqrt{-ab^2-4cd^2+\left(b^2+4ac\right)d}\arctan\left(-\frac{\left(2ab^2+\left(b^2c+4ac^2-8c^2d\right)x^2-\left(b^2+4ac\right)d+\left(b^3+4abc-8bcd\right)x\right)\sqrt{-ab^2-4cd^2+\left(b^2+4ac\right)d}\sqrt{cx^2+bx+a}}{2\left(a^2b^3+4abcd^2+2\left(ab^2c^2+4c^3d^2-\left(b^2c^2+4ac^3\right)d\right)x^3+3\left(ab^3c+4bc^2d^2-\left(b^3c+4abc^2\right)d\right)x^2-\left(ab^3+4a^2bc\right)d+\left(ab^4+2a^2b^2c+4\left(b^2c+2ac^2\right)d^2-\left(b^4+6ab^2c+8a^2c^2\right)d\right)x\right)}\right)}{ab^2+4cd^2-\left(b^2+4ac\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 -
32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 12
8*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*
c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^
2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*
c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3
 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 +
 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)
*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a
*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)
*x^2 + d^2))/sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d), -sqrt(-a*b^2 - 4*c*d^
2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2
 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (
b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2
 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3
*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(
b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x))/(a*b^2 + 4*c*d^
2 - (b^2 + 4*a*c)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(bx+cx^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(56) = 112.
time = 3.45, size = 703, normalized size = 10.65

$$-\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2b^2c-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2ac^2+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2c^2d-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)b^3\sqrt{c}-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)abc^{\frac{3}{2}}+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bc^{\frac{3}{2}}d-3ab^2c+4\sqrt{ab^2-b^2d-4acd+4cd^2}\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2c^{\frac{3}{2}}+4a^2c^2+2b^2cd+4\sqrt{ab^2-b^2d-4acd+4cd^2}\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bc+\sqrt{ab^2-b^2d-4acd+4cd^2}b^2\sqrt{c}\right|\right)}{\sqrt{ab^2-b^2d-4acd+4cd^2}}+\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2b^2c-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2ac^2+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2c^2d-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)b^3\sqrt{c}-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)abc^{\frac{3}{2}}+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bc^{\frac{3}{2}}d-3ab^2c-4\sqrt{ab^2-b^2d-4acd+4cd^2}\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2c^{\frac{3}{2}}+4a^2c^2+2b^2cd-4\sqrt{ab^2-b^2d-4acd+4cd^2}\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bc-\sqrt{ab^2-b^2d-4acd+4cd^2}b^2\sqrt{c}\right|\right)}{\sqrt{ab^2-b^2d-4acd+4cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d
- (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2
)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2 - b^2
*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(a*b^
2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d +
 4*c*d^2) + log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sqrt(
a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
- sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d
- 4*a*c*d + 4*c*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)), x)
```

3.4 $\int \frac{1}{\sqrt{a+bx+cx^2} \left(d+bx+cx^2\right)^2} \, dx$

Optimal. Leaf size=129

$$-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)}+\frac{\left(b^2+4c(a-2d)\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}$$

```
[Out] (b^2+4*c*(a-2*d))*arctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*
x+a)^(1/2))/(a-d)^(3/2)/(b^2-4*c*d)^(3/2)-(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-
d)/(b^2-4*c*d)/(c*x^2+b*x+d)
```

Rubi [A]
time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {988, 12, 996, 214}

$$\frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] -(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*x
^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
- 4*c*d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
```

```
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{\int -\frac{c^2(b^2+4c(a-2d))(a-d)}{2\sqrt{a+bx+cx^2}\,(d+bx+cx^2)} \cdots}{c^2(a-d)^2(b^2-4cd)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} - \frac{(b^2+4c(a-2d))\int \frac{\cdots}{\sqrt{a+bx \cdots}}}{2(a-d)(b^2- \cdots}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{(b(b^2+4c(a-2d)))\,\mathrm{Subst}\left(\int \cdots\right.}{(a \cdots}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{(b^2+4c(a-2d))\tanh^{-1}\left(\frac{\cdots}{\sqrt{\cdots}}\right.}{(a-d)^{3/2}(b^2 \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.69, size = 215, normalized size = 1.67

$$\frac{\frac{2(b+2cx)\sqrt{a+x(b+cx)}}{d+x(b+cx)}+(b^2+4c(a-2d))\,\mathrm{RootSum}\left[-ab^2+a^2c+b^2d+2ab\sqrt{c}\,\#1-4b\sqrt{c}\,d\#1+b^2\#1^2-2ac\#1^2+4cd\#1^2-2b\sqrt{c}\,\#1^3+c\#1^4\&,\frac{\log\left(-\sqrt{c}\,x+\sqrt{a+bx+cx^2}-\#1\right)}{a\sqrt{c}-2\sqrt{c}\,d+b\#1-\sqrt{c}\,\#1^2}\&\right]}{2(a-d)(-b^2+4cd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] ((2*(b + 2*c*x)*Sqrt[a + x*(b + c*x)])/(d + x*(b + c*x)) + (b^2 + 4*c*(a -
2*d))*RootSum[-(a*b^2) + a^2*c + b^2*d + 2*a*b*Sqrt[c]*#1 - 4*b*Sqrt[c]*d*#
1 + b^2*#1^2 - 2*a*c*#1^2 + 4*c*d*#1^2 - 2*b*Sqrt[c]*#1^3 + c*#1^4 & , Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]/(a*Sqrt[c] - 2*Sqrt[c]*d + b*#1
- Sqrt[c]*#1^2) & ])/(2*(a - d)*(-b^2 + 4*c*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(117) = 234$.
time = 0.17, size = 827, normalized size = 6.41

method	result
default	$-\frac{2c\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)^2 c-\sqrt{b^2-4cd}\bigl(}{x+\frac{b+\sqrt{b^2-4cd}}{2c}}\right)}{(b^2-4cd)^{\frac{3}{2}}\sqrt{a-d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+
(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b
^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-
4*c*d)^(1/2))/c))+1/(b^2-4*c*d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*
((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d+(b^2
-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+
a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))+2/(b^2-4*c*d)^(3/2)*c/(a-d)^
(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d
)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))+1/(b^2-
4*c*d)*(-1/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)*((x+1/2*(b+(b^2-4*c*d)^(1/
2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-1/2
*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2
-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4
*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*
d)^(1/2))/c)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(117) = 234$.
time = 3.59, size = 1544, normalized size = 11.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 -
 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*log((8*a^2*b
^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a
*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*
(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 +
 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c +
 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8
*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (
b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 +
 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 +
 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b
*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*
(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c +
4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c +
4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c
)*d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a
*b^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5
+ 16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c
^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x), -1/2*(sqrt(-a*b^2 - 4*c*d^2 + (b^2
 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d -
 (b^3 + 4*a*b*c - 8*b*c*d)*x)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c
^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*
c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(
a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d
^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^
2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(a
*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c + 4*
```

```
a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c + 4*
a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c)*
d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a*b
^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5 +
16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(bx+cx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. 2(117) = 234.
time = 5.01, size = 1166, normalized size = 9.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((b^2 + 4*a*c - 8*c*d)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b
^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c)
 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d
+ 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^
2*c*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*c + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt
(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) - (b^2 + 4*a*c - 8*c*d)*log(abs((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*
b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c -
 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*
c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c - sqrt(a*b^2 - b^2*d - 4*a*c
*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))/(a*b^2
 - b^2*d - 4*a*c*d + 4*c*d^2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*
sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2) - 8*(sqrt(c)*x
```

```
- sqrt(c*x^2 + b*x + a))^2*c^(3/2)*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c - 8*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*b*c*d + 3*a*b^2*sqrt(c) - 4*a^2*c^(3/2) - 2*b^2*sqrt(c)*d)/(
((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b*sqrt(c) + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2 - 2*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*sqrt(c) + 4*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b*sqrt(c)*d - a*b^2 + a^2*c + b^2*d)*(a*b^2 - b^2*d -
4*a*c*d + 4*c*d^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2), x)
```

3.5 $\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx$

Optimal. Leaf size=224

$$-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^2}+\frac{3(b^2+4c(a-2d))\,(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2\,(b^2-4cd)^2\,(d+bx+cx^2)}-\frac{(3b^4+8b^2c(a-4d)+}{}$$

```
[Out] -1/4*(3*b^4+8*b^2*c*(a-4*d)+16*c^2*(3*a^2-8*a*d+8*d^2))*arctanh((2*c*x+b)*(
a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-d)^(5/2)/(b^2-4*c*d)^(
5/2)-1/2*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d)/(c*x^2+b*x+d)^2+3/
4*(b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^2/(b^2-4*c*d)^2/(c*
x^2+b*x+d)
```

Rubi [A]
time = 0.27, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {988, 1074, 12, 996, 214}

$$-\frac{(16c^2(3a^2-8ad+8d^2)+8b^2c(a-4d)+3b^4)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\,(b^2-4cd)^{5/2}}+\frac{3(b+2cx)\,(4c(a-2d)+b^2)\,\sqrt{a+bx+cx^2}}{4(a-d)^2\,(b^2-4cd)^2\,(bx+cx^2+d)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\,(b^2-4cd)\,(bx+cx^2+d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] -1/2*((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x +
c*x^2)^2) + (3*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*
(a - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)) - ((3*b^4 + 8*b^2*c*(a - 4*d)
+ 16*c^2*(3*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b
^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(4*(a - d)^(5/2)*(b^2 - 4*c*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
```

```
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f))))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^2} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)(3b^2+12ac-16cd)-4}{\sqrt{a+bx+cx^2}}}{2c^2(a-d)^2\,(b}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))\,(b+2c}{4(a-d)^2\,(b^2-4cd)^2\,(}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))\,(b+2c}{4(a-d)^2\,(b^2-4cd)^2\,(}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))\,(b+2c}{4(a-d)^2\,(b^2-4cd)^2\,(}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))\,(b+2c}{4(a-d)^2\,(b^2-4cd)^2\,(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 486 vs. $2(224) = 448$.
time = 10.44, size = 486, normalized size = 2.17

$$\frac{-2\sqrt{a-d}\,\sqrt{b^2-4cd}\,(b+2cx)\sqrt{a+x(b+cx)}\,(2(a-d)\,(b^2-4cd)-3(b^2+4c(a-2d))\,(d+x(b+cx)))+(3b^4+8b^2c(a-4d)+16c^2(3a^2-8ad+8d^2))\,(d+x(b+cx))^2\log\left(b-\sqrt{b^2-4cd}+2cx\right)-(3b^4+8b^2c(a-4d)+16c^2(3a^2-8ad+8d^2))\,(d+x(b+cx))^2\log\left(b+\sqrt{b^2-4cd}+2cx\right)+(3b^4+8b^2c(a-4d)+16c^2(3a^2-8ad+8d^2))\,(d+x(b+cx))^2\log\left(b^2+b\sqrt{b^2-4cd}+2c\left(-2a+\sqrt{b^2-4cd}\,x-2\sqrt{a-d}\,\sqrt{a+x(b+cx)}\right)\right)-(3b^4+8b^2c(a-4d)+16c^2(3a^2-8ad+8d^2))\,(d+x(b+cx))^2\log\left(-b^2+b\sqrt{b^2-4cd}+2c\left(2a+\sqrt{b^2-4cd}\,x+2\sqrt{a-d}\,\sqrt{a+x(b+cx)}\right)\right)}{8(a-d)^{5/2}\,(b^2-4cd)^{5/2}\,(d+x(b+cx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] (-2*Sqrt[a - d]*Sqrt[b^2 - 4*c*d]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(2*(a -
 d)*(b^2 - 4*c*d) - 3*(b^2 + 4*c*(a - 2*d))*(d + x*(b + c*x))) + (3*b^4 + 8
*b^2*c*(a - 4*d) + 16*c^2*(3*a^2 - 8*a*d + 8*d^2))*(d + x*(b + c*x))^2*Log[
b - Sqrt[b^2 - 4*c*d] + 2*c*x] - (3*b^4 + 8*b^2*c*(a - 4*d) + 16*c^2*(3*a^2
 - 8*a*d + 8*d^2))*(d + x*(b + c*x))^2*Log[b + Sqrt[b^2 - 4*c*d] + 2*c*x] +
 (3*b^4 + 8*b^2*c*(a - 4*d) + 16*c^2*(3*a^2 - 8*a*d + 8*d^2))*(d + x*(b + c
*x))^2*Log[b^2 + b*Sqrt[b^2 - 4*c*d] + 2*c*(-2*a + Sqrt[b^2 - 4*c*d]*x - 2*
Sqrt[a - d]*Sqrt[a + x*(b + c*x)])] - (3*b^4 + 8*b^2*c*(a - 4*d) + 16*c^2*(
3*a^2 - 8*a*d + 8*d^2))*(d + x*(b + c*x))^2*Log[-b^2 + b*Sqrt[b^2 - 4*c*d]
+ 2*c*(2*a + Sqrt[b^2 - 4*c*d]*x + 2*Sqrt[a - d]*Sqrt[a + x*(b + c*x)]])])/(
8*(a - d)^(5/2)*(b^2 - 4*c*d)^(5/2)*(d + x*(b + c*x))^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. $2(204) = 408$.
time = 0.15, size = 1868, normalized size = 8.34

method	result	size
default	Expression too large to display	1868

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b^2-4*c*d)^(3/2)*(-1/2/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*((x-1/2*
(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c)+a-d)^(1/2)-3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2
*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln(
(2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(
(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))+1/2*c/(a-d)^(3/
2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(
1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^
2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))-1/(b^2-4*
c*d)^(3/2)*(-1/2/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*((x+1/2*(b+(b^2-4*
c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(
1/2)+3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)*
((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d
)^(1/2))/c)+a-d)^(1/2)-1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4
*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4
*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^
(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)))+1/2*c/(a-d)^(3/2)*ln((2*a-2*d-(b^2
-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2
-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d
)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)))+6*c^2/(b^2-4*c*d)^(5/2)/(a-d)^(1
/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(
1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-
4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))-3/(b^2-4*c*d)
^2*c*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/
2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/
2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^
2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2
-4*c*d)^(1/2))/c)))-6*c^2/(b^2-4*c*d)^(5/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*
c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-
4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d
)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-3/(b^2-4*c*d)^2*c*(-1/(a-d)/(x+1
/2*(b+(b^2-4*c*d)^(1/2))/c)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d
)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-1/2*(b^2-4*c*d)^(1/2)/(a
-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(
```

```
a-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b
+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(204) = 408$.
time = 6.55, size = 3818, normalized size = 17.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d^2 -
 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c + 8*
a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3
+ (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^2*c^
2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c^2 -
 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (3*b^
5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)
*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(
b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*
c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)
*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 -
2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2
+ 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4
*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c
*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d +
 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3
*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2
 + d^2)) - 4*(2*a^2*b^5 + 128*b*c^2*d^4 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a
*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c
^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 + 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3
)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3
*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 +
 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*
```

```
b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c
^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*
a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*
a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 +
 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 1
44*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b
^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b
^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12
*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*
a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*
c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 -
32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 +
 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a
*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8
*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b
*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2
*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(
a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)
*x), -1/8*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d
^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c
+ 8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*
x^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^
2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c
^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (
3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*
c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c
)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)
*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 -
(b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)
*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2
)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(2*a^2*b^5 + 128*b*c^2*d^4
 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 +
 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3
+ 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32
*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*
b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^
3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3
)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(
a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^
2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)
*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^
5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^
5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 144*a^2*b^2*...
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2986 vs. 2(204) = 408.
time = 4.94, size = 2986, normalized size = 13.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d + 128*c^2*
d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^
2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^
(3/2)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2 -
 b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(
a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c
*d + 4*c*d^2) - (3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d
+ 128*c^2*d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sq
rt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b
*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2
*d - 4*a*c*d + 4*c*d^2))/(a^2*b^4 - 2*a*b^4*d - 8*a^2*b^2*c*d + b^4*d^2 + 1
6*a*b^2*c*d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*c^2*d^3 + 16*c^2*d^4) -
 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(3/2) + 8*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(5/2) + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*a^2*c^(7/2) - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(5/2)*d
 - 128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(7/2)*d + 128*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^6*c^(7/2)*d^2 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*b^5*c + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^2 + 144*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^3 - 96*(sqrt(c)*x - sqrt(c*x^2 + b
```

```
*x + a))^5*b^3*c^2*d - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^3*d
+ 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^3*d^2 + 9*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*b^6*sqrt(c) + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a*b^4*c^(3/2) + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(5/2)
- 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(7/2) - 78*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*b^4*c^(3/2)*d - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a*b^2*c^(5/2)*d + 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(7
/2)*d + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(5/2)*d^2 - 1152*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(7/2)*d^2 + 768*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*c^(7/2)*d^3 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^
7 - 10*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c - 288*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^3*b*c^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b
^5*c*d + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^2*d + 1344*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^3*d - 256*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b^3*c^2*d^2 - 2304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*
c^3*d^2 + 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d^3 - 14*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*sqrt(c) - 71*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*a^2*b^4*c^(3/2) - 200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^
3*b^2*c^(5/2) + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(7/2) + 23*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*sqrt(c)*d + 280*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*b^4*c^(3/2)*d + 1168*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*a^2*b^2*c^(5/2)*d - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^
(7/2)*d - 272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(3/2)*d^2 - 2048*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(5/2)*d^2 + 640*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a^2*c^(7/2)*d^2 + 1152*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*b^2*c^(5/2)*d^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7 -
47*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c - 56*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^3*b^3*c^2 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*
c^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*d + 136*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b^5*c*d + 496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^
3*c^2*d - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^3*d - 80*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*b^5*c*d^2 - 896*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a*b^3*c^2*d^2 + 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^3*d^2
+ 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d^3 - 11*a^2*b^6*sqrt(c)
- 11*a^3*b^4*c^(3/2) + 72*a^4*b^2*c^(5/2) - 48*a^5*c^(7/2) + 17*a*b^6*sqrt(
c)*d + 118*a^2*b^4*c^(3/2)*d - 256*a^3*b^2*c^(5/2)*d + 96*a^4*c^(7/2)*d - 6
*b^6*sqrt(c)*d^2 - 152*a*b^4*c^(3/2)*d^2 + 160*a^2*b^2*c^(5/2)*d^2 + 48*b^4
*c^(3/2)*d^3)/((a^2*b^4 - 2*a*b^4*d - 8*a^2*b^2*c*d + b^4*d^2 + 16*a*b^2*c*
d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*c^2*d^3 + 16*c^2*d^4)*((sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^4*c + 2*(sqrt(c)*x - ...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3), x)
```

3.6 $\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx$

Optimal. Leaf size=328

$$-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3}+\frac{5(b^2+4c(a-2d))\,(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2\,(b^2-4cd)^2\,(d+bx+cx^2)^2}-\frac{(15b^4+8b^2c(7a-22d)}{24(}$$

```
[Out] 1/8*(b^2+4*c*(a-2*d))*(5*b^4-8*b^2*c*(a+4*d)+16*c^2*(5*a^2-8*a*d+8*d^2))*ar
ctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-d)^(7
/2)/(b^2-4*c*d)^(7/2)-1/3*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d)/(
c*x^2+b*x+d)^3+5/12*(b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^2
/(b^2-4*c*d)^2/(c*x^2+b*x+d)^2-1/24*(15*b^4+8*b^2*c*(7*a-22*d)+16*c^2*(15*a
^2-44*a*d+44*d^2))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^3/(b^2-4*c*d)^3/(c*x
^2+b*x+d)
```

Rubi [A]
time = 0.60, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {988, 1074, 12, 996, 214}

$$-\frac{(b+2cx)\,(16c^2(15a^2-44ad+44d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3\,(b^2-4cd)^3\,(bx+cx^2+d)}+\frac{(4c(a-2d)+b^2)\,(16c^2(5a^2-8ad+8d^2)-8b^2c(a+4d)+5b^4)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{8(a-d)^{7/2}\,(b^2-4cd)^{7/2}}+\frac{5(b+2cx)\,(4c(a-2d)+b^2)\sqrt{a+bx+cx^2}}{12(a-d)^2\,(b^2-4cd)^2\,(bx+cx^2+d)^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(bx+cx^2+d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] -1/3*((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x +
c*x^2)^3) + (5*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(12
*(a - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)^2) - ((15*b^4 + 8*b^2*c*(7*a -
 22*d) + 16*c^2*(15*a^2 - 44*a*d + 44*d^2))*(b + 2*c*x)*Sqrt[a + b*x + c*x^
2])/(24*(a - d)^3*(b^2 - 4*c*d)^3*(d + b*x + c*x^2)) + ((b^2 + 4*c*(a - 2*d
))*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2*(5*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sq
rt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(8*(a -
d)^(7/2)*(b^2 - 4*c*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
```

```
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)(5b^2+20ac-24cd)-8bc}{\sqrt{a+bx+cx^2}\,(d}}{3c^2(a-d)^2\,(b^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))\,(b+2cx}{12(a-d)^2\,(b^2-4cd)^2\,(d}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))\,(b+2cx}{12(a-d)^2\,(b^2-4cd)^2\,(d}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))\,(b+2cx}{12(a-d)^2\,(b^2-4cd)^2\,(d}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))\,(b+2cx}{12(a-d)^2\,(b^2-4cd)^2\,(d}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))\,(b+2cx}{12(a-d)^2\,(b^2-4cd)^2\,(d}$$

Mathematica [A]
time = 11.82, size = 604, normalized size = 1.84

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] (-2*Sqrt[a - d]*Sqrt[b^2 - 4*c*d]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(8*(a -
 d)^2*(b^2 - 4*c*d)^2 - 10*(b^2 + 4*c*(a - 2*d))*(a - d)*(b^2 - 4*c*d)*(d +
 x*(b + c*x)) + (15*b^4 + 8*b^2*c*(7*a - 22*d) + 16*c^2*(15*a^2 - 44*a*d +
44*d^2))*(d + x*(b + c*x))^2) - 3*(b^2 + 4*c*(a - 2*d))*(5*b^4 - 8*b^2*c*(a
 + 4*d) + 16*c^2*(5*a^2 - 8*a*d + 8*d^2))*(d + x*(b + c*x))^3*Log[b - Sqrt[
b^2 - 4*c*d] + 2*c*x] + 3*(b^2 + 4*c*(a - 2*d))*(5*b^4 - 8*b^2*c*(a + 4*d)
+ 16*c^2*(5*a^2 - 8*a*d + 8*d^2))*(d + x*(b + c*x))^3*Log[b + Sqrt[b^2 - 4*
c*d] + 2*c*x] - 3*(b^2 + 4*c*(a - 2*d))*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2
```

```
*(5*a^2 - 8*a*d + 8*d^2))*(d + x*(b + c*x))^3*Log[b^2 + b*Sqrt[b^2 - 4*c*d]
 + 2*c*(-2*a + Sqrt[b^2 - 4*c*d]*x - 2*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])]
+ 3*(b^2 + 4*c*(a - 2*d))*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2*(5*a^2 - 8*a*
d + 8*d^2))*(d + x*(b + c*x))^3*Log[-b^2 + b*Sqrt[b^2 - 4*c*d] + 2*c*(2*a +
 Sqrt[b^2 - 4*c*d]*x + 2*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/(48*(a - d)^(
7/2)*(b^2 - 4*c*d)^(7/2)*(d + x*(b + c*x))^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3648 vs. $2(304) = 608$.
time = 0.17, size = 3649, normalized size = 11.12

method	result	size
default	Expression too large to display	3649

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(b^2-4*c*d)^(5/2)*c*(-1/2/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*((x+1/2
*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2)
)/c)+a-d)^(1/2)+3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/(a-d)/(x+1/2*(b+(b^2-4*c*d)
^(1/2))/c)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b
+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a
-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/
2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2
))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)))+1/2*c/(a-d)^(3/2)*ln((2
*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+
1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1
/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)))+1/(b^2-4*c*d)^2*(-1/3
/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^3*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)
^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-5/6*(b^2
-4*c*d)^(1/2)/(a-d)*(-1/2/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*((x-1/2*
(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c)+a-d)^(1/2)-3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2
*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln(
(2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(
(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))+1/2*c/(a-d)^(3/
2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(
1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^
2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))-2/3*c/(a-
d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2)
)/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*
(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2
-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-
```

```
4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4
*c*d)^(1/2))/c)))-20*c^3/(b^2-4*c*d)^(7/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*
c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4*
c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(
1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))+20*c^3/(b^2-4*c*d)^(7/2)/(a-d)^(1/2)
*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/
2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-
4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))+10*c^2/(b^2-
4*c*d)^3*(-1/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)*((x+1/2*(b+(b^2-4*c*d)^(
1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-1
/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b
^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2
-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*
c*d)^(1/2))/c)))+10*c^2/(b^2-4*c*d)^3*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*
d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*
(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))+1/(b^2-4*c*d)^2*(-1/3/
(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^3*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*
c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+5/6*(b^2-4*c
*d)^(1/2)/(a-d)*(-1/2/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*((x+1/2*(b+(b
^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a
-d)^(1/2)+3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2)
)/c)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-
4*c*d)^(1/2))/c)+a-d)^(1/2)-1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(
b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(
b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+
a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)))+1/2*c/(a-d)^(3/2)*ln((2*a-2*d
-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b
+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c
)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)))-2/3*c/(a-d)*(-1/(a-d)/(x+1/
2*(b+(b^2-4*c*d)^(1/2))/c)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)
^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-1/2*(b^2-4*c*d)^(1/2)/(a-
d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a
-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+
(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))))-4/(b^2
-4*c*d)^(5/2)*c*(-1/2/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*((x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-...
```

<u>Maxima</u> **[F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3962 vs. $2(304) = 608$.
time = 28.66, size = 8134, normalized size = 24.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(1024*c^3*d^6 - (5*b^6*c^3 + 12*a*b^4*c^4 + 48*a^2*b^2*c^5 + 320*a
^3*c^6 - 1024*c^6*d^3 + 384*(b^2*c^5 + 4*a*c^6)*d^2 - 24*(3*b^4*c^4 + 8*a*b
^2*c^5 + 48*a^2*c^6)*d)*x^6 - 384*(b^2*c^2 + 4*a*c^3)*d^5 - 3*(5*b^7*c^2 +
12*a*b^5*c^3 + 48*a^2*b^3*c^4 + 320*a^3*b*c^5 - 1024*b*c^5*d^3 + 384*(b^3*c
^4 + 4*a*b*c^5)*d^2 - 24*(3*b^5*c^3 + 8*a*b^3*c^4 + 48*a^2*b*c^5)*d)*x^5 +
24*(3*b^4*c + 8*a*b^2*c^2 + 48*a^2*c^3)*d^4 - 3*(5*b^8*c + 12*a*b^6*c^2 + 4
8*a^2*b^4*c^3 + 320*a^3*b^2*c^4 - 1024*c^5*d^4 - 128*(5*b^2*c^4 - 12*a*c^5)
*d^3 + 24*(13*b^4*c^3 + 56*a*b^2*c^4 - 48*a^2*c^5)*d^2 - (67*b^6*c^2 + 180*
a*b^4*c^3 + 1104*a^2*b^2*c^4 - 320*a^3*c^5)*d)*x^4 - (5*b^6 + 12*a*b^4*c +
48*a^2*b^2*c^2 + 320*a^3*c^3)*d^3 - (5*b^9 + 12*a*b^7*c + 48*a^2*b^5*c^2 +
320*a^3*b^3*c^3 - 6144*b*c^4*d^4 + 256*(5*b^3*c^3 + 36*a*b*c^4)*d^3 - 48*(b
^5*c^2 - 8*a*b^3*c^3 + 144*a^2*b*c^4)*d^2 - 6*(7*b^7*c + 20*a*b^5*c^2 + 144
*a^2*b^3*c^3 - 320*a^3*b*c^4)*d)*x^3 + 3*(1024*c^4*d^5 + 128*(5*b^2*c^3 - 1
2*a*c^4)*d^4 - 24*(13*b^4*c^2 + 56*a*b^2*c^3 - 48*a^2*c^4)*d^3 + (67*b^6*c
+ 180*a*b^4*c^2 + 1104*a^2*b^2*c^3 - 320*a^3*c^4)*d^2 - (5*b^8 + 12*a*b^6*c
 + 48*a^2*b^4*c^2 + 320*a^3*b^2*c^3)*d)*x^2 + 3*(1024*b*c^3*d^5 - 384*(b^3*
c^2 + 4*a*b*c^3)*d^4 + 24*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3)*d^3 - (5*b
^7 + 12*a*b^5*c + 48*a^2*b^3*c^2 + 320*a^3*b*c^3)*d^2)*x)*sqrt(a*b^2 + 4*c*
d^2 - (b^2 + 4*a*c)*d)*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^
4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 +
 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24
*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2
*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*
(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b
*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*
d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*
b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d
^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d
*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*(8*a^3*b^7 + 4608*b*c^3*d^6 - 2592*(b^3*
c^2 + 4*a*b*c^3)*d^5 + 2*(15*a*b^6*c^3 + 56*a^2*b^4*c^4 + 240*a^3*b^2*c^5 +
 2816*c^6*d^4 - 1408*(b^2*c^5 + 4*a*c^6)*d^3 + 4*(59*b^4*c^4 + 584*a*b^2*c^
5 + 944*a^2*c^6)*d^2 - (15*b^6*c^3 + 292*a*b^4*c^4 + 1168*a^2*b^2*c^5 + 960
*a^3*c^6)*d)*x^5 + 4*(123*b^5*c + 1352*a*b^3*c^2 + 1968*a^2*b*c^3)*d^4 + 5*
```

```
(15*a*b^7*c^2 + 56*a^2*b^5*c^3 + 240*a^3*b^3*c^4 + 2816*b*c^5*d^4 - 1408*(b
^3*c^4 + 4*a*b*c^5)*d^3 + 4*(59*b^5*c^3 + 584*a*b^3*c^4 + 944*a^2*b*c^5)*d^
2 - (15*b^7*c^2 + 292*a*b^5*c^3 + 1168*a^2*b^3*c^4 + 960*a^3*b*c^5)*d)*x^4
- (33*b^7 + 940*a*b^5*c + 3760*a^2*b^3*c^2 + 2112*a^3*b*c^3)*d^3 + 4*(15*a*
b^8*c + 51*a^2*b^6*c^2 + 220*a^3*b^4*c^3 + 3456*c^5*d^5 + 16*(63*b^2*c^4 -
452*a*c^5)*d^4 - 4*(273*b^4*c^3 + 584*a*b^2*c^4 - 1264*a^2*c^5)*d^3 + 8*(27
*b^6*c^2 + 233*a*b^4*c^3 + 236*a^2*b^2*c^4 - 160*a^3*c^5)*d^2 - (15*b^8*c +
 267*a*b^6*c^2 + 992*a^2*b^4*c^3 + 560*a^3*b^2*c^4)*d)*x^3 + (59*a*b^7 + 58
4*a^2*b^5*c + 944*a^3*b^3*c^2)*d^2 + (15*a*b^9 + 26*a^2*b^7*c + 120*a^3*b^5
*c^2 + 20736*b*c^4*d^5 - 32*(251*b^3*c^3 + 1356*a*b*c^4)*d^4 + 8*(61*b^5*c^
2 + 1768*a*b^3*c^3 + 3792*a^2*b*c^4)*d^3 + 4*(29*b^7*c - 124*a*b^5*c^2 - 18
88*a^2*b^3*c^3 - 1920*a^3*b*c^4)*d^2 - (15*b^9 + 142*a*b^7*c + 112*a^2*b^5*
c^2 - 1440*a^3*b^3*c^3)*d)*x^2 - 34*(a^2*b^7 + 4*a^3*b^5*c)*d - 2*(5*a^2*b^
8 + 12*a^3*b^6*c - 4608*c^4*d^6 - 864*(b^2*c^3 - 12*a*c^4)*d^5 + 4*(329*b^4
*c^2 + 456*a*b^2*c^3 - 1968*a^2*c^4)*d^4 - (283*b^6*c + 2356*a*b^4*c^2 + 12
96*a^2*b^2*c^3 - 2112*a^3*c^4)*d^3 + (20*b^8 + 413*a*b^6*c + 1304*a^2*b^4*c
^2 + 336*a^3*b^2*c^3)*d^2 - (25*a*b^8 + 142*a^2*b^6*c + 264*a^3*b^4*c^2)*d)
*x)*sqrt(c*x^2 + b*x + a))/(a^4*b^8*d^3 + 256*c^4*d^11 - 256*(b^2*c^3 + 4*a
*c^4)*d^10 + 32*(3*b^4*c^2 + 32*a*b^2*c^3 + 48*a^2*c^4)*d^9 - 16*(b^6*c + 2
4*a*b^4*c^2 + 96*a^2*b^2*c^3 + 64*a^3*c^4)*d^8 + (b^8 + 64*a*b^6*c + 576*a^
2*b^4*c^2 + 1024*a^3*b^2*c^3 + 256*a^4*c^4)*d^7 - 4*(a*b^8 + 24*a^2*b^6*c +
 96*a^3*b^4*c^2 + 64*a^4*b^2*c^3)*d^6 + (a^4*b^8*c^3 + 256*c^7*d^8 - 256*(b
^2*c^6 + 4*a*c^7)*d^7 + 32*(3*b^4*c^5 + 32*a*b^2*c^6 + 48*a^2*c^7)*d^6 - 16
*(b^6*c^4 + 24*a*b^4*c^5 + 96*a^2*b^2*c^6 + 64*a^3*c^7)*d^5 + (b^8*c^3 + 64
*a*b^6*c^4 + 576*a^2*b^4*c^5 + 1024*a^3*b^2*c^6 + 256*a^4*c^7)*d^4 - 4*(a*b
^8*c^3 + 24*a^2*b^6*c^4 + 96*a^3*b^4*c^5 + 64*a^4*b^2*c^6)*d^3 + 2*(3*a^2*b
^8*c^3 + 32*a^3*b^6*c^4 + 48*a^4*b^4*c^5)*d^2 - 4*(a^3*b^8*c^3 + 4*a^4*b^6*
c^4)*d)*x^6 + 2*(3*a^2*b^8 + 32*a^3*b^6*c + 48*a^4*b^4*c^2)*d^5 + 3*(a^4*b^
9*c^2 + 256*b*c^6*d^8 - 256*(b^3*c^5 + 4*a*b*c^6)*d^7 + 32*(3*b^5*c^4 + 32*
a*b^3*c^5 + 48*a^2*b*c^6)*d^6 - 16*(b^7*c^3 + 24*a*b^5*c^4 + 96*a^2*b^3*c^5
 + 64*a^3*b*c^6)*d^5 + (b^9*c^2 + 64*a*b^7*c^3 + 576*a^2*b^5*c^4 + 1024*a^3
*b^3*c^5 + 256*a^4*b*c^6)*d^4 - 4*(a*b^9*c^2 + 24*a^2*b^7*c^3 + 96*a^3*b^5*
c^4 + 64*a^4*b^3*c^5)*d^3 + 2*(3*a^2*b^9*c^2 + 32*a^3*b^7*c^3 + 48*a^4*b^5*
c^4)*d^2 - 4*(a^3*b^9*c^2 + 4*a^4*b^7*c^3)*d)*x...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 30280 vs. $2(304) = 608$.

time = 10.03, size = 30280, normalized size = 92.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*((5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a^3*c^3 - 72*b^4*c*d - 19
2*a*b^2*c^2*d - 1152*a^2*c^3*d + 384*b^2*c^2*d^2 + 1536*a*c^3*d^2 - 1024*c^
3*d^3)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c
^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
^(3/2)*d + 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d + 4*sqrt(a*b^2
- b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt
(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*
c*d + 4*c*d^2) - (5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a^3*c^3 - 72*b^
4*c*d - 192*a*b^2*c^2*d - 1152*a^2*c^3*d + 384*b^2*c^2*d^2 + 1536*a*c^3*d^2
 - 1024*c^3*d^3)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b*c^(3/2)*d + 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d - 4*s
qrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^
2*d - 4*a*c*d + 4*c*d^2))/(a^3*b^6 - 3*a^2*b^6*d - 12*a^3*b^4*c*d + 3*a*b^6
*d^2 + 36*a^2*b^4*c*d^2 + 48*a^3*b^2*c^2*d^2 - b^6*d^3 - 36*a*b^4*c*d^3 - 1
44*a^2*b^2*c^2*d^3 - 64*a^3*c^3*d^3 + 12*b^4*c*d^4 + 144*a*b^2*c^2*d^4 + 19
2*a^2*c^3*d^4 - 48*b^2*c^2*d^5 - 192*a*c^3*d^5 + 64*c^3*d^6) + 1/24*(15*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^11*b^6*c^(5/2) + 36*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^10*a^12*b^4*c^(7/2) + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^10*a^13*b^2*c^(9/2) + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^14
*c^(11/2) - 165*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^10*b^6*c^(5/2)*d -
 612*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^11*b^4*c^(7/2)*d - 2160*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^10*a^12*b^2*c^(9/2)*d - 14016*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^10*a^13*c^(11/2)*d + 825*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^10*a^9*b^6*c^(5/2)*d^2 + 4356*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^1
0*a^10*b^4*c^(7/2)*d^2 + 15408*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^11*
b^2*c^(9/2)*d^2 + 95424*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^12*c^(11/2
)*d^2 - 2475*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^8*b^6*c^(5/2)*d^3 - 1
7820*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^9*b^4*c^(7/2)*d^3 - 68112*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^10*b^2*c^(9/2)*d^3 - 402240*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^10*a^11*c^(11/2)*d^3 + 4950*(sqrt(c)*x - sqrt(c*
```

```
x^2 + b*x + a))^10*a^7*b^6*c^(5/2)*d^4 + 47520*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^10*a^8*b^4*c^(7/2)*d^4 + 205920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^10*a^9*b^2*c^(9/2)*d^4 + 1174272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^
10*c^(11/2)*d^4 - 6930*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^6*b^6*c^(5/
2)*d^5 - 87912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^7*b^4*c^(7/2)*d^5 -
 446688*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^8*b^2*c^(9/2)*d^5 - 251328
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^9*c^(11/2)*d^5 + 6930*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^10*a^5*b^6*c^(5/2)*d^6 + 116424*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^10*a^6*b^4*c^(7/2)*d^6 + 712800*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^10*a^7*b^2*c^(9/2)*d^6 + 4067712*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^10*a^8*c^(11/2)*d^6 - 4950*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4
*b^6*c^(5/2)*d^7 - 111672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^5*b^4*c^
(7/2)*d^7 - 845856*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^6*b^2*c^(9/2)*d
^7 - 5056128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^7*c^(11/2)*d^7 + 2475
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^6*c^(5/2)*d^8 + 77220*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^10*a^4*b^4*c^(7/2)*d^8 + 746064*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^10*a^5*b^2*c^(9/2)*d^8 + 4847040*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^10*a^6*c^(11/2)*d^8 - 825*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^10*a^2*b^6*c^(5/2)*d^9 - 37620*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^
3*b^4*c^(7/2)*d^9 - 483120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^4*b^2*c
^(9/2)*d^9 - 3562944*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^5*c^(11/2)*d^
9 + 165*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^6*c^(5/2)*d^10 + 12276*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^4*c^(7/2)*d^10 + 223344*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^10*a^3*b^2*c^(9/2)*d^10 + 1974720*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^10*a^4*c^(11/2)*d^10 - 15*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^10*b^6*c^(5/2)*d^11 - 2412*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^1
0*a*b^4*c^(7/2)*d^11 - 69840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^2
*c^(9/2)*d^11 - 799296*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*c^(11/2)*
d^11 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4), x)
```

3.7 $\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx$

Optimal. Leaf size=162

$$-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)}-\frac{(8aef-b(e^2+4df))\tanh^{-1}\left(\frac{\sqrt{bd-ae}\,(e+2fx)}{\sqrt{e}\,\sqrt{be-4af}\,\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}$$

```
[Out] -(8*a*e*f-b*(4*d*f+e^2))*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)/e^(1/2)/(-4*a*f
+b*e)^(1/2)/(f*x^2+e*x+d)^(1/2))/e^(3/2)/(-a*e+b*d)^(3/2)/(-4*a*f+b*e)^(3/2
)-b*(2*f*x+e)*(f*x^2+e*x+d)^(1/2)/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e*x+
a*e)
```

Rubi [A]
time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {988, 12, 996, 214}

$$-\frac{(8aef-b(4df+e^2))\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\,\sqrt{be-4af}\,\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -((b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e +
 b*e*x + b*f*x^2))) - ((8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]
*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(e^(3/2)*
(b*d - a*e)^(3/2)*(b*e - 4*a*f)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
```

```
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx &= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)} + \frac{\int \frac{b(bd-ae)f^2(8aef \cdots}{2\sqrt{d+ex+fx^2 \cdots}}}{be(bd-ae)^2 f \cdots}\\ &= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)} + \frac{(8aef-b(e^2+4d \cdots}{2e \cdots}\\ &= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)} - \frac{(8aef-b(e^2+4d \cdots}{\cdots}\\ &= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)} - \frac{(8aef-b(e^2+4d \cdots}{\cdots}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.01, size = 241, normalized size = 1.49

$$\frac{\frac{2b(e+2fx)\sqrt{d+x(e+fx)}}{ae+bx(e+fx)}+(-8aef+b(e^2+4df))\,\text{RootSum}\left[-bde^2+ae^3+bd^2f+2bde\sqrt{f}\,\#1-4ae^2\sqrt{f}\,\#1+be^2\#1^2-2bdf\#1^2+4aef\#1^2-2be\sqrt{f}\,\#1^3+bf\#1^4\&,\frac{\log\left(-\sqrt{f}\,x+\sqrt{d+ex+fx^2}-\#1\right)}{bd\sqrt{f}-2ae\sqrt{f}+be\#1-b\sqrt{f}\,\#1^2}\&\right]}{2e(-bd+ae)(be-4af)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] ((2*b*(e + 2*f*x)*Sqrt[d + x*(e + f*x)])/(a*e + b*x*(e + f*x)) + (-8*a*e*f
+ b*(e^2 + 4*d*f))*RootSum[-(b*d*e^2) + a*e^3 + b*d^2*f + 2*b*d*e*Sqrt[f]*#
1 - 4*a*e^2*Sqrt[f]*#1 + b*e^2*#1^2 - 2*b*d*f*#1^2 + 4*a*e*f*#1^2 - 2*b*e*S
qrt[f]*#1^3 + b*f*#1^4 & , Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]/(
b*d*Sqrt[f] - 2*a*e*Sqrt[f] + b*e*#1 - b*Sqrt[f]*#1^2) & ])/(2*e*(-(b*d) +
a*e)*(b*e - 4*a*f))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. $2(146) = 292$.
time = 0.15, size = 1352, normalized size = 8.35

method	result
default	$\frac{2f\ln\left(\frac{-\frac{2(ae-bd)}{b}-\frac{\sqrt{-eb(4fa-eb)}\left(x+\frac{eb+\sqrt{-eb(4fa-eb)}}{2bf}\right)}{b}+2\sqrt{-\frac{ae-bd}{b}}\sqrt{\left(x+\frac{eb+\sqrt{-eb(4fa}{2bf}\right.}}{x+\frac{eb+\sqrt{-eb(4fa-eb)}}{2bf}}\right.}{e(4fa-eb)\sqrt{-eb(4fa-eb)}\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e/(4*a*f-b*e)*f/(-e*b*(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*
(a*e-b*d)-(-e*b*(4*a*f-b*e))^(1/2)/b*(x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/
b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)^2
*f-(-e*b*(4*a*f-b*e))^(1/2)/b*(x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)-1/
b*(a*e-b*d))^(1/2))/(x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f))-1/e/(4*a*f-
b*e)/b*(b/(a*e-b*d)/(x-1/2*(-e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)*((x-1/2*(-e
*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-e*b*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-e
*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)-1/2*(-e*b*(4*a*f-b*e
))^(1/2)/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-e*b*(4*a*f-b
*e))^(1/2)/b*(x-1/2*(-e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))
^(1/2)*((x-1/2*(-e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-e*b*(4*a*f-b*e))^
(1/2)/b*(x-1/2*(-e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(
x-1/2*(-e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)))-1/e/(4*a*f-b*e)/b*(b/(a*e-b*d)
/(x+1/2*(e*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)*((x+1/2*(e*b+(-e*b*(4*a*f-b*e))
^(1/2))/b/f)^2*f-(-e*b*(4*a*f-b*e))^(1/2)/b*(x+1/2*(e*b+(-e*b*(4*a*f-b*e))^
```

```
(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)+1/2*(-e*b*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-1
/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-e*b*(4*a*f-b*e))^(1/2)/b*(x+1/2*(e
*b+(-e*b*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(e*b+(-e
*b*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-e*b*(4*a*f-b*e))^(1/2)/b*(x+1/2*(e*b+(-e*
b*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(e*b+(-e*b*(4*a*f-b
*e))^(1/2))/b/f)))-2/e/(4*a*f-b*e)*f/(-e*b*(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*
d))^(1/2)*ln((-2/b*(a*e-b*d)+(-e*b*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-e*b+(-e*b*
(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-e*b+(-e*b*(4*a*
f-b*e))^(1/2))/b/f)^2*f+(-e*b*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-e*b+(-e*b*(4*a*
f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-e*b+(-e*b*(4*a*f-b*e))^(
1/2))/b/f))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*f*x^2 + b*x*e + a*e)^2*sqrt(f*x^2 + x*e + d)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(155) = 310.
time = 4.54, size = 1919, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*((4*b^2*d*f^2*x^2 + (b^2*x + a*b)*e^3 + (b^2*f*x^2 - 8*a*b*f*x - 8*a^
2*f)*e^2 - 4*(2*a*b*f^2*x^2 - b^2*d*f*x - a*b*d*f)*e)*sqrt(-4*a*b*d*f*e - a
*b*e^3 + (b^2*d + 4*a^2*f)*e^2)*log((16*b^2*d^2*f^4*x^4 - 4*(8*b*d*f^3*x^3
+ (b*x - a)*e^4 + (3*b*f*x^2 - 10*a*f*x + 2*b*d)*e^3 + 2*(b*f^2*x^3 - 12*a*
f^2*x^2 + 4*b*d*f*x - 2*a*d*f)*e^2 - 4*(4*a*f^3*x^3 - 3*b*d*f^2*x^2 + 2*a*d
*f^2*x)*e)*sqrt(-4*a*b*d*f*e - a*b*e^3 + (b^2*d + 4*a^2*f)*e^2)*sqrt(f*x^2
+ x*e + d) + (b^2*x^2 - 6*a*b*x + a^2)*e^6 + 2*(b^2*f*x^3 - 19*a*b*f*x^2 -
4*a*b*d + 4*(b^2*d + 4*a^2*f)*x)*e^5 + (b^2*f^2*x^4 - 64*a*b*f^2*x^3 - 80*a
*b*d*f*x + 8*b^2*d^2 + 24*a^2*d*f + 32*(b^2*d*f + 5*a^2*f^2)*x^2)*e^4 - 16*
(2*a*b*f^3*x^4 + 13*a*b*d*f^2*x^2 + 2*a*b*d^2*f - (3*b^2*d*f^2 + 16*a^2*f^3
)*x^3 - 2*(b^2*d^2*f + 4*a^2*d*f^2)*x)*e^3 - 8*(32*a*b*d*f^3*x^3 + 12*a*b*d
^2*f^2*x - 2*a^2*d^2*f^2 - (3*b^2*d*f^3 + 16*a^2*f^4)*x^4 - 2*(3*b^2*d^2*f^
2 + 8*a^2*d*f^3)*x^2)*e^2 - 32*(4*a*b*d*f^4*x^4 - b^2*d^2*f^3*x^3 + 3*a*b*d
```

```
^2*f^3*x^2)*e)/(b^2*f^2*x^4 + (b^2*x^2 + 2*a*b*x + a^2)*e^2 + 2*(b^2*f*x^3
+ a*b*f*x^2)*e)) - 4*(8*a*b^2*d*f^2*x*e + a*b^2*e^4 + (2*a*b^2*f*x - b^3*d
- 4*a^2*b*f)*e^3 + 2*(2*a*b^2*d*f - (b^3*d*f + 4*a^2*b*f^2)*x)*e^2)*sqrt(f*
x^2 + x*e + d))/(16*a^2*b^3*d^2*f^3*x^2*e^2 + (a^2*b^3*x + a^3*b^2)*e^7 + (
a^2*b^3*f*x^2 - 2*a^2*b^3*d - 8*a^4*b*f - 2*(a*b^4*d + 4*a^3*b^2*f)*x)*e^6
+ (a*b^4*d^2 + 16*a^3*b^2*d*f + 16*a^5*f^2 - 2*(a*b^4*d*f + 4*a^3*b^2*f^2)*
x^2 + (b^5*d^2 + 16*a^2*b^3*d*f + 16*a^4*b*f^2)*x)*e^5 - (8*a^2*b^3*d^2*f +
 32*a^4*b*d*f^2 - (b^5*d^2*f + 16*a^2*b^3*d*f^2 + 16*a^4*b*f^3)*x^2 + 8*(a*
b^4*d^2*f + 4*a^3*b^2*d*f^2)*x)*e^4 + 8*(2*a^2*b^3*d^2*f^2*x + 2*a^3*b^2*d^
2*f^2 - (a*b^4*d^2*f^2 + 4*a^3*b^2*d*f^3)*x^2)*e^3), 1/2*((4*b^2*d*f^2*x^2
+ (b^2*x + a*b)*e^3 + (b^2*f*x^2 - 8*a*b*f*x - 8*a^2*f)*e^2 - 4*(2*a*b*f^2*
x^2 - b^2*d*f*x - a*b*d*f)*e)*sqrt(4*a*b*d*f*e + a*b*e^3 - (b^2*d + 4*a^2*f
)*e^2)*arctan(1/2*(4*b*d*f^2*x^2 + (b*x - a)*e^3 + (b*f*x^2 - 8*a*f*x + 2*b
*d)*e^2 - 4*(2*a*f^2*x^2 - b*d*f*x + a*d*f)*e)*sqrt(4*a*b*d*f*e + a*b*e^3 -
 (b^2*d + 4*a^2*f)*e^2)*sqrt(f*x^2 + x*e + d)/(a*b*x*e^5 + (3*a*b*f*x^2 + a
*b*d - (b^2*d + 4*a^2*f)*x)*e^4 + (2*a*b*f^2*x^3 + 6*a*b*d*f*x - b^2*d^2 -
4*a^2*d*f - 3*(b^2*d*f + 4*a^2*f^2)*x^2)*e^3 + 2*(6*a*b*d*f^2*x^2 + 2*a*b*d
^2*f - (b^2*d*f^2 + 4*a^2*f^3)*x^3 - (b^2*d^2*f + 4*a^2*d*f^2)*x)*e^2 + 8*(
a*b*d*f^3*x^3 + a*b*d^2*f^2*x)*e)) + 2*(8*a*b^2*d*f^2*x*e + a*b^2*e^4 + (2*
a*b^2*f*x - b^3*d - 4*a^2*b*f)*e^3 + 2*(2*a*b^2*d*f - (b^3*d*f + 4*a^2*b*f^
2)*x)*e^2)*sqrt(f*x^2 + x*e + d))/(16*a^2*b^3*d^2*f^3*x^2*e^2 + (a^2*b^3*x
+ a^3*b^2)*e^7 + (a^2*b^3*f*x^2 - 2*a^2*b^3*d - 8*a^4*b*f - 2*(a*b^4*d + 4*
a^3*b^2*f)*x)*e^6 + (a*b^4*d^2 + 16*a^3*b^2*d*f + 16*a^5*f^2 - 2*(a*b^4*d*f
 + 4*a^3*b^2*f^2)*x^2 + (b^5*d^2 + 16*a^2*b^3*d*f + 16*a^4*b*f^2)*x)*e^5 -
(8*a^2*b^3*d^2*f + 32*a^4*b*d*f^2 - (b^5*d^2*f + 16*a^2*b^3*d*f^2 + 16*a^4*
b*f^3)*x^2 + 8*(a*b^4*d^2*f + 4*a^3*b^2*d*f^2)*x)*e^4 + 8*(2*a^2*b^3*d^2*f^
2*x + 2*a^3*b^2*d^2*f^2 - (a*b^4*d^2*f^2 + 4*a^3*b^2*d*f^3)*x^2)*e^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: TypeError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[2]%%%},[8,2,0,0,0]%%%}+%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,
%%%{-1
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\,f\,x^2+b\,e\,x+a\,e)^2\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

3.8 $\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {996, 210}

$$\frac{\text{ArcTan}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\left(4\text{Subst}\left(\int \frac{1}{-24-2x^2}\,dx, x, \frac{2+2x}{\sqrt{5+2x+x^2}}\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 39, normalized size = 1.39

$$-\frac{\tan^{-1}\left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
[Out] -(ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]]/Sqrt[3])
```

Maple [A]
time = 0.28, size = 27, normalized size = 0.96

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^2+2x+5}}\right)}{3}$
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+3\right)x^2+3\sqrt{x^2+2x+5}\,x+2\,\mathrm{RootOf}\left(\_Z^2+3\right)x+3\sqrt{x^2+2x+5}+7\,\mathrm{RootOf}\left(\_Z^2+3\right)}{x^2+2x+4}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

Fricas [A]
time = 1.56, size = 38, normalized size = 1.36

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^2+2\,x+5}\,(x+1)-\frac{1}{3}\sqrt{3}\,\left(x^2+2\,x+4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)*(x + 1) - 1/3*sqrt(3)*(x
^2 + 2*x + 4))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.
time = 6.76, size = 52, normalized size = 1.86

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}+2\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + 1/3*sqrt(
3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(x^2+2\,x+4)\,\sqrt{x^2+2\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

```
[Out] int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

3.9 $\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx$

Optimal. Leaf size=136

$$-\frac{2^{1+q}\left(-\frac{e-\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-1-p-q}(2a+ex+2cx^2)^{1+p+q}\,{}_2F_1\left(-p-q,1+p+q;2+p+q;\frac{e+\sqrt{-16ac+e^2}+4cx}{2\sqrt{-16ac+e^2}}\right)}{\sqrt{-16ac+e^2}\,(1+p+q)}$$

```
[Out] -2^(1+q)*(2*c*x^2+e*x+2*a)^(1+p+q)*hypergeom([-p-q, 1+p+q],[2+p+q],1/2*(e+4
*c*x+(-16*a*c+e^2)^(1/2))/(-16*a*c+e^2)^(1/2))*((-e-4*c*x+(-16*a*c+e^2)^(1/
2))/(-16*a*c+e^2)^(1/2))^(-1-p-q)/(1+p+q)/(-16*a*c+e^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {981, 638}

$$-\frac{2^{q+1}\left(-\frac{-\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q-1}(2a+2cx^2+ex)^{p+q+1}\,{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}}\right)}{(p+q+1)\sqrt{e^2-16ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

```
[Out] -((2^(1 + q)*(-((e - Sqrt[-16*a*c + e^2] + 4*c*x)/Sqrt[-16*a*c + e^2]))^(-1
 - p - q)*(2*a + e*x + 2*c*x^2)^(1 + p + q)*Hypergeometric2F1[-p - q, 1 + p
 + q, 2 + p + q, (e + Sqrt[-16*a*c + e^2] + 4*c*x)/(2*Sqrt[-16*a*c + e^2])]
)/(Sqrt[-16*a*c + e^2]*(1 + p + q)))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 981

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_) + (e_.)*(x_) + (f_.)*(x_
)^2)^(q_.), x_Symbol] :> Dist[(c/f)^p, Int[(d + e*x + f*x^2)^(p + q), x], x
] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a
*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && ( !IntegerQ[q] || LeafCount[d + e
*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

Rubi steps

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q\,dx = 2^{-p}\int \left(2a+ex+2cx^2\right)^{p+q}\,dx$$
$$= -\frac{2^{1+q}\left(-\frac{e-\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-1-p-q}\left(2a+ex+2cx^2\right)^{1+p+q}}{\sqrt{-16ac+e^2}}$$

Mathematica [A]
time = 0.14, size = 142, normalized size = 1.04

$$\frac{2^{-2+q}\left(e-\sqrt{-16ac+e^2}+4cx\right)\left(\frac{e+\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-p-q}(2a+x(e+2cx))^{p+q}\,{}_2F_1\left(-p-q,1+p+q;2+p+q;\frac{-e+\sqrt{-16ac+e^2}-4cx}{2\sqrt{-16ac+e^2}}\right)}{c(1+p+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
[Out] (2^(-2 + q)*(e - Sqrt[-16*a*c + e^2] + 4*c*x)*((e + Sqrt[-16*a*c + e^2] + 4
*c*x)/Sqrt[-16*a*c + e^2])^(-p - q)*(2*a + x*(e + 2*c*x))^(p + q)*Hypergeom
etric2F1[-p - q, 1 + p + q, 2 + p + q, (-e + Sqrt[-16*a*c + e^2] - 4*c*x)/(
2*Sqrt[-16*a*c + e^2])])/(c*(1 + p + q))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a+\frac{1}{2}ex+c\,x^2\right)^p \left(2c\,x^2+ex+2a\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
[Out] int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="maxima")
[Out] integrate((2*c*x^2 + x*e + 2*a)^q*(c*x^2 + 1/2*x*e + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="fricas")
```

```
[Out] integral((2*c*x^2 + x*e + 2*a)^q*(c*x^2 + 1/2*x*e + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x**2)**p*(2*c*x**2+e*x+2*a)**q,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^2 + x*e + 2*a)^q*(c*x^2 + 1/2*x*e + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,x^2+\frac{e\,x}{2}+a\right)}^p\,{\left(2\,c\,x^2+e\,x+2\,a\right)}^q \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x)
```

```
[Out] int((a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q, x)
```

3.10 $\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx$

Optimal. Leaf size=200

$$-\frac{2^{1+p+q}\sqrt{c}\left(-\frac{\sqrt{c}\left(e-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-1-p-q}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{1+q}{}_2F_1\left(-p-q,1\right.}{\sqrt{ce^2-4af^2}\,(1+p+q)}$$

```
[Out] -2^(1+p+q)*(a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^(1+q)*hypergeom([-p-q, 1+p
+q],[2+p+q],1/2*c^(1/2)*(e+2*f*x+(-4*a*f^2+c*e^2)^(1/2)/c^(1/2))/(-4*a*f^2+
c*e^2)^(1/2))*c^(1/2)*(-c^(1/2)*(e+2*f*x-(-4*a*f^2+c*e^2)^(1/2)/c^(1/2))/(-
4*a*f^2+c*e^2)^(1/2))^(-1-p-q)/(1+p+q)/(-4*a*f^2+c*e^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {982, 638}

$$-\frac{\sqrt{c}\,2^{p+q+1}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{q+1}\left(-\frac{\sqrt{c}\left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+e+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-p-q-1}{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{\sqrt{c}\left(e+2fx+\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}\right)}{2\sqrt{ce^2-4af^2}}\right)}{(p+q+1)\sqrt{ce^2-4af^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

```
[Out] -((2^(1 + p + q)*Sqrt[c]*(-((Sqrt[c]*(e - Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2
*f*x))/Sqrt[c*e^2 - 4*a*f^2]))^(-1 - p - q)*(a + (c*e*x)/f + c*x^2)^p*((a*f
)/c + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q,
(Sqrt[c]*(e + Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f
^2])])/(Sqrt[c*e^2 - 4*a*f^2]*(1 + p + q)))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 982

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x + c*x^2)^FracPart[p]/(d
^IntPart[p]*(d + e*x + f*x^2)^FracPart[p])), Int[(d + e*x + f*x^2)^(p + q),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b
*d - a*e, 0] &&  !IntegerQ[p] &&  !IntegerQ[q] &&  !GtQ[c/f, 0]
```

Rubi steps

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx = \left(\left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^{-p}\right) \int \left(\frac{af}{c}+ex+\ldots\right.$$

$$= -\frac{2^{1+p+q}\sqrt{c}\left(-\frac{\sqrt{c}\left(e-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-1-p-q} \left(a+\ldots\right.}{\ldots}$$

Mathematica [A]
time = 0.28, size = 172, normalized size = 0.86

$$\frac{2^{-1+p+q}\left(\frac{af}{c}+x(e+fx)\right)^q \left(a+\frac{cx(e+fx)}{f}\right)^p \left(-\sqrt{ce^2-4af^2}+\sqrt{c}\,(e+2fx)\right)\left(1+\frac{\sqrt{c}\,(e+2fx)}{\sqrt{ce^2-4af^2}}\right)^{-p-q} {}_2F_1\left(-p-q,1+p+q;2+p+q;\frac{1}{2}-\frac{\sqrt{c}\,(e+2fx)}{2\sqrt{ce^2-4af^2}}\right)}{\sqrt{c}\,f(1+p+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

```
[Out] (2^(-1 + p + q)*((a*f)/c + x*(e + f*x))^q*(a + (c*x*(e + f*x))/f)^p*(-Sqrt[
c*e^2 - 4*a*f^2] + Sqrt[c]*(e + 2*f*x))*(1 + (Sqrt[c]*(e + 2*f*x))/Sqrt[c*e
^2 - 4*a*f^2])^(-p - q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q, 1/2
 - (Sqrt[c]*(e + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f^2])])/(Sqrt[c]*f*(1 + p + q)
)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(a+\frac{cex}{f}+c\,x^2\right)^p \left(\frac{af}{c}+ex+f\,x^2\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

```
[Out] int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="maxima")
[Out] integrate((c*x^2 + c*x*e/f + a)^p*(f*x^2 + x*e + a*f/c)^q, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="fricas")
[Out] integral((((c*f*x^2 + c*x*e + a*f)/c)^q*((c*f*x^2 + c*x*e + a*f)/f)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + fx^2\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x**2)**p*(a*f/c+e*x+f*x**2)**q,x)
[Out] Integral((a + c*e*x/f + c*x**2)**p*(a*f/c + e*x + f*x**2)**q, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="giac")
[Out] integrate((c*x^2 + c*x*e/f + a)^p*(f*x^2 + x*e + a*f/c)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,x+f\,x^2+\frac{a\,f}{c}\right)}^q\,{\left(a+c\,x^2+\frac{c\,e\,x}{f}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x + f*x^2 + (a*f)/c)^q*(a + c*x^2 + (c*e*x)/f)^p,x)
[Out] int((e*x + f*x^2 + (a*f)/c)^q*(a + c*x^2 + (c*e*x)/f)^p, x)
```

3.11 $\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=48

$$\frac{\sqrt{1+x^2}\,\sqrt{1+2x+x^2}}{1+x}+\frac{\sqrt{1+2x+x^2}\,\sinh^{-1}(x)}{1+x}$$

```
[Out] arcsinh(x)*((1+x)^2)^(1/2)/(1+x)+(x^2+1)^(1/2)*((1+x)^2)^(1/2)/(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {984, 655, 221}

$$\frac{\sqrt{x^2+1}\,\sqrt{x^2+2x+1}}{x+1}+\frac{\sqrt{x^2+2x+1}\,\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[1 + 2*x + x^2])/(1 + x) + (Sqrt[1 + 2*x + x^2]*ArcSinh[
x])/(1 + x)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{1+2x+x^2}\int \frac{2+2x}{\sqrt{1+x^2}}\,dx}{2+2x}$$
$$= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x} + \frac{\left(2\sqrt{1+2x+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}$$
$$= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x} + \frac{\sqrt{1+2x+x^2}\sinh^{-1}(x)}{1+x}$$

Mathematica [A]
time = 0.07, size = 37, normalized size = 0.77

$$\frac{\sqrt{(1+x)^2}\left(\sqrt{1+x^2}+\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)}{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[(1 + x)^2]*(Sqrt[1 + x^2] + ArcTanh[x/Sqrt[1 + x^2]]))/(1 + x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.09, size = 16, normalized size = 0.33

method	result	size
default	$\mathrm{csgn}(1+x)\left(\mathrm{arcsinh}\left(x\right)+\sqrt{x^2+1}\right)$	16
risch	$\frac{\mathrm{arcsinh}(x)\sqrt{\left(1+x\right)^2}}{1+x}+\frac{\sqrt{x^2+1}\,\sqrt{\left(1+x\right)^2}}{1+x}$	37
meijerg	$\frac{\mathrm{arcsinh}(x)\sqrt{\left(1+x\right)^2}}{1+x}+\frac{\sqrt{\left(1+x\right)^2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^2+1}\right)}{2(1+x)\sqrt{\pi}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(1+x)*(arcsinh(x)+(x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x + 1)^2)/sqrt(x^2 + 1), x)
```

Fricas [A]
time = 1.19, size = 22, normalized size = 0.46

$$\sqrt{x^2+1} - \log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)**2)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt((x + 1)**2)/sqrt(x**2 + 1), x)
```

Giac [A]
time = 2.26, size = 49, normalized size = 1.02

$$-\left(\sqrt{2} - \log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1) - \log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1) + \sqrt{x^2+1}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
1) + sqrt(x^2 + 1)*sgn(x + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.12 $\int \frac{1}{(-1+x^2)^2 \sqrt{-1+x+x^2}} \, dx$

Optimal. Leaf size=70

$$\frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{8}\tan^{-1}\left(\frac{3+x}{2\sqrt{-1+x+x^2}}\right) - \frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)$$

```
[Out] -1/8*arctan(1/2*(3+x)/(x^2+x-1)^(1/2))-5/8*arctanh(1/2*(1-3*x)/(x^2+x-1)^(1
/2))+1/2*(x^2+x-1)^(1/2)/(-x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {990, 1047, 738, 212, 210}

$$-\frac{1}{8}\text{ArcTan}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right) + \frac{\sqrt{x^2+x-1}}{2(1-x^2)} - \frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] Sqrt[-1 + x + x^2]/(2*(1 - x^2)) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/8
 - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p + 1
)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*((-c)*e*(2*
p + q + 4)))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x^2)^2\sqrt{-1+x+x^2}}\,dx &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)}-\frac{1}{4}\int \frac{3+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx\\ &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)}+\frac{1}{8}\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx-\frac{5}{8}\int \frac{1}{(-1+x)\sqrt{-1\\ &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,\frac{-3-x}{\sqrt{-1+x+x^2}}\right)+\frac{5}{4}\text{Sub\\ &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)}-\frac{1}{8}\tan^{-1}\left(\frac{3+x}{2\sqrt{-1+x+x^2}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3}{2\sqrt{-1+x}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 62, normalized size = 0.89

$$-\frac{\sqrt{-1+x+x^2}}{2(-1+x^2)}-\frac{1}{4}\tan^{-1}\left(1+x-\sqrt{-1+x+x^2}\right)+\frac{5}{4}\tanh^{-1}\left(1-x+\sqrt{-1+x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -1/2*Sqrt[-1 + x + x^2]/(-1 + x^2) - ArcTan[1 + x - Sqrt[-1 + x + x^2]]/4 +
 (5*ArcTanh[1 - x + Sqrt[-1 + x + x^2]])/4
```

Maple [A]
time = 0.20, size = 84, normalized size = 1.20

method	result
risch	$-\frac{\sqrt{x^2+x-1}}{2(x^2-1)}+\frac{5\,\operatorname{arctanh}\left(\frac{-1+3x}{2\sqrt{(-1+x)^2-2+3x}}\right)}{8}+\frac{\arctan\left(\frac{-3-x}{2\sqrt{(1+x)^2-2-x}}\right)}{8}$
default	$-\frac{\sqrt{(-1+x)^2-2+3x}}{4(-1+x)}+\frac{5\,\operatorname{arctanh}\left(\frac{-1+3x}{2\sqrt{(-1+x)^2-2+3x}}\right)}{8}+\frac{\sqrt{(1+x)^2-2-x}}{4+4x}+$ arctan(
trager	$-\frac{\sqrt{x^2+x-1}}{2(x^2-1)}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-x\operatorname{RootOf}\left(\_Z^2+1\right)+2\sqrt{x^2+x-1}-3\operatorname{RootOf}\left(\_Z^2+1\right)}{1+x}\right)}{8}+$ 5 ln(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2/(x^2+x-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-1+x)*((-1+x)^2-2+3*x)^(1/2)+5/8*arctanh(1/2*(-1+3*x)/((-1+x)^2-2+3*x
)^(1/2))+1/4/(1+x)*((1+x)^2-2-x)^(1/2)+1/8*arctan(1/2*(-3-x)/((1+x)^2-2-x)^
(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + x - 1)*(x^2 - 1)^2), x)
```

Fricas [A]
time = 1.66, size = 82, normalized size = 1.17

$$\frac{2\,(x^2-1)\arctan\left(-x+\sqrt{x^2+x-1}-1\right)+5\,(x^2-1)\log\left(-x+\sqrt{x^2+x-1}+2\right)-5\,(x^2-1)\log\left(-x+\sqrt{x^2+x-1}\right)-4\sqrt{x^2+x-1}}{8\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(x^2 - 1)*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5*(x^2 - 1)*log(-x +
sqrt(x^2 + x - 1) + 2) - 5*(x^2 - 1)*log(-x + sqrt(x^2 + x - 1)) - 4*sqrt(x
^2 + x - 1))/(x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)^2 (x+1)^2 \sqrt{x^2+x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral(1/((x - 1)**2*(x + 1)**2*sqrt(x**2 + x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(52) = 104.
time = 3.64, size = 143, normalized size = 2.04

$$\frac{2 \left(x-\sqrt{x^2+x-1}\right)^3+3 \left(x-\sqrt{x^2+x-1}\right)^2-x+\sqrt{x^2+x-1}-1}{2 \left(\left(x-\sqrt{x^2+x-1}\right)^4-2 \left(x-\sqrt{x^2+x-1}\right)^2-4 x+4 \sqrt{x^2+x-1}\right)}+\frac{1}{4} \arctan\left(-x+\sqrt{x^2+x-1}-1\right)+\frac{5}{8} \log\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right)-\frac{5}{8} \log\left(\left|-x+\sqrt{x^2+x-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(x - sqrt(x^2 + x - 1))^3 + 3*(x - sqrt(x^2 + x - 1))^2 - x + sqrt(x
^2 + x - 1) - 1)/((x - sqrt(x^2 + x - 1))^4 - 2*(x - sqrt(x^2 + x - 1))^2 -
 4*x + 4*sqrt(x^2 + x - 1)) + 1/4*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5/8*
log(abs(-x + sqrt(x^2 + x - 1) + 2)) - 5/8*log(abs(-x + sqrt(x^2 + x - 1)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2-1\right)^2 \sqrt{x^2+x-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^2*(x + x^2 - 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 - 1)^2*(x + x^2 - 1)^(1/2)), x)
```

3.13 $\int \frac{1}{\sqrt{a+bx+cx^2}\,\sqrt{d+fx^2}}\,dx$

Optimal. Leaf size=1077

$$-\frac{\sqrt[4]{b^2d+b\sqrt{b^2-4ac}\,d-2a(cd-af)}\,\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\,\sqrt{\frac{}{\left(\left(b+\sqrt{}\right.\right.}}}{}$$

```
[Out] -(cos(2*arctan((2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/2))^(
1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2)))^2)^(1/2)/cos(2*arctan((2*c^2*d-2*
a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2
)/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/2))^(1/4)/(b+2*c*x+(-4*a*c+b^2)
^(1/2))^(1/2)))*EllipticF(sin(2*arctan((2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)
^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d-2*a*(-a*f+c*d)+b
*d*(-4*a*c+b^2)^(1/2))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2))),1/2*(2+2*
(a*f+c*d)*(b+(-4*a*c+b^2)^(1/2))/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/
2))^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/2))^(1/2))*(b+2*c
*x+(-4*a*c+b^2)^(1/2))^(3/2)*(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/2))^
(1/4)*(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2
)^(1/2)))^(1/2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*
a*c+b^2)^(1/2))^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((f*x^2+d)*(4*a
*c-(b+(-4*a*c+b^2)^(1/2))^2)^2/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f+d*(b
+(-4*a*c+b^2)^(1/2))^2))^(1/2)*((1-4*(a*f+c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+
x*(b+(-4*a*c+b^2)^(1/2)))/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(4*a^2*f+d*(b+(-4*a*
c+b^2)^(1/2))^2)+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^2*(4*c^2*d+f*(b+(-4*a*c+b^2
)^(1/2))^2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f+d*(b+(-4*a*c+b^2)^(1/2)
)^2))/(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2
)^(1/2)))^(1/2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*
a*c+b^2)^(1/2))^(1/2))^2)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2))
)^(1/4)/(4*a*c-(b+(-4*a*c+b^2)^(1/2))^2)/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2
)/(1-4*(a*f+c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))/(b+2
*c*x+(-4*a*c+b^2)^(1/2))/(4*a^2*f+d*(b+(-4*a*c+b^2)^(1/2))^2)+(2*a+x*(b+(-4
```

```
*a*c+b^2)^(1/2)))^2*(4*c^2*d+f*(b+(-4*a*c+b^2)^(1/2))^2)/(b+2*c*x+(-4*a*c+b
^2)^(1/2))^2/(4*a^2*f+d*(b+(-4*a*c+b^2)^(1/2))^2))^(1/2)
```

Rubi [A]
time = 2.04, antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1007, 950, 1117}

$$-\frac{\sqrt[4]{db^2+\sqrt{b^2-4ac}\,db-2a(cd-af)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2-4ac}\right)^2\right)^2(fx^2+d)}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4ac}\right)^2}}\left(\frac{\sqrt{2dc^2-2afc+b\left(b+\sqrt{b^2-4ac}\right)f}\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\sqrt{db^2+\sqrt{b^2-4ac}\,db-2a(cd-af)}\left(b+2cx+\sqrt{b^2-4ac}\right)}+1\right)\sqrt{\frac{\frac{\left(4dc^2+\left(b+\sqrt{b^2-4ac}\right)^2f\right)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)^2}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4ac}\right)^2}-\frac{4\left(b+\sqrt{b^2-4ac}\right)(cd+af)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4ac}\right)}+1}{\left(\frac{\sqrt{2dc^2-2afc+b\left(b+\sqrt{b^2-4ac}\right)f}\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\sqrt{db^2+\sqrt{b^2-4ac}\,db-2a(cd-af)}\left(b+2cx+\sqrt{b^2-4ac}\right)}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{2dc^2-2afc+b\left(b+\sqrt{b^2-4ac}\right)f}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}}{\sqrt[4]{db^2+\sqrt{b^2-4ac}\,db-2a(cd-af)}\sqrt{b+2cx+\sqrt{b^2-4ac}}}\right)|\frac{1}{2}\left(\frac{\left(b+\sqrt{b^2-4ac}\right)(cd+af)}{\sqrt{2dc^2-2afc+b\left(b+\sqrt{b^2-4ac}\right)f}\sqrt{db^2+\sqrt{b^2-4ac}\,db-2a(cd-af)}}+1\right)\right)}{\left(4ac-\left(b+\sqrt{b^2-4ac}\right)^2\right)\sqrt[4]{2dc^2-2afc+b\left(b+\sqrt{b^2-4ac}\right)f}\sqrt{cx^2+bx+a}\sqrt{fx^2+d}\sqrt{\frac{\left(4dc^2+\left(b+\sqrt{b^2-4ac}\right)^2f\right)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)^2}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4ac}\right)^2}-\frac{4\left(b+\sqrt{b^2-4ac}\right)(cd+af)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4ac}\right)}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]
```

```
[Out] -(((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*(b + Sqrt[b^2 -
4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c -
(b + Sqrt[b^2 - 4*a*c])^2)^2*(d + f*x^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4
*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - 2*a*c*f +
b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d
 + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
))*Sqrt[(1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt[b^2 -
4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2
- 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*
f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d -
 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[(
(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[2*a + (b + Sqr
t[b^2 - 4*a*c])*x])/((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4
)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (1 + ((b + Sqrt[b^2 - 4*a*c])*(c*d
 + a*f))/(Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*Sqrt[b^2*d
+ b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]))/2])/((4*a*c - (b + Sqrt[b^2 -
4*a*c])^2)*(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[a +
 b*x + c*x^2]*Sqrt[d + f*x^2]*Sqrt[1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*
f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2
*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^
2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4
*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]))
```

Rule 950

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*
(x_)^2]), x_Symbol] :> Dist[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*((a + c*x^2)/(
(c*f^2 + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqrt[a + c*x^2])), Subst[Int[1/
Sqrt[1 - (2*c*d*f + 2*a*e*g)*(x^2/(c*f^2 + a*g^2)) + (c*d^2 + a*e^2)*(x^4/(
c*f^2 + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
```

```
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1007

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (f_.)*(x_)^2]), x
_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b + r + 2*c*x]*(Sqrt[2
*a + (b + r)*x]/Sqrt[a + b*x + c*x^2]), Int[1/(Sqrt[b + r + 2*c*x]*Sqrt[2*a
 + (b + r)*x]*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ
[b^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\sqrt{d+fx^2}}\,dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}+2cx}\,\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\right)\int \frac{}{\sqrt{b+\sqrt{b^2-}}}}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\,\sqrt{\frac{}{\left(\left(b+\right.\right.}}\right.}{\sqrt[4]{b^2d+b\sqrt{b^2-4ac}\,d-2a(cd-af)}\,\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a}}$$

$$= -$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.13, size = 600, normalized size = 0.56

$$-\frac{2\sqrt{2}\left(-b+\sqrt{b^2-4ac}-2cx\right)\left(-i\sqrt{d}+\sqrt{f}x\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(i\sqrt{d}+\sqrt{f}x\right)}{\left(-2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\sqrt{\frac{c\left(-i\sqrt{d}\left(\sqrt{b^2-4ac}+2cx\right)+\sqrt{f}\left(-2a+\sqrt{b^2-4ac}\,x\right)+b\left(-i\sqrt{d}-\sqrt{f}x\right)\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\left(-2ic\sqrt{d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\right)\middle|\frac{cd-i\sqrt{b^2-4ac}\sqrt{d}\sqrt{f}+af}{cd+i\sqrt{b^2-4ac}\sqrt{d}\sqrt{f}+af}\right)}{\left(-2ic\sqrt{d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\sqrt{\frac{ic\sqrt{b^2-4ac}\left(\sqrt{d}+i\sqrt{f}x\right)}{\left(2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\sqrt{d+fx^2}\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]
```

```
[Out] (-2*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*((-I)*Sqrt[d] + Sqrt[f]*x)*Sqr
t[-((c*Sqrt[b^2 - 4*a*c]*(I*Sqrt[d] + Sqrt[f]*x))/(((-2*I)*c*Sqrt[d] + (b +
```

```
 Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*Sqrt[(c*((
-I)*Sqrt[d]*(Sqrt[b^2 - 4*a*c] + 2*c*x) + Sqrt[f]*(-2*a + Sqrt[b^2 - 4*a*c]
*x) + b*((-I)*Sqrt[d] - Sqrt[f]*x)))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*
a*c])*Sqrt[f])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*EllipticF[ArcSin[Sqrt[(((
-2*I)*c*Sqrt[d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x))]], (c*d - I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)/
(c*d + I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)])/(((-2*I)*c*Sqrt[d] + (-
b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*Sqrt[(I*c*Sqrt[b^2 - 4*a*c]*(Sqrt[d] + I*Sq
rt[f]*x))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x))]*Sqrt[d + f*x^2]*Sqrt[a + x*(b + c*x)])
```

Maple [A]
time = 0.27, size = 714, normalized size = 0.66

method	result
default	$\frac{16\left(bcf\,x^{2}-2c^{2}x^{2}\sqrt{-df}-cf\,x^{2}\sqrt{-4ac+b^{2}}+4acfx-2bcx\sqrt{-df}-2cx\sqrt{-4ac+b^{2}}\,\sqrt{-df}+abf+2ac\sqrt{-df}+\cdots\right.}{\cdots}$
elliptic	$\frac{2\sqrt{\left(c\,x^{2}+bx+a\right)\left(f\,x^{2}+d\right)}\,\left(\frac{-b+\sqrt{-4ac+b^{2}}}{2c}+\frac{\sqrt{-df}}{f}\right)\sqrt{\frac{\left(-\frac{\sqrt{-df}}{f}+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)\left(x-\frac{-b+\cdots}{\cdots}\right.}{\left(-\frac{\sqrt{-df}}{f}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}\right)\left(x+\frac{b+\cdots}{\cdots}\right.}}\cdots}{\sqrt{c\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16*(b*c*f*x^2-2*c^2*x^2*(-d*f)^(1/2)-c*f*x^2*(-4*a*c+b^2)^(1/2)+4*a*c*f*x-2
*b*c*x*(-d*f)^(1/2)-2*c*x*(-4*a*c+b^2)^(1/2)*(-d*f)^(1/2)+a*b*f+2*a*c*(-d*f
)^(1/2)+a*f*(-4*a*c+b^2)^(1/2)-b^2*(-d*f)^(1/2)-b*(-4*a*c+b^2)^(1/2)*(-d*f)
^(1/2))*EllipticF(((f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f)*(-b-2*c*x+(-
4*a*c+b^2)^(1/2))/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)/(b+2*c*x+(-4*
a*c+b^2)^(1/2)))^(1/2),((f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c+b*f)*(f*(-4*
a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)/(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c
-b*f)/(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f))^(1/2))*((-4*a*c+b^2)^(1/
2)*(f*x+(-d*f)^(1/2))*c/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)/(b+2*c*
x+(-4*a*c+b^2)^(1/2)))^(1/2)*(-(-4*a*c+b^2)^(1/2)*(-f*x+(-d*f)^(1/2))*c/(f*
(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c-b*f)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2
)*((f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f)*(-b-2*c*x+(-4*a*c+b^2)^(1/2)
```

```
)/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))
^(1/2)*(c*x^2+b*x+a)^(1/2)*(f*x^2+d)^(1/2)/(1/c/f*(-b-2*c*x+(-4*a*c+b^2)^(1
/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2))*(-f*x+(-d*f)^(1/2))*(f*x+(-d*f)^(1/2)))^(
1/2)/(-4*a*c+b^2)^(1/2)/(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f)/((c*x^2
+b*x+a)*(f*x^2+d))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)/(c*f*x^4 + b*f*x^3 + b*d*x +
 (c*d + a*f)*x^2 + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + fx^2}\,\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+d)**(1/2),x)
[Out] Integral(1/(sqrt(d + f*x**2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f\,x^2+d}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.14 $\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx$

Optimal. Leaf size=98

$$-\frac{1}{2}\sin^{-1}(2+x)-\frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] -1/2*arcsin(2+x)-1/2*arctanh(x/(-x^2-4*x-3)^(1/2))-1/2*arctan(1/2*(1+(-3-x)
/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^
(1/2))*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1003, 633, 222, 1042, 1000, 12, 1040, 1175, 632, 210, 1041, 212}

$$-\frac{1}{2}\text{ArcSin}(x+2)-\frac{\text{ArcTan}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] -1/2*ArcSin[2 + x] - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqr
t[2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh
[x/Sqrt[-3 - 4*x - x^2]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1003

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
```

```
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1042

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx\right) - \frac{1}{2}\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right) + \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}$$
$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{4}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx + \frac{1}{4}\int -\frac{}{\sqrt{-3-4x}}$$
$$= -\frac{1}{2}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{}{\sqrt{-}}\right.$$
$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - 8\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\right.$$
$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx,\right.$$
$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\right.$$
$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right.}{\sqrt{2}}$$

Mathematica [A]
time = 0.16, size = 75, normalized size = 0.77

$$\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{\sqrt{2}} + \tan^{-1}\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]/Sqrt[2] + ArcTan[Sqrt[-3 -
 4*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(84) = 168$.
time = 0.44, size = 341, normalized size = 3.48

method	result

trager	$-\frac{\ln\left(\frac{16\,\mathrm{RootOf}\left(16\_Z^2+8\_Z+3\right)^2 x-8\,\mathrm{RootOf}\left(16\_Z^2+8\_Z+3\right)x-24\,\mathrm{RootOf}\left(16\_Z^2+8\_Z+3\right)-6\sqrt{-x^2-4x-3}-3x-6}{4\,\mathrm{RootOf}\left(16\_Z^2+8\_Z+3\right)x+3x+3}\right)}{2}$
default	$-\frac{\arcsin(x+2)}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}}}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\,\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsin(x+2)+1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*
arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2
/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-
3/2-x))-1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+
x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^
2-12)^(1/2)*2^(1/2))+1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/
2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*
x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x
/(-3/2-x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 - 4*x - 3)/(2*x^2 + 4*x + 3), x)
```

Fricas [A]
time = 1.27, size = 161, normalized size = 1.64

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\,x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}\,(x+2)}{x^2+4x+3}\right)+\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}\,x+4x+3}{x^2}\right)-\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}\,x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
```

```
/(2*x + 3)) + 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/
8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4
*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x+1)(x+3)}}{2x^2+4x+3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3),x)
```

```
[Out] Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(82) = 164.
time = 3.77, size = 171, normalized size = 1.74

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{1}{2}\arcsin\left(x+2\right)-\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{\left(x+2\right)^2}+1\right)+\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{\left(x+2\right)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^2-4\,x-3}}{2\,x^2+4\,x+3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3),x)
```

```
[Out] int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3), x)
```

3.15 $\int (3-x+2x^2)(2+3x+5x^2)^4\,dx$

Optimal. Leaf size=68

$$48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x^8}{4}+\frac{5075x^9}{9}+\frac{475x^{10}}{2}+\frac{1250x^{11}}{11}$$

```
[Out] 48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+5
075/9*x^9+475/2*x^10+1250/11*x^11
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1671}

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx = \int (48+272x+1064x^2+2624x^3+5099x^4+7131x^5+8232x^6+68$$
$$= 48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{341}{4}$$

Mathematica [A]
time = 0.00, size = 68, normalized size = 1.00

$$48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x^8}{4}+\frac{5075x^9}{9}+\frac{475x^{10}}{2}+\frac{1250x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

Maple [A]
time = 0.10, size = 55, normalized size = 0.81

method	result
gosper	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{1250}{11}x^{11}$
default	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{1250}{11}x^{11}$
norman	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{1250}{11}x^{11}$
risch	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{1250}{11}x^{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+5
075/9*x^9+475/2*x^10+1250/11*x^11
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.79

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^9+\frac{3415}{4}\,x^8+1176\,x^7+\frac{2377}{2}\,x^6+\frac{5099}{5}\,x^5+656\,x^4+\frac{1064}{3}\,x^3+136\,x^2+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

Fricas [A]
time = 5.34, size = 54, normalized size = 0.79

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^9+\frac{3415}{4}\,x^8+1176\,x^7+\frac{2377}{2}\,x^6+\frac{5099}{5}\,x^5+656\,x^4+\frac{1064}{3}\,x^3+136\,x^2+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

Sympy [A]
time = 0.02, size = 65, normalized size = 0.96

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**4,x)
```

```
[Out] 1250*x**11/11 + 475*x**10/2 + 5075*x**9/9 + 3415*x**8/4 + 1176*x**7 + 2377*
x**6/2 + 5099*x**5/5 + 656*x**4 + 1064*x**3/3 + 136*x**2 + 48*x
```

Giac [A]
time = 3.55, size = 54, normalized size = 0.79

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^{9}+\frac{3415}{4}\,x^{8}+1176\,x^{7}+\frac{2377}{2}\,x^{6}+\frac{5099}{5}\,x^{5}+656\,x^{4}+\frac{1064}{3}\,x^{3}+136\,x^{2}+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

Mupad [B]
time = 0.06, size = 54, normalized size = 0.79

$$\frac{1250\,x^{11}}{11}+\frac{475\,x^{10}}{2}+\frac{5075\,x^{9}}{9}+\frac{3415\,x^{8}}{4}+1176\,x^{7}+\frac{2377\,x^{6}}{2}+\frac{5099\,x^{5}}{5}+656\,x^{4}+\frac{1064\,x^{3}}{3}+136\,x^{2}+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

3.16 $\int (3-x+2x^2)(2+3x+5x^2)^3\,dx$

Optimal. Leaf size=56

$$24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{250x^9}{9}$$

```
[Out] 24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/9
*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1671}

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (3-x+2x^2)(2+3x+5x^2)^3\,dx = \int (24+100x+322x^2+579x^3+876x^4+804x^5+720x^6+325x^7$$
$$= 24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.00

$$24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{250x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

Maple [A]
time = 0.10, size = 45, normalized size = 0.80

method	result	size
gosper	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
default	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
norman	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
risch	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/9
*x^9
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.79

$$\frac{250}{9}\,x^9+\frac{325}{8}\,x^8+\frac{720}{7}\,x^7+134\,x^6+\frac{876}{5}\,x^5+\frac{579}{4}\,x^4+\frac{322}{3}\,x^3+50\,x^2+24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

Fricas [A]
time = 4.22, size = 44, normalized size = 0.79

$$\frac{250}{9}\,x^9+\frac{325}{8}\,x^8+\frac{720}{7}\,x^7+134\,x^6+\frac{876}{5}\,x^5+\frac{579}{4}\,x^4+\frac{322}{3}\,x^3+50\,x^2+24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

Sympy [A]
time = 0.01, size = 53, normalized size = 0.95

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**3,x)
```

```
[Out] 250*x**9/9 + 325*x**8/8 + 720*x**7/7 + 134*x**6 + 876*x**5/5 + 579*x**4/4 +
 322*x**3/3 + 50*x**2 + 24*x
```

Giac [A]
time = 2.94, size = 44, normalized size = 0.79

$$\frac{250}{9}\,x^{9} + \frac{325}{8}\,x^{8} + \frac{720}{7}\,x^{7} + 134\,x^{6} + \frac{876}{5}\,x^{5} + \frac{579}{4}\,x^{4} + \frac{322}{3}\,x^{3} + 50\,x^{2} + 24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

Mupad [B]
time = 0.03, size = 44, normalized size = 0.79

$$\frac{250\,x^9}{9}+\frac{325\,x^8}{8}+\frac{720\,x^7}{7}+134\,x^6+\frac{876\,x^5}{5}+\frac{579\,x^4}{4}+\frac{322\,x^3}{3}+50\,x^2+24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

3.17 $\int (3-x+2x^2)(2+3x+5x^2)^2\,dx$

Optimal. Leaf size=44

$$12x+16x^2+\frac{83x^3}{3}+\frac{85x^4}{4}+\frac{103x^5}{5}+\frac{35x^6}{6}+\frac{50x^7}{7}$$

```
[Out] 12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{1671\}$

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (3-x+2x^2)(2+3x+5x^2)^2\,dx &= \int (12+32x+83x^2+85x^3+103x^4+35x^5+50x^6)\,dx\\ &= 12x+16x^2+\frac{83x^3}{3}+\frac{85x^4}{4}+\frac{103x^5}{5}+\frac{35x^6}{6}+\frac{50x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$12x+16x^2+\frac{83x^3}{3}+\frac{85x^4}{4}+\frac{103x^5}{5}+\frac{35x^6}{6}+\frac{50x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

Maple [A]
time = 0.08, size = 35, normalized size = 0.80

method	result	size
gosper	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
default	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
norman	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
risch	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

Maxima [A]
time = 0.30, size = 34, normalized size = 0.77

$$\frac{50}{7}\,x^7+\frac{35}{6}\,x^6+\frac{103}{5}\,x^5+\frac{85}{4}\,x^4+\frac{83}{3}\,x^3+16\,x^2+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

Fricas [A]
time = 5.54, size = 34, normalized size = 0.77

$$\frac{50}{7}\,x^7+\frac{35}{6}\,x^6+\frac{103}{5}\,x^5+\frac{85}{4}\,x^4+\frac{83}{3}\,x^3+16\,x^2+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

Sympy [A]
time = 0.01, size = 41, normalized size = 0.93

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**2,x)
```

```
[Out] 50*x**7/7 + 35*x**6/6 + 103*x**5/5 + 85*x**4/4 + 83*x**3/3 + 16*x**2 + 12*x
```

Giac [A]
time = 2.20, size = 34, normalized size = 0.77

$$\frac{50}{7}\,x^{7}+\frac{35}{6}\,x^{6}+\frac{103}{5}\,x^{5}+\frac{85}{4}\,x^{4}+\frac{83}{3}\,x^{3}+16\,x^{2}+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

Mupad [B]
time = 0.03, size = 34, normalized size = 0.77

$$\frac{50\,x^{7}}{7}+\frac{35\,x^{6}}{6}+\frac{103\,x^{5}}{5}+\frac{85\,x^{4}}{4}+\frac{83\,x^{3}}{3}+16\,x^{2}+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

3.18 $\int (3-x+2x^2)(2+3x+5x^2)\,dx$

Optimal. Leaf size=30

$$6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5$$

```
[Out] 6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1671\}$

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (3-x+2x^2)(2+3x+5x^2)\,dx &= \int (6+7x+16x^2+x^3+10x^4)\,dx\\ &= 6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

Maple [A]
time = 0.03, size = 25, normalized size = 0.83

method	result	size
gosper	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4+2x^5$	25
default	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4+2x^5$	25
norman	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4+2x^5$	25
risch	$6x+\frac{7}{2}x^2+\frac{16}{3}x^3+\frac{1}{4}x^4+2x^5$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$2\,x^5+\frac{1}{4}\,x^4+\frac{16}{3}\,x^3+\frac{7}{2}\,x^2+6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

Fricas [A]
time = 3.27, size = 24, normalized size = 0.80

$$2\,x^5+\frac{1}{4}\,x^4+\frac{16}{3}\,x^3+\frac{7}{2}\,x^2+6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2),x)
```

```
[Out] 2*x**5 + x**4/4 + 16*x**3/3 + 7*x**2/2 + 6*x
```

Giac [A]
time = 3.45, size = 24, normalized size = 0.80

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="giac")
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

Mupad [B]
time = 0.02, size = 24, normalized size = 0.80

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2),x)
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

3.19 $\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx$

Optimal. Leaf size=42

$$\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(2+3x+5x^2\right)$$

```
[Out] 2/5*x-11/50*ln(5*x^2+3*x+2)+143/775*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{143\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx &= \int \left(\frac{2}{5}+\frac{11(1-x)}{5\left(2+3x+5x^2\right)}\right) dx\\ &= \frac{2x}{5}+\frac{11}{5}\int \frac{1-x}{2+3x+5x^2}\,dx\\ &= \frac{2x}{5}-\frac{11}{50}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{143}{50}\int \frac{1}{2+3x+5x^2}\,dx\\ &= \frac{2x}{5}-\frac{11}{50}\log\left(2+3x+5x^2\right)-\frac{143}{25}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right)\\ &= \frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(2+3x+5x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(2+3x+5x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

Maple [A]
time = 0.22, size = 34, normalized size = 0.81

method	result	size

default	$\frac{2x}{5}-\frac{11\ln(5x^2+3x+2)}{50}+\frac{143\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{775}$	34
risch	$\frac{2x}{5}-\frac{11\ln(100x^2+60x+40)}{50}+\frac{143\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{775}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x-11/50*ln(5*x^2+3*x+2)+143/775*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.52, size = 33, normalized size = 0.79

$$\frac{143}{775}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

Fricas [A]
time = 4.10, size = 33, normalized size = 0.79

$$\frac{143}{775}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

Sympy [A]
time = 0.04, size = 49, normalized size = 1.17

$$\frac{2x}{5}-\frac{11\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{50}+\frac{143\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2),x)
```

```
[Out] 2*x/5 - 11*log(x**2 + 3*x/5 + 2/5)/50 + 143*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/775
```

Giac [A]
time = 3.98, size = 33, normalized size = 0.79

$$\frac{143}{775}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

Mupad [B]
time = 3.39, size = 35, normalized size = 0.83

$$\frac{2\,x}{5}-\frac{11\,\ln\left(5\,x^2+3\,x+2\right)}{50}+\frac{143\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2),x)
```

```
[Out] (2*x)/5 - (11*log(3*x + 5*x^2 + 2))/50 + (143*31^(1/2)*atan((10*31^(1/2)*x)
/31 + (3*31^(1/2))/31))/775
```

3.20 $\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=43

$$\frac{11(7+13x)}{155\,(2+3x+5x^2)}+\frac{82\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}$$

```
[Out] 11/155*(7+13*x)/(5*x^2+3*x+2)+82/961*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2
)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1674, 12, 632, 210}

$$\frac{82\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{11(13x+7)}{155\,(5x^2+3x+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{1}{31}\int \frac{41}{2+3x+5x^2}\,dx\\ &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{41}{31}\int \frac{1}{2+3x+5x^2}\,dx\\ &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}-\frac{82}{31}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right)\\ &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{82\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 43, normalized size = 1.00

$$\frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{82\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

Maple [**A**]
time = 0.12, size = 34, normalized size = 0.79

method	result	size
default	$\frac{\frac{143x}{775}+\frac{77}{775}}{x^2+\frac{3}{5}x+\frac{2}{5}}+\frac{82\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{961}$	34

risch	$\frac{\frac{143x}{775}+\frac{77}{775}}{x^2+\frac{3}{5}x+\frac{2}{5}}+\frac{82\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{961}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (143/775*x+77/775)/(x^2+3/5*x+2/5)+82/961*arctan(1/31*(3+10*x)*31^(1/2))*31
^(1/2)
```

Maxima [A]
time = 0.50, size = 36, normalized size = 0.84

$$\frac{82}{961}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{11\,(13\,x+7)}{155\,(5\,x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2
 + 3*x + 2)
```

Fricas [A]
time = 3.03, size = 45, normalized size = 1.05

$$\frac{410\sqrt{31}\,(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+4433\,x+2387}{4805\,(5\,x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4805*(410*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4
433*x + 2387)/(5*x^2 + 3*x + 2)
```

Sympy [A]
time = 0.05, size = 42, normalized size = 0.98

$$\frac{143x+77}{775x^2+465x+310}+\frac{82\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

```
[Out] (143*x + 77)/(775*x**2 + 465*x + 310) + 82*sqrt(31)*atan(10*sqrt(31)*x/31 +
 3*sqrt(31)/31)/961
```

Giac [A]
time = 5.82, size = 36, normalized size = 0.84

$$\frac{82}{961}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{11\,(13\,x+7)}{155\,(5\,x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2
 + 3*x + 2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{\frac{143\,x}{775}+\frac{77}{775}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\frac{82\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] ((143*x)/775 + 77/775)/((3*x)/5 + x^2 + 2/5) + (82*31^(1/2)*atan((10*31^(1/
2)*x)/31 + (3*31^(1/2))/31))/961
```

3.21 $\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx$

Optimal. Leaf size=64

$$\frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{1106\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

```
[Out] 11/310*(7+13*x)/(5*x^2+3*x+2)^2+553/9610*(3+10*x)/(5*x^2+3*x+2)+1106/29791*
arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1674, 12, 628, 632, 210}

$$\frac{1106\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}+\frac{553(10x+3)}{9610\left(5x^2+3x+2\right)}+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*(3 + 10*x))/(9610*(2 + 3*x
 + 5*x^2)) + (1106*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^3}\,dx &= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{1}{62}\int \frac{553}{5\left(2+3x+5x^2\right)^2}\,dx\\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553}{310}\int \frac{1}{\left(2+3x+5x^2\right)^2}\,dx\\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{553}{961}\int \frac{1}{2+3x+5x^2}\,dx\\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}-\frac{1106}{961}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+\right.\\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{1106\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.83

$$\frac{\frac{31\left(1141+4094x+4977x^2+5530x^3\right)}{\left(2+3x+5x^2\right)^2}+2212\sqrt{31}\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{59582}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

[Out] ((31*(1141 + 4094*x + 4977*x^2 + 5530*x^3))/(2 + 3*x + 5*x^2)^2 + 2212*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]])/59582

Maple [A]
time = 0.10, size = 47, normalized size = 0.73

method	result	size
default	$\frac{\frac{2765}{961}x^3+\frac{4977}{1922}x^2+\frac{2047}{961}x+\frac{1141}{1922}}{(5x^2+3x+2)^2}+\frac{1106\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47
risch	$\frac{\frac{2765}{961}x^3+\frac{4977}{1922}x^2+\frac{2047}{961}x+\frac{1141}{1922}}{(5x^2+3x+2)^2}+\frac{1106\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)

[Out] 25*(553/4805*x^3+4977/48050*x^2+2047/24025*x+1141/48050)/(5*x^2+3*x+2)^2+1106/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)

Maxima [A]
time = 0.51, size = 56, normalized size = 0.88

$$\frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{5530x^3+4977x^2+4094x+1141}{1922(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4977*x^2 + 4094*x + 1141)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

Fricas [A]
time = 1.81, size = 75, normalized size = 1.17

$$\frac{171430x^3+2212\sqrt{31}(25x^4+30x^3+29x^2+12x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+154287x^2+126914x+35371}{59582(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 1/59582*(171430*x^3 + 2212*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 154287*x^2 + 126914*x + 35371)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

Sympy [A]
time = 0.07, size = 63, normalized size = 0.98

$$\frac{5530x^3 + 4977x^2 + 4094x + 1141}{48050x^4 + 57660x^3 + 55738x^2 + 23064x + 7688} + \frac{1106\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (5530*x**3 + 4977*x**2 + 4094*x + 1141)/(48050*x**4 + 57660*x**3 + 55738*x*
*2 + 23064*x + 7688) + 1106*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)
/29791
```

Giac [A]
time = 3.61, size = 46, normalized size = 0.72

$$\frac{1106}{29791}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x + 3)\right) + \frac{5530\,x^3 + 4977\,x^2 + 4094\,x + 1141}{1922\,(5\,x^2 + 3\,x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4
977*x^2 + 4094*x + 1141)/(5*x^2 + 3*x + 2)^2
```

Mupad [B]
time = 0.05, size = 55, normalized size = 0.86

$$\frac{1106\,\sqrt{31}\,\operatorname{atan}\left(\frac{10\,\sqrt{31}\,x}{31} + \frac{3\,\sqrt{31}}{31}\right)}{29791} + \frac{\frac{553\,x^3}{4805} + \frac{4977\,x^2}{48050} + \frac{2047\,x}{24025} + \frac{1141}{48050}}{x^4 + \frac{6\,x^3}{5} + \frac{29\,x^2}{25} + \frac{12\,x}{25} + \frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (1106*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((2047*x
)/24025 + (4977*x^2)/48050 + (553*x^3)/4805 + 1141/48050)/((12*x)/25 + (29*
x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

3.22 $\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx$

Optimal. Leaf size=80

$$144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3315x^8+\frac{24859x^9}{9}+1571x^{10}+\frac{11525x^{11}}{11}+\frac{875x}{3}$$

```
[Out] 144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315*
x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1671}

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384x^2+144x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \int (144+768x+3016x^2+7352x^3+14801x^4+21542x^5+27763x^6$$

$$= 144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}$$

Mathematica [A]
time = 0.00, size = 80, normalized size = 1.00

$$144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3315x^8+\frac{24859x^9}{9}+1571x^{10}+\frac{11525x^{11}}{11}+\frac{875x^{12}}{3}+\frac{2500x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

Maple [A]
time = 0.12, size = 65, normalized size = 0.81

method	result
gosper	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x^{10}+$
default	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x^{10}+$
norman	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x^{10}+$
risch	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x^{10}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315*
x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.80

$$\frac{2500}{13}\,x^{13}+\frac{875}{3}\,x^{12}+\frac{11525}{11}\,x^{11}+1571\,x^{10}+\frac{24859}{9}\,x^{9}+3315\,x^{8}+\frac{27763}{7}\,x^{7}+\frac{10771}{3}\,x^{6}+\frac{14801}{5}\,x^{5}+1838\,x^{4}+\frac{3016}{3}\,x^{3}+384\,x^{2}+144\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

Fricas [A]
time = 1.70, size = 64, normalized size = 0.80

$$\frac{2500}{13}\,x^{13}+\frac{875}{3}\,x^{12}+\frac{11525}{11}\,x^{11}+1571\,x^{10}+\frac{24859}{9}\,x^{9}+3315\,x^{8}+\frac{27763}{7}\,x^{7}+\frac{10771}{3}\,x^{6}+\frac{14801}{5}\,x^{5}+1838\,x^{4}+\frac{3016}{3}\,x^{3}+384\,x^{2}+144\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

Sympy [A]
time = 0.02, size = 76, normalized size = 0.95

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**4,x)
```

```
[Out] 2500*x**13/13 + 875*x**12/3 + 11525*x**11/11 + 1571*x**10 + 24859*x**9/9 +
3315*x**8 + 27763*x**7/7 + 10771*x**6/3 + 14801*x**5/5 + 1838*x**4 + 3016*x
**3/3 + 384*x**2 + 144*x
```

Giac [A]
time = 5.55, size = 64, normalized size = 0.80

$$\frac{2500}{13}\,x^{13} + \frac{875}{3}\,x^{12} + \frac{11525}{11}\,x^{11} + 1571\,x^{10} + \frac{24859}{9}\,x^9 + 3315\,x^8 + \frac{27763}{7}\,x^7 + \frac{10771}{3}\,x^6 + \frac{14801}{5}\,x^5 + 1838\,x^4 + \frac{3016}{3}\,x^3 + 384\,x^2 + 144\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

Mupad [B]
time = 0.08, size = 64, normalized size = 0.80

$$\frac{2500\,x^{13}}{13} + \frac{875\,x^{12}}{3} + \frac{11525\,x^{11}}{11} + 1571\,x^{10} + \frac{24859\,x^9}{9} + 3315\,x^8 + \frac{27763\,x^7}{7} + \frac{10771\,x^6}{3} + \frac{14801\,x^5}{5} + 1838\,x^4 + \frac{3016\,x^3}{3} + 384\,x^2 + 144\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

3.23 $\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx$

Optimal. Leaf size=66

$$72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6+444x^7+\frac{1863x^8}{8}+\frac{1865x^9}{9}+40x^{10}+\frac{500x^{11}}{11}$$

```
[Out] 72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+186
5/9*x^9+40*x^10+500/11*x^11
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{1671\}$

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \int \left(72+276x+914x^2+1615x^3+2693x^4+2694x^5+3108x^6+1\right.$$

$$= 72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6+444x^7+\frac{186}{8}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.00

$$72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6+444x^7+\frac{1863x^8}{8}+\frac{1865x^9}{9}+40x^{10}+\frac{500x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

Maple [A]
time = 0.11, size = 55, normalized size = 0.83

method	result	siz
gosper	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^{11}$	55
default	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^{11}$	55
norman	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^{11}$	55
risch	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^{11}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+186
5/9*x^9+40*x^10+500/11*x^11
```

Maxima [A]
time = 0.30, size = 54, normalized size = 0.82

$$\frac{500}{11}\,x^{11}+40\,x^{10}+\frac{1865}{9}\,x^9+\frac{1863}{8}\,x^8+444\,x^7+449\,x^6+\frac{2693}{5}\,x^5+\frac{1615}{4}\,x^4+\frac{914}{3}\,x^3+138\,x^2+72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

Fricas [A]
time = 2.49, size = 54, normalized size = 0.82

$$\frac{500}{11}\,x^{11}+40\,x^{10}+\frac{1865}{9}\,x^9+\frac{1863}{8}\,x^8+444\,x^7+449\,x^6+\frac{2693}{5}\,x^5+\frac{1615}{4}\,x^4+\frac{914}{3}\,x^3+138\,x^2+72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

Sympy [A]
time = 0.02, size = 63, normalized size = 0.95

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**3,x)
```

```
[Out] 500*x**11/11 + 40*x**10 + 1865*x**9/9 + 1863*x**8/8 + 444*x**7 + 449*x**6 +
 2693*x**5/5 + 1615*x**4/4 + 914*x**3/3 + 138*x**2 + 72*x
```

Giac [A]
time = 3.99, size = 54, normalized size = 0.82

$$\frac{500}{11}\,x^{11}+40\,x^{10}+\frac{1865}{9}\,x^{9}+\frac{1863}{8}\,x^{8}+444\,x^{7}+449\,x^{6}+\frac{2693}{5}\,x^{5}+\frac{1615}{4}\,x^{4}+\frac{914}{3}\,x^{3}+138\,x^{2}+72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

Mupad [B]
time = 0.05, size = 54, normalized size = 0.82

$$\frac{500\,x^{11}}{11}+40\,x^{10}+\frac{1865\,x^{9}}{9}+\frac{1863\,x^{8}}{8}+444\,x^{7}+449\,x^{6}+\frac{2693\,x^{5}}{5}+\frac{1615\,x^{4}}{4}+\frac{914\,x^{3}}{3}+138\,x^{2}+72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

3.24 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=54

$$36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5+\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}$$

```
[Out] 36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1671}

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx &= \int \left(36+84x+241x^2+236x^3+390x^4+172x^5+321x^6+20x^7+1\right.\\ &= 36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5+\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 54, normalized size = 1.00

$$36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5+\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

Maple [A]
time = 0.13, size = 45, normalized size = 0.83

method	result	size
gosper	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
default	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
norman	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
risch	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.81

$$\frac{100}{9}\,x^9+\frac{5}{2}\,x^8+\frac{321}{7}\,x^7+\frac{86}{3}\,x^6+78\,x^5+59\,x^4+\frac{241}{3}\,x^3+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

Fricas [A]
time = 1.76, size = 44, normalized size = 0.81

$$\frac{100}{9}\,x^9+\frac{5}{2}\,x^8+\frac{321}{7}\,x^7+\frac{86}{3}\,x^6+78\,x^5+59\,x^4+\frac{241}{3}\,x^3+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

Sympy [A]
time = 0.01, size = 51, normalized size = 0.94

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**2,x)
```

```
[Out] 100*x**9/9 + 5*x**8/2 + 321*x**7/7 + 86*x**6/3 + 78*x**5 + 59*x**4 + 241*x*
*3/3 + 42*x**2 + 36*x
```

Giac [A]
time = 4.34, size = 44, normalized size = 0.81

$$\frac{100}{9}\,x^9+\frac{5}{2}\,x^8+\frac{321}{7}\,x^7+\frac{86}{3}\,x^6+78\,x^5+59\,x^4+\frac{241}{3}\,x^3+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

Mupad [B]
time = 0.03, size = 44, normalized size = 0.81

$$\frac{100\,x^9}{9}+\frac{5\,x^8}{2}+\frac{321\,x^7}{7}+\frac{86\,x^6}{3}+78\,x^5+59\,x^4+\frac{241\,x^3}{3}+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

3.25 $\int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) \, dx$

Optimal. Leaf size=46

$$18x + \frac{15x^2}{2} + \frac{53x^3}{3} + \frac{x^4}{4} + \frac{61x^5}{5} - \frac{4x^6}{3} + \frac{20x^7}{7}$$

```
[Out] 18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1671}

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) \, dx &= \int (18 + 15x + 53x^2 + x^3 + 61x^4 - 8x^5 + 20x^6) \, dx \\ &= 18x + \frac{15x^2}{2} + \frac{53x^3}{3} + \frac{x^4}{4} + \frac{61x^5}{5} - \frac{4x^6}{3} + \frac{20x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 46, normalized size = 1.00

$$18x + \frac{15x^2}{2} + \frac{53x^3}{3} + \frac{x^4}{4} + \frac{61x^5}{5} - \frac{4x^6}{3} + \frac{20x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/7

Maple [A]
time = 0.10, size = 35, normalized size = 0.76

method	result	size
gosper	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35
default	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35
norman	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35
risch	$18x+\frac{15}{2}x^2+\frac{53}{3}x^3+\frac{1}{4}x^4+\frac{61}{5}x^5-\frac{4}{3}x^6+\frac{20}{7}x^7$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)

[Out] 18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7

Maxima [A]
time = 0.27, size = 34, normalized size = 0.74

$$\frac{20}{7}\,x^7-\frac{4}{3}\,x^6+\frac{61}{5}\,x^5+\frac{1}{4}\,x^4+\frac{53}{3}\,x^3+\frac{15}{2}\,x^2+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="maxima")

[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x

Fricas [A]
time = 2.55, size = 34, normalized size = 0.74

$$\frac{20}{7}\,x^7-\frac{4}{3}\,x^6+\frac{61}{5}\,x^5+\frac{1}{4}\,x^4+\frac{53}{3}\,x^3+\frac{15}{2}\,x^2+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="fricas")

[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x

Sympy [A]
time = 0.01, size = 41, normalized size = 0.89

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2),x)
```

```
[Out] 20*x**7/7 - 4*x**6/3 + 61*x**5/5 + x**4/4 + 53*x**3/3 + 15*x**2/2 + 18*x
```

Giac [A]
time = 3.31, size = 34, normalized size = 0.74

$$\frac{20}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{61}{5}\,x^5 + \frac{1}{4}\,x^4 + \frac{53}{3}\,x^3 + \frac{15}{2}\,x^2 + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

Mupad [B]
time = 0.03, size = 34, normalized size = 0.74

$$\frac{20\,x^7}{7} - \frac{4\,x^6}{3} + \frac{61\,x^5}{5} + \frac{x^4}{4} + \frac{53\,x^3}{3} + \frac{15\,x^2}{2} + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2),x)
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

3.26 $\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx$

Optimal. Leaf size=56

$$\frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{8349\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}}-\frac{1573\log\left(2+3x+5x^2\right)}{1250}$$

```
[Out] 381/125*x-16/25*x^2+4/15*x^3-1573/1250*ln(5*x^2+3*x+2)+8349/19375*arctan(1/
31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{8349\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}+\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log\left(5x^2+3x+2\right)}{1250}+\frac{381x}{125}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] (381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]])
/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx &= \int \left(\frac{381}{125}-\frac{32x}{25}+\frac{4x^2}{5}+\frac{121(3-13x)}{125\,(2+3x+5x^2)}\right) dx \\
&= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{121}{125}\int \frac{3-13x}{2+3x+5x^2}\,dx \\
&= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\int \frac{3+10x}{2+3x+5x^2}\,dx}{1250}+\frac{8349\int \frac{1}{2+3x+5x^2}\,dx}{1250} \\
&= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\log\left(2+3x+5x^2\right)}{1250}-\frac{8349}{625}\mathrm{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3\right. \\
&= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{8349\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}}-\frac{1573\log\left(2+3x+5x^2\right)}{1250}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.95

$$\frac{8349\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}}+\frac{10x(1143-240x+100x^2)-4719\log\left(2+3x+5x^2\right)}{3750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] (8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) + (10*x*(1143 - 240*x + 1
00*x^2) - 4719*Log[2 + 3*x + 5*x^2])/3750
```

Maple [A]
time = 0.12, size = 44, normalized size = 0.79

method	result	size
default	$\frac{381x}{125}-\frac{16x^{2}}{25}+\frac{4x^{3}}{15}-\frac{1573\ln\left(5x^{2}+3x+2\right)}{1250}+\frac{8349\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{19375}$	44
risch	$\frac{4x^{3}}{15}-\frac{16x^{2}}{25}+\frac{381x}{125}-\frac{1573\ln\left(100x^{2}+60x+40\right)}{1250}+\frac{8349\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{19375}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 381/125*x-16/25*x^2+4/15*x^3-1573/1250*ln(5*x^2+3*x+2)+8349/19375*arctan(1/
31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.49, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^{3}-\frac{16}{25}\,x^{2}+\frac{8349}{19375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{381}{125}\,x-\frac{1573}{1250}\,\log\left(5\,x^{2}+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

Fricas [A]
time = 2.38, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^{3}-\frac{16}{25}\,x^{2}+\frac{8349}{19375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{381}{125}\,x-\frac{1573}{1250}\,\log\left(5\,x^{2}+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

Sympy [A]
time = 0.05, size = 63, normalized size = 1.12

$$\frac{4x^{3}}{15}-\frac{16x^{2}}{25}+\frac{381x}{125}-\frac{1573\log\left(x^{2}+\frac{3x}{5}+\frac{2}{5}\right)}{1250}+\frac{8349\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{19375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

```
[Out] 4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*log(x**2 + 3*x/5 + 2/5)/1250 + 83
49*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19375
```

Giac [A]
time = 2.87, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

Mupad [B]
time = 3.45, size = 45, normalized size = 0.80

$$\frac{381\,x}{125} - \frac{1573\,\ln\left(5\,x^2+3\,x+2\right)}{1250} + \frac{8349\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{19375} - \frac{16\,x^2}{25} + \frac{4\,x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2),x)
```

```
[Out] (381*x)/125 - (1573*log(3*x + 5*x^2 + 2))/1250 + (8349*31^(1/2)*atan((10*31
^(1/2)*x)/31 + (3*31^(1/2))/31))/19375 - (16*x^2)/25 + (4*x^3)/15
```

3.27 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx$

Optimal. Leaf size=63

$$\frac{4x}{25}+\frac{121(61+69x)}{3875(2+3x+5x^2)}+\frac{41932\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}}-\frac{22}{125}\log\left(2+3x+5x^2\right)$$

```
[Out] 4/25*x+121/3875*(61+69*x)/(5*x^2+3*x+2)-22/125*ln(5*x^2+3*x+2)+41932/120125
*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{41932\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}+\frac{121(69x+61)}{3875(5x^2+3x+2)}-\frac{22}{125}\log\left(5x^2+3x+2\right)+\frac{4x}{25}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 + (121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (41932*ArcTan[(3 +
10*x)/Sqrt[31]])/(3875*Sqrt[31]) - (22*Log[2 + 3*x + 5*x^2])/125
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx &= \frac{121(61+69x)}{3875\,(2+3x+5x^2)}+\frac{1}{31}\int \frac{\frac{4032}{25}-\frac{992x}{25}+\frac{124x^2}{5}}{2+3x+5x^2}\,dx \\
&= \frac{121(61+69x)}{3875\,(2+3x+5x^2)}+\frac{1}{31}\int \left(\frac{124}{25}+\frac{44(86-31x)}{25\,(2+3x+5x^2)}\right)dx \\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875\,(2+3x+5x^2)}+\frac{44}{775}\int \frac{86-31x}{2+3x+5x^2}\,dx \\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875\,(2+3x+5x^2)}-\frac{22}{125}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{20966\int \frac{1}{2+3x+5x^2}\,dx}{3875} \\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875\,(2+3x+5x^2)}-\frac{22}{125}\log\left(2+3x+5x^2\right)-\frac{41932\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x
\end{aligned}
$$

$$
= \frac{4x}{25}+\frac{121(61+69x)}{3875\,(2+3x+5x^2)}+\frac{41932\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}}-\frac{22}{125}\log\left(2+3x+5x^2\right)
$$

Mathematica [**A**]

time = 0.02, size = 59, normalized size = 0.94

$$\frac{19220x + \frac{3751(61+69x)}{2+3x+5x^2} + 41932\sqrt{31}\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right) - 21142\log\left(2+3x+5x^2\right)}{120125}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]

[Out] (19220*x + (3751*(61 + 69*x))/(2 + 3*x + 5*x^2) + 41932*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] - 21142*Log[2 + 3*x + 5*x^2])/120125

Maple [A]
time = 0.14, size = 51, normalized size = 0.81

method	result	size
risch	$\frac{4x}{25}+\frac{\frac{8349x}{19375}+\frac{7381}{19375}}{x^2+\frac{3}{5}x+\frac{2}{5}}-\frac{22\ln\left(100x^2+60x+40\right)}{125}+\frac{41932\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{120125}$	50
default	$\frac{4x}{25}-\frac{11\left(-\frac{759x}{775}-\frac{671}{775}\right)}{25\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{22\ln\left(5x^2+3x+2\right)}{125}+\frac{41932\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{120125}$	51

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)

[Out] 4/25*x-11/25*(-759/775*x-671/775)/(x^2+3/5*x+2/5)-22/125*ln(5*x^2+3*x+2)+41932/120125*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)

Maxima [A]
time = 0.49, size = 52, normalized size = 0.83

$$\frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{4}{25}\,x+\frac{121\,(69\,x+61)}{3875\,(5\,x^2+3\,x+2)}-\frac{22}{125}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] 41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875*(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)

Fricas [A]
time = 2.64, size = 78, normalized size = 1.24

$$\frac{96100\,x^3+41932\,\sqrt{31}\,(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+57660\,x^2-21142\,(5\,x^2+3\,x+2)\log\left(5\,x^2+3\,x+2\right)+297259\,x+228811}{120125\,(5\,x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/120125*(96100*x^3 + 41932*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)
*(10*x + 3)) + 57660*x^2 - 21142*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 2
97259*x + 228811)/(5*x^2 + 3*x + 2)
```

Sympy [A]
time = 0.07, size = 65, normalized size = 1.03

$$\frac{4x}{25}+\frac{8349x+7381}{19375x^2+11625x+7750}-\frac{22\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{125}+\frac{41932\sqrt{31}\ \mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] 4*x/25 + (8349*x + 7381)/(19375*x**2 + 11625*x + 7750) - 22*log(x**2 + 3*x/
5 + 2/5)/125 + 41932*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/120125
```

Giac [A]
time = 3.02, size = 52, normalized size = 0.83

$$\frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{4}{25}\,x+\frac{121\,(69\,x+61)}{3875\,(5\,x^2+3\,x+2)}-\frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875*
(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.81

$$\frac{4\,x}{25}-\frac{22\,\ln\left(5\,x^2+3\,x+2\right)}{125}+\frac{\frac{8349\,x}{19375}+\frac{7381}{19375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\frac{41932\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (4*x)/25 - (22*log(3*x + 5*x^2 + 2))/125 + ((8349*x)/19375 + 7381/19375)/((
3*x)/5 + x^2 + 2/5) + (41932*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2)
)/31))/120125
```

3.28 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=64

$$\frac{121(61+69x)}{7750\,(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250\,(2+3x+5x^2)}+\frac{4330\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

```
[Out] 121/7750*(61+69*x)/(5*x^2+3*x+2)^2+11/240250*(17557+45710*x)/(5*x^2+3*x+2)+
4330/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1674, 12, 632, 210}

$$\frac{4330\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}+\frac{121(69x+61)}{7750\,(5x^2+3x+2)^2}+\frac{11(45710x+17557)}{240250\,(5x^2+3x+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + (11*(17557 + 45710*x))/(2402
50*(2 + 3*x + 5*x^2)) + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx &= \frac{121(61+69x)}{7750\,(2+3x+5x^2)^2}+\frac{1}{62}\int \frac{\frac{48669}{125}-\frac{1984x}{25}+\frac{248x^2}{5}}{(2+3x+5x^2)^2}\,dx\\
&= \frac{121(61+69x)}{7750\,(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250\,(2+3x+5x^2)}+\frac{\int \frac{4330}{2+3x+5x^2}\,dx}{1922}\\
&= \frac{121(61+69x)}{7750\,(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250\,(2+3x+5x^2)}+\frac{2165}{961}\int \frac{1}{2+3x+5x^2}\,dx\\
&= \frac{121(61+69x)}{7750\,(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250\,(2+3x+5x^2)}-\frac{4330}{961}\mathrm{Subst}\left(\int \frac{1}{-31-x^2}\,dx, x, 3\right.\\
&= \frac{121(61+69x)}{7750\,(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250\,(2+3x+5x^2)}+\frac{4330\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.83

$$\frac{11(11183+33524x+44983x^2+45710x^3)}{48050\,(2+3x+5x^2)^2}+\frac{4330\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(11183 + 33524*x + 44983*x^2 + 45710*x^3))/(48050*(2 + 3*x + 5*x^2)^2)
+ (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Maple [A]
time = 0.11, size = 47, normalized size = 0.73

method	result	size

default	$\frac{\frac{50281}{4805}x^3+\frac{494813}{48050}x^2+\frac{184382}{24025}x+\frac{123013}{48050}}{(5x^2+3x+2)^2}+\frac{4330\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47
risch	$\frac{\frac{50281}{4805}x^3+\frac{494813}{48050}x^2+\frac{184382}{24025}x+\frac{123013}{48050}}{(5x^2+3x+2)^2}+\frac{4330\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 25*(50281/120125*x^3+494813/1201250*x^2+184382/600625*x+123013/1201250)/(5*
x^2+3*x+2)^2+4330/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.50, size = 56, normalized size = 0.88

$$\frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{11(45710x^3+44983x^2+33524x+11183)}{48050(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
+ 44983*x^2 + 33524*x + 11183)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Fricas [A]
time = 2.32, size = 75, normalized size = 1.17

$$\frac{15587110x^3+216500\sqrt{31}(25x^4+30x^3+29x^2+12x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+15339203x^2+11431684x+3813403}{1489550(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/1489550*(15587110*x^3 + 216500*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
+ 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 15339203*x^2 + 11431684*x + 3813403
)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [A]
time = 0.07, size = 63, normalized size = 0.98

$$\frac{502810x^3+494813x^2+368764x+123013}{1201250x^4+1441500x^3+1393450x^2+576600x+192200}+\frac{4330\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] (502810*x**3 + 494813*x**2 + 368764*x + 123013)/(1201250*x**4 + 1441500*x**
3 + 1393450*x**2 + 576600*x + 192200) + 4330*sqrt(31)*atan(10*sqrt(31)*x/31
 + 3*sqrt(31)/31)/29791
```

Giac [A]
time = 3.33, size = 46, normalized size = 0.72

$$\frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{11(45710x^3+44983x^2+33524x+11183)}{48050(5x^2+3x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
+ 44983*x^2 + 33524*x + 11183)/(5*x^2 + 3*x + 2)^2
```

Mupad [B]
time = 3.44, size = 55, normalized size = 0.86

$$\frac{4330\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}+\frac{\frac{50281x^3}{120125}+\frac{494813x^2}{1201250}+\frac{184382x}{600625}+\frac{123013}{1201250}}{x^4+\frac{6x^3}{5}+\frac{29x^2}{25}+\frac{12x}{25}+\frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (4330*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((184382
*x)/600625 + (494813*x^2)/1201250 + (50281*x^3)/120125 + 123013/1201250)/((
12*x)/25 + (29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

3.29 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} \, dx$

Optimal. Leaf size=85

$$\frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}+\frac{66752\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

```
[Out] 121/11625*(61+69*x)/(5*x^2+3*x+2)^3+11/120125*(4579+12060*x)/(5*x^2+3*x+2)^
2+16688/148955*(3+10*x)/(5*x^2+3*x+2)+66752/923521*arctan(1/31*(3+10*x)*31^
(1/2))*31^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1674, 12, 628, 632, 210}

$$\frac{66752\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}+\frac{16688(10x+3)}{148955(5x^2+3x+2)}+\frac{11(12060x+4579)}{120125(5x^2+3x+2)^2}+\frac{121(69x+61)}{11625(5x^2+3x+2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (11*(4579 + 12060*x))/(1201
25*(2 + 3*x + 5*x^2)^2) + (16688*(3 + 10*x))/(148955*(2 + 3*x + 5*x^2)) + (
66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(29791*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

```
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx &= \frac{121(61+69x)}{11625\,(2+3x+5x^2)^3}+\frac{1}{93}\int \frac{\frac{77178}{125}-\frac{2976x}{25}+\frac{372x^2}{5}}{(2+3x+5x^2)^3}\,dx\\
&= \frac{121(61+69x)}{11625\,(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125\,(2+3x+5x^2)^2}+\frac{\int \frac{100128}{5(2+3x+5x^2)^2}\,dx}{5766}\\
&= \frac{121(61+69x)}{11625\,(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125\,(2+3x+5x^2)^2}+\frac{16688\int \frac{1}{(2+3x+5x^2)^2}\,dx}{4805}\\
&= \frac{121(61+69x)}{11625\,(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125\,(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955\,(2+3x+5x^2)}+\frac{3337}{\ }\\
&= \frac{121(61+69x)}{11625\,(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125\,(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955\,(2+3x+5x^2)}-\frac{6675}{\ }\\
&= \frac{121(61+69x)}{11625\,(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125\,(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955\,(2+3x+5x^2)}+\frac{6675}{\ }
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.74

$$\frac{1259239+5674908x+12780597x^2+21491796x^3+18774000x^4+12516000x^5}{446865\,(2+3x+5x^2)^3}+\frac{66752\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (1259239 + 5674908*x + 12780597*x^2 + 21491796*x^3 + 18774000*x^4 + 1251600
0*x^5)/(446865*(2 + 3*x + 5*x^2)^3) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(
29791*Sqrt[31])
```

Maple [A]
time = 0.12, size = 57, normalized size = 0.67

method	result	size
default	$\frac{\frac{834400}{29791}x^5+\frac{1251600}{29791}x^4+\frac{7163932}{148955}x^3+\frac{4260199}{148955}x^2+\frac{1891636}{148955}x+\frac{1259239}{446865}}{(5x^2+3x+2)^3}+\frac{66752\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{923521}$	57
risch	$\frac{\frac{834400}{29791}x^5+\frac{1251600}{29791}x^4+\frac{7163932}{148955}x^3+\frac{4260199}{148955}x^2+\frac{1891636}{148955}x+\frac{1259239}{446865}}{(5x^2+3x+2)^3}+\frac{66752\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{923521}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 125*(33376/148955*x^5+50064/148955*x^4+7163932/18619375*x^3+4260199/1861937
5*x^2+1891636/18619375*x+1259239/55858125)/(5*x^2+3*x+2)^3+66752/923521*arc
tan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.51, size = 76, normalized size = 0.89

$$\frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+5674908\,x+1259239}{446865\,(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\,x+8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000
*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(1
25*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

Fricas [A]
time = 1.88, size = 105, normalized size = 1.24

$$\frac{387996000\,x^5+581994000\,x^4+666245676\,x^3+1001280\,\sqrt{31}\,(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\,x+8)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+396198507\,x^2+175922148\,x+39036409}{13852815\,(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\,x+8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/13852815*(387996000*x^5 + 581994000*x^4 + 666245676*x^3 + 1001280*sqrt(31
)*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*arctan(1/31*
sqrt(31)*(10*x + 3)) + 396198507*x^2 + 175922148*x + 39036409)/(125*x^6 + 2
25*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

Sympy [A]
time = 0.09, size = 83, normalized size = 0.98

$$\frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{55858125x^6 + 100544625x^5 + 127356525x^4 + 92501055x^3 + 50942610x^2 + 16087140x + 3574920} + \frac{66752\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{923521}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**4,x)
```

```
[Out] (12516000*x**5 + 18774000*x**4 + 21491796*x**3 + 12780597*x**2 + 5674908*x
+ 1259239)/(55858125*x**6 + 100544625*x**5 + 127356525*x**4 + 92501055*x**3
 + 50942610*x**2 + 16087140*x + 3574920) + 66752*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/923521
```

Giac [A]
time = 3.65, size = 56, normalized size = 0.66

$$\frac{66752}{923521}\sqrt{31}\,\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right) + \frac{12516000\,x^5 + 18774000\,x^4 + 21491796\,x^3 + 12780597\,x^2 + 5674908\,x + 1259239}{446865\,(5\,x^2+3\,x+2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000
*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(5
*x^2 + 3*x + 2)^3
```

Mupad [B]
time = 3.47, size = 75, normalized size = 0.88

$$\frac{66752\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{923521} + \frac{\frac{33376\,x^5}{148955} + \frac{50064\,x^4}{148955} + \frac{7163932\,x^3}{18619375} + \frac{4260199\,x^2}{18619375} + \frac{1891636\,x}{18619375} + \frac{1259239}{55858125}}{x^6 + \frac{9\,x^5}{5} + \frac{57\,x^4}{25} + \frac{207\,x^3}{125} + \frac{114\,x^2}{125} + \frac{36\,x}{125} + \frac{8}{125}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] (66752*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/923521 + ((1891
636*x)/18619375 + (4260199*x^2)/18619375 + (7163932*x^3)/18619375 + (50064*
x^4)/148955 + (33376*x^5)/148955 + 1259239/55858125)/((36*x)/125 + (114*x^2
)/125 + (207*x^3)/125 + (57*x^4)/25 + (9*x^5)/5 + x^6 + 8/125)
```

3.30 $\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx$

Optimal. Leaf size=96

$$432x+1080x^2+2856x^3+5144x^4+\frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}+\frac{94881x^8}{8}+\frac{103583x^9}{9}+\frac{75311x^{10}}{10}+\frac{68583x^{11}}{11}+$$

```
[Out] 432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881/
8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^13+
2250/7*x^14+1000/3*x^15
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{1671\}$

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+\frac{43083x^5}{5}+5144x^4+2856x^3+1080x^2+432x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = \int (432+2160x+8568x^2+20576x^3+43083x^4+64529x^5+91349$$
$$= 432x+1080x^2+2856x^3+5144x^4+\frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x}{7}$$

Mathematica [A]
time = 0.00, size = 96, normalized size = 1.00

$$432x+1080x^2+2856x^3+5144x^4+\frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}+\frac{94881x^8}{8}+\frac{103583x^9}{9}+\frac{75311x^{10}}{10}+\frac{68583x^{11}}{11}+\frac{30395x^{12}}{12}+\frac{27050x^{13}}{13}+\frac{2250x^{14}}{7}+\frac{1000x^{15}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]

[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3

Maple [A]
time = 0.10, size = 75, normalized size = 0.78

method	result
gosper	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{75311}{10}x^{10}$
default	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{75311}{10}x^{10}$
norman	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{75311}{10}x^{10}$
risch	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{75311}{10}x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)

[Out] 432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881/
8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^13+
2250/7*x^14+1000/3*x^15

Maxima [A]
time = 0.27, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="maxima")

[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x

Fricas [A]
time = 2.05, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="fricas")

[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x

Sympy [A]
time = 0.02, size = 92, normalized size = 0.96

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^{9}}{9}+\frac{94881x^{8}}{8}+\frac{91349x^{7}}{7}+\frac{64529x^{6}}{6}+\frac{43083x^{5}}{5}+5144x^{4}+2856x^{3}+1080x^{2}+432x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**4,x)
```

```
[Out] 1000*x**15/3 + 2250*x**14/7 + 27050*x**13/13 + 30395*x**12/12 + 68583*x**11
/11 + 75311*x**10/10 + 103583*x**9/9 + 94881*x**8/8 + 91349*x**7/7 + 64529*
x**6/6 + 43083*x**5/5 + 5144*x**4 + 2856*x**3 + 1080*x**2 + 432*x
```

Giac [A]
time = 3.76, size = 74, normalized size = 0.77

$$\frac{1000}{3}\,x^{15}+\frac{2250}{7}\,x^{14}+\frac{27050}{13}\,x^{13}+\frac{30395}{12}\,x^{12}+\frac{68583}{11}\,x^{11}+\frac{75311}{10}\,x^{10}+\frac{103583}{9}\,x^{9}+\frac{94881}{8}\,x^{8}+\frac{91349}{7}\,x^{7}+\frac{64529}{6}\,x^{6}+\frac{43083}{5}\,x^{5}+5144\,x^{4}+2856\,x^{3}+1080\,x^{2}+432\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

Mupad [B]
time = 0.12, size = 74, normalized size = 0.77

$$\frac{1000\,x^{15}}{3}+\frac{2250\,x^{14}}{7}+\frac{27050\,x^{13}}{13}+\frac{30395\,x^{12}}{12}+\frac{68583\,x^{11}}{11}+\frac{75311\,x^{10}}{10}+\frac{103583\,x^{9}}{9}+\frac{94881\,x^{8}}{8}+\frac{91349\,x^{7}}{7}+\frac{64529\,x^{6}}{6}+\frac{43083\,x^{5}}{5}+5144\,x^{4}+2856\,x^{3}+1080\,x^{2}+432\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

3.31 $\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx$

Optimal. Leaf size=82

$$216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{7869x^8}{8}+\frac{3316x^9}{3}+\frac{3061x^{10}}{10}+\frac{4830x^{11}}{11}+25x^{12}+$$

```
[Out] 216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*x
^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1671}

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+216x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx = \int \left(216+756x+2610x^2+4483x^3+8292x^4+8619x^5+12016x^6\right.$$

$$=216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}$$

Mathematica [A]
time = 0.00, size = 82, normalized size = 1.00

$$216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{7869x^8}{8}+\frac{3316x^9}{3}+\frac{3061x^{10}}{10}+\frac{4830x^{11}}{11}+25x^{12}+\frac{1000x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

Maple [A]
time = 0.11, size = 65, normalized size = 0.79

method	result
gosper	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+\frac{4830}{11}x^{11}$
default	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+\frac{4830}{11}x^{11}$
norman	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+\frac{4830}{11}x^{11}$
risch	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+\frac{4830}{11}x^{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*x
^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^9+\frac{7869}{8}x^8+\frac{12016}{7}x^7+\frac{2873}{2}x^6+\frac{8292}{5}x^5+\frac{4483}{4}x^4+870x^3+378x^2+216x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

Fricas [A]
time = 2.25, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^9+\frac{7869}{8}x^8+\frac{12016}{7}x^7+\frac{2873}{2}x^6+\frac{8292}{5}x^5+\frac{4483}{4}x^4+870x^3+378x^2+216x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

Sympy [A]
time = 0.02, size = 78, normalized size = 0.95

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+216x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**3,x)
```

```
[Out] 1000*x**13/13 + 25*x**12 + 4830*x**11/11 + 3061*x**10/10 + 3316*x**9/3 + 78
69*x**8/8 + 12016*x**7/7 + 2873*x**6/2 + 8292*x**5/5 + 4483*x**4/4 + 870*x*
*3 + 378*x**2 + 216*x
```

Giac [A]
time = 4.37, size = 64, normalized size = 0.78

$$\frac{1000}{13}\,x^{13}+25\,x^{12}+\frac{4830}{11}\,x^{11}+\frac{3061}{10}\,x^{10}+\frac{3316}{3}\,x^9+\frac{7869}{8}\,x^8+\frac{12016}{7}\,x^7+\frac{2873}{2}\,x^6+\frac{8292}{5}\,x^5+\frac{4483}{4}\,x^4+870\,x^3+378\,x^2+216\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

Mupad [B]
time = 0.08, size = 64, normalized size = 0.78

$$\frac{1000\,x^{13}}{13}+25\,x^{12}+\frac{4830\,x^{11}}{11}+\frac{3061\,x^{10}}{10}+\frac{3316\,x^9}{3}+\frac{7869\,x^8}{8}+\frac{12016\,x^7}{7}+\frac{2873\,x^6}{2}+\frac{8292\,x^5}{5}+\frac{4483\,x^4}{4}+870\,x^3+378\,x^2+216\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

3.32 $\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx$

Optimal. Leaf size=68

$$108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+\frac{922x^9}{9}-6x^{10}+\frac{200x^{11}}{11}$$

```
[Out] 108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+92
2/9*x^9-6*x^10+200/11*x^11
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1671}

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx &= \int \left(108+216x+711x^2+635x^3+1416x^4+598x^5+1571x^6+83x\right. \\ &= 108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 68, normalized size = 1.00

$$108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+\frac{922x^9}{9}-6x^{10}+\frac{200x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

Maple [A]
time = 0.11, size = 55, normalized size = 0.81

method	result	size
gosper	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$	55
default	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$	55
norman	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$	55
risch	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+92
2/9*x^9-6*x^10+200/11*x^11
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.79

$$\frac{200}{11}\,x^{11}-6\,x^{10}+\frac{922}{9}\,x^{9}+\frac{83}{8}\,x^{8}+\frac{1571}{7}\,x^{7}+\frac{299}{3}\,x^{6}+\frac{1416}{5}\,x^{5}+\frac{635}{4}\,x^{4}+237\,x^{3}+108\,x^{2}+108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

Fricas [A]
time = 3.04, size = 54, normalized size = 0.79

$$\frac{200}{11}\,x^{11}-6\,x^{10}+\frac{922}{9}\,x^{9}+\frac{83}{8}\,x^{8}+\frac{1571}{7}\,x^{7}+\frac{299}{3}\,x^{6}+\frac{1416}{5}\,x^{5}+\frac{635}{4}\,x^{4}+237\,x^{3}+108\,x^{2}+108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

Sympy [A]
time = 0.01, size = 65, normalized size = 0.96

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^{9}}{9}+\frac{83x^{8}}{8}+\frac{1571x^{7}}{7}+\frac{299x^{6}}{3}+\frac{1416x^{5}}{5}+\frac{635x^{4}}{4}+237x^{3}+108x^{2}+108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**2,x)
```

```
[Out] 200*x**11/11 - 6*x**10 + 922*x**9/9 + 83*x**8/8 + 1571*x**7/7 + 299*x**6/3
+ 1416*x**5/5 + 635*x**4/4 + 237*x**3 + 108*x**2 + 108*x
```

Giac [A]
time = 3.63, size = 54, normalized size = 0.79

$$\frac{200}{11}\,x^{11}-6\,x^{10}+\frac{922}{9}\,x^{9}+\frac{83}{8}\,x^{8}+\frac{1571}{7}\,x^{7}+\frac{299}{3}\,x^{6}+\frac{1416}{5}\,x^{5}+\frac{635}{4}\,x^{4}+237\,x^{3}+108\,x^{2}+108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

Mupad [B]
time = 0.05, size = 54, normalized size = 0.79

$$\frac{200\,x^{11}}{11}-6\,x^{10}+\frac{922\,x^{9}}{9}+\frac{83\,x^{8}}{8}+\frac{1571\,x^{7}}{7}+\frac{299\,x^{6}}{3}+\frac{1416\,x^{5}}{5}+\frac{635\,x^{4}}{4}+237\,x^{3}+108\,x^{2}+108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

3.33 $\int (3-x+2x^2)^3 (2+3x+5x^2)\, dx$

<u>**Optimal**</u>. Leaf size=56

$$54x+\frac{27x^2}{2}+60x^3-5x^4+\frac{288x^5}{5}-\frac{83x^6}{6}+\frac{190x^7}{7}-\frac{9x^8}{2}+\frac{40x^9}{9}$$

```
[Out] 54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9
```

<u>**Rubi**</u> [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1671}

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

<u>Rule 1671</u>

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

<u>Rubi steps</u>

$$\begin{aligned}\int (3-x+2x^2)^3 (2+3x+5x^2)\, dx &= \int (54+27x+180x^2-20x^3+288x^4-83x^5+190x^6-36x^7+40 \\ &= 54x+\frac{27x^2}{2}+60x^3-5x^4+\frac{288x^5}{5}-\frac{83x^6}{6}+\frac{190x^7}{7}-\frac{9x^8}{2}+\frac{40x^9}{9}\end{aligned}$$

<u>**Mathematica**</u> [A]
time = 0.00, size = 56, normalized size = 1.00

$$54x+\frac{27x^2}{2}+60x^3-5x^4+\frac{288x^5}{5}-\frac{83x^6}{6}+\frac{190x^7}{7}-\frac{9x^8}{2}+\frac{40x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7 - (9*x^8)/2 + (40*x^9)/9

Maple [A]
time = 0.10, size = 45, normalized size = 0.80

method	result	size
gosper	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
default	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
norman	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
risch	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)

[Out] 54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9

Maxima [A]
time = 0.28, size = 44, normalized size = 0.79

$$\frac{40}{9}\,x^9-\frac{9}{2}\,x^8+\frac{190}{7}\,x^7-\frac{83}{6}\,x^6+\frac{288}{5}\,x^5-5\,x^4+60\,x^3+\frac{27}{2}\,x^2+54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="maxima")

[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x

Fricas [A]
time = 3.05, size = 44, normalized size = 0.79

$$\frac{40}{9}\,x^9-\frac{9}{2}\,x^8+\frac{190}{7}\,x^7-\frac{83}{6}\,x^6+\frac{288}{5}\,x^5-5\,x^4+60\,x^3+\frac{27}{2}\,x^2+54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="fricas")

[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x

Sympy [A]
time = 0.01, size = 53, normalized size = 0.95

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2),x)
```

```
[Out] 40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x*
*3 + 27*x**2/2 + 54*x
```

Giac [A]
time = 5.85, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

Mupad [B]
time = 0.03, size = 44, normalized size = 0.79

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2),x)
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

3.34 $\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx$

Optimal. Leaf size=70

$$\frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}+\frac{328757\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}}-\frac{158389\log\left(2+3x+5x^2\right)}{31250}$$

```
[Out] 49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln(
5*x^2+3*x+2)+328757/484375*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{328757\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}+\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] (49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/2
5 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2 +
 3*x + 5*x^2])/31250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx &= \int \left(\frac{49508}{3125}-\frac{7451x}{625}+\frac{1222x^2}{125}-\frac{84x^3}{25}+\frac{8x^4}{5}-\frac{1331(11+119x)}{3125\,(2+3x+5x^2)}\right)dx \\
&= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{1331\int \frac{11+119x}{2+3x+5x^2}\,dx}{3125} \\
&= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\int \frac{3+10x}{2+3x+5x^2}\,dx}{31250}+\frac{328757\int \frac{}{2+3}}{3125} \\
&= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\log(2+3x+5x^2)}{31250}-\frac{328757}{} \\
&= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}+\frac{328757\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}}-\frac{158389\log}{3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 0.90

$$\frac{1972542\sqrt{31}\,\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)+31(5x(297048-111765x+61100x^2-15750x^3+6000x^4)-475167\log(2+3x+5x^2))}{2906250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] (1972542*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 31*(5*x*(297048 - 111765*x
+ 61100*x^2 - 15750*x^3 + 6000*x^4) - 475167*Log[2 + 3*x + 5*x^2]))/2906250
```

Maple [A]
time = 0.12, size = 54, normalized size = 0.77

method	result	si

default	$\frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\ln(5x^2+3x+2)}{31250}+\frac{328757\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{484375}$	54
risch	$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}+\frac{49508x}{3125}-\frac{158389\ln(100x^2+60x+40)}{31250}+\frac{328757\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{484375}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln(
5*x^2+3*x+2)+328757/484375*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.50, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^5-\frac{21}{25}\,x^4+\frac{1222}{375}\,x^3-\frac{7451}{1250}\,x^2+\frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

Fricas [A]
time = 2.36, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^5-\frac{21}{25}\,x^4+\frac{1222}{375}\,x^3-\frac{7451}{1250}\,x^2+\frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

Sympy [A]
time = 0.05, size = 76, normalized size = 1.09

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}+\frac{49508x}{3125}-\frac{158389\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{31250}+\frac{328757\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{484375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] 8*x**5/25 - 21*x**4/25 + 1222*x**3/375 - 7451*x**2/1250 + 49508*x/3125 - 15
8389*log(x**2 + 3*x/5 + 2/5)/31250 + 328757*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/484375
```

Giac [A]
time = 5.40, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^{5}-\frac{21}{25}\,x^{4}+\frac{1222}{375}\,x^{3}-\frac{7451}{1250}\,x^{2}+\frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^{2}+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

Mupad [B]
time = 0.04, size = 55, normalized size = 0.79

$$\frac{49508\,x}{3125}-\frac{158389\,\ln\left(5\,x^{2}+3\,x+2\right)}{31250}+\frac{328757\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{484375}-\frac{7451\,x^{2}}{1250}+\frac{1222\,x^{3}}{375}-\frac{21\,x^{4}}{25}+\frac{8\,x^{5}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2),x)
```

```
[Out] (49508*x)/3125 - (158389*log(3*x + 5*x^2 + 2))/31250 + (328757*31^(1/2)*ata
n((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/484375 - (7451*x^2)/1250 + (1222*x
^3)/375 - (21*x^4)/25 + (8*x^5)/25
```

3.35 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx$

Optimal. Leaf size=77

$$\frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\,(2+3x+5x^2)}+\frac{3819607\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}}-\frac{10769\log\left(2+3x+5x^2\right)}{6250}$$

```
[Out] 1466/625*x-54/125*x^2+8/75*x^3+1331/96875*(443+247*x)/(5*x^2+3*x+2)-10769/6
250*ln(5*x^2+3*x+2)+3819607/3003125*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{3819607\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}+\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875\,(5x^2+3x+2)}-\frac{10769\log\left(5x^2+3x+2\right)}{6250}+\frac{1466x}{625}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx &= \frac{1331(443+247x)}{96875\,(2+3x+5x^2)} + \frac{1}{31}\int \frac{\frac{372701}{625}-\frac{230981x}{625}+\frac{37882x^2}{125}-\frac{2604x^3}{25}+\frac{248x^4}{5}}{2+3x+5x^2}\,dx \\
&= \frac{1331(443+247x)}{96875\,(2+3x+5x^2)} + \frac{1}{31}\int \left(\frac{45446}{625}-\frac{3348x}{125}+\frac{248x^2}{25}+\frac{121(2329-2759x)}{625\,(2+3x+5x^2)}\right) \\
&= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\,(2+3x+5x^2)}+\frac{121\int \frac{2329-2759x}{2+3x+5x^2}\,dx}{19375} \\
&= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\,(2+3x+5x^2)}-\frac{10769\int \frac{3+10x}{2+3x+5x^2}\,dx}{6250}+\frac{3819607\int}{19} \\
&= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\,(2+3x+5x^2)}-\frac{10769\log(2+3x+5x^2)}{6250}-\frac{38196}{} \\
&= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\,(2+3x+5x^2)}+\frac{3819607\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}}-\frac{10769}{}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 77, normalized size = 1.00

$$\frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\,(2+3x+5x^2)}+\frac{3819607\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}}-\frac{10769\log\left(2+3x+5x^2\right)}{6250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

Maple [A]
time = 0.12, size = 61, normalized size = 0.79

method	result	size
risch	$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}+\frac{\frac{328757x}{484375}+\frac{589633}{484375}}{x^2+\frac{3}{5}x+\frac{2}{5}}-\frac{10769\ln\left(100x^2+60x+40\right)}{6250}+\frac{3819607\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{3003125}$	60
default	$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}-\frac{121\left(-\frac{2717x}{775}-\frac{4873}{775}\right)}{625\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{10769\ln\left(5x^2+3x+2\right)}{6250}+\frac{3819607\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{3003125}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/75*x^3-54/125*x^2+1466/625*x-121/625*(-2717/775*x-4873/775)/(x^2+3/5*x+2/
5)-10769/6250*ln(5*x^2+3*x+2)+3819607/3003125*arctan(1/31*(3+10*x)*31^(1/2)
)*31^(1/2)
```

Maxima [A]
time = 0.51, size = 62, normalized size = 0.81

$$\frac{8}{75}\,x^3-\frac{54}{125}\,x^2+\frac{3819607}{3003125}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{1466}{625}\,x+\frac{1331\,(247\,x+443)}{96875\,(5\,x^2+3\,x+2)}-\frac{10769}{6250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x
 + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/62
50*log(5*x^2 + 3*x + 2)
```

Fricas [A]
time = 2.30, size = 88, normalized size = 1.14

$$\frac{9610000\,x^5-33154500\,x^4+191815600\,x^3+22917642\,\sqrt{31}\,(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+111226140\,x^2-31047027\,(5\,x^2+3\,x+2)\log\left(5\,x^2+3\,x+2\right)+145678362\,x+109671738}{18018750\,(5\,x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/18018750*(9610000*x^5 - 33154500*x^4 + 191815600*x^3 + 22917642*sqrt(31)*
(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 111226140*x^2 - 310470
27*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 145678362*x + 109671738)/(5*x^2
 + 3*x + 2)
```

Sympy [A]
time = 0.07, size = 78, normalized size = 1.01

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{328757x + 589633}{484375x^2 + 290625x + 193750} - \frac{10769\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{6250} + \frac{3819607\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{3003125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

```
[Out] 8*x**3/75 - 54*x**2/125 + 1466*x/625 + (328757*x + 589633)/(484375*x**2 + 2
90625*x + 193750) - 10769*log(x**2 + 3*x/5 + 2/5)/6250 + 3819607*sqrt(31)*a
tan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3003125
```

Giac [A]
time = 2.33, size = 62, normalized size = 0.81

$$\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10x+3)\right) + \frac{1466}{625}x + \frac{1331\,(247x+443)}{96875\,(5x^2+3x+2)} - \frac{10769}{6250}\log\left(5x^2+3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x
 + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/62
50*log(5*x^2 + 3*x + 2)
```

Mupad [B]
time = 3.43, size = 61, normalized size = 0.79

$$\frac{1466\,x}{625} - \frac{10769\,\ln\left(5x^2+3x+2\right)}{6250} + \frac{\frac{328757\,x}{484375} + \frac{589633}{484375}}{x^2 + \frac{3x}{5} + \frac{2}{5}} + \frac{3819607\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{3003125} - \frac{54\,x^2}{125} + \frac{8\,x^3}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (1466*x)/625 - (10769*log(3*x + 5*x^2 + 2))/6250 + ((328757*x)/484375 + 589
633/484375)/((3*x)/5 + x^2 + 2/5) + (3819607*31^(1/2)*atan((10*31^(1/2)*x)/
31 + (3*31^(1/2))/31))/3003125 - (54*x^2)/125 + (8*x^3)/75
```

3.36 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=84

$$\frac{8x}{125}+\frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)}+\frac{11341176\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}}-\frac{66}{625}\log\left(2+3x+5x^2\right)$$

```
[Out] 8/125*x+1331/193750*(443+247*x)/(5*x^2+3*x+2)^2+121/6006250*(188381+342840*
x)/(5*x^2+3*x+2)-66/625*ln(5*x^2+3*x+2)+11341176/18619375*arctan(1/31*(3+10
*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{11341176\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}+\frac{121(342840x+188381)}{6006250\,(5x^2+3x+2)}+\frac{1331(247x+443)}{193750\,(5x^2+3x+2)^2}-\frac{66}{625}\log\left(5x^2+3x+2\right)+\frac{8x}{125}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (8*x)/125 + (1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + (121*(18838
1 + 342840*x))/(6006250*(2 + 3*x + 5*x^2)) + (11341176*ArcTan[(3 + 10*x)/Sq
rt[31]])/(600625*Sqrt[31]) - (66*Log[2 + 3*x + 5*x^2])/625
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{1}{62}\int \frac{\frac{4055767}{3125}-\frac{461962x}{625}+\frac{75764x^2}{125}-\frac{5208x^3}{25}+\frac{496x^4}{5}}{(2+3x+5x^2)^2}\,dx$$

$$= \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)}+\frac{\int \frac{\frac{2222876}{125}-\frac{207576x}{125}+\frac{15376x^2}{25}}{2+3x+5x^2}\,dx}{1922}$$

$$= \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)}+\frac{\int \left(\frac{15376}{125}+\frac{132(16607-1922x)}{125(2+3x+5x^2)}\right) dx}{1922}$$

$$= \frac{8x}{125}+\frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)}+\frac{66\int \frac{16607-1922x}{2+3x+5x^2}\,dx}{120125}$$

$$= \frac{8x}{125}+\frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)}-\frac{66}{625}\int \frac{3+10x}{2+3x+5x^2}\,dx$$

$$= \frac{8x}{125}+\frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)}-\frac{66}{625}\log\left(2+3x+5x^2\right)$$

$$= \frac{8x}{125}+\frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2}+\frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)}+\frac{11341176\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.93

$$\frac{11916400x+\frac{1279091(443+247x)}{(2+3x+5x^2)^2}+\frac{3751(188381+342840x)}{2+3x+5x^2}+113411760\sqrt{31}\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)-19662060\log\left(2+3x+5x^2\right)}{186193750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11916400*x + (1279091*(443 + 247*x))/(2 + 3*x + 5*x^2)^2 + (3751*(188381 +
 342840*x))/(2 + 3*x + 5*x^2) + 113411760*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[3
1]] - 19662060*Log[2 + 3*x + 5*x^2])/186193750
```

Maple [A]
time = 0.13, size = 63, normalized size = 0.75

method	result	size
default	$\frac{8x}{125}-\frac{11\left(-\frac{377124}{24025}x^3-\frac{866987}{48050}x^2-\frac{293711}{24025}x-\frac{232243}{48050}\right)}{5(5x^2+3x+2)^2}-\frac{66\ln(5x^2+3x+2)}{625}+\frac{11341176\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{18619375}$	63

risch	$\frac{8x}{125}+\frac{\frac{4148364}{120125}x^3+\frac{9536857}{240250}x^2+\frac{3230821}{120125}x+\frac{2554673}{240250}}{(5x^2+3x+2)^2}-\frac{66\ln(100x^2+60x+40)}{625}+\frac{11341176\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{18619375}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/125*x-11/5*(-377124/24025*x^3-866987/48050*x^2-293711/24025*x-232243/4805
0)/(5*x^2+3*x+2)^2-66/625*ln(5*x^2+3*x+2)+11341176/18619375*arctan(1/31*(3+
10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.50, size = 72, normalized size = 0.86

$$\frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{8}{125}x+\frac{121(68568x^3+78817x^2+53402x+21113)}{240250(25x^4+30x^3+29x^2+12x+4)}-\frac{66}{625}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121
/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(25*x^4 + 30*x^3 + 29*x^2
 + 12*x + 4) - 66/625*log(5*x^2 + 3*x + 2)
```

Fricas [A]
time = 3.41, size = 118, normalized size = 1.40

$$\frac{59582000x^5+71498400x^4+1355107960x^3+22682352\sqrt{31}(25x^4+30x^3+29x^2+12x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+1506812195x^2-3932412(25x^4+30x^3+29x^2+12x+4)\log(5x^2+3x+2)+1011087630x+395974315}{37238750(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/37238750*(59582000*x^5 + 71498400*x^4 + 1355107960*x^3 + 22682352*sqrt(31
)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) +
1506812195*x^2 - 3932412*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 +
3*x + 2) + 1011087630*x + 395974315)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [A]
time = 0.09, size = 85, normalized size = 1.01

$$\frac{8x}{125}+\frac{8296728x^3+9536857x^2+6461642x+2554673}{6006250x^4+7207500x^3+6967250x^2+2883000x+961000}-\frac{66\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{625}+\frac{11341176\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{18619375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

```
[Out] 8*x/125 + (8296728*x**3 + 9536857*x**2 + 6461642*x + 2554673)/(6006250*x**4
 + 7207500*x**3 + 6967250*x**2 + 2883000*x + 961000) - 66*log(x**2 + 3*x/5
+ 2/5)/625 + 11341176*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/18619
375
```

Giac [A]
time = 5.61, size = 62, normalized size = 0.74

$$\frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{8}{125}\,x+\frac{121\,(68568\,x^3+78817\,x^2+53402\,x+21113)}{240250\,(5\,x^2+3\,x+2)^2}-\frac{66}{625}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121
/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(5*x^2 + 3*x + 2)^2 - 66/
625*log(5*x^2 + 3*x + 2)
```

Mupad [B]
time = 3.43, size = 71, normalized size = 0.85

$$\frac{8\,x}{125}-\frac{66\,\ln\left(5\,x^2+3\,x+2\right)}{625}+\frac{11341176\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{18619375}+\frac{\frac{4148364\,x^3}{3003125}+\frac{9536857\,x^2}{6006250}+\frac{3230821\,x}{3003125}+\frac{2554673}{6006250}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (8*x)/125 - (66*log(3*x + 5*x^2 + 2))/625 + (11341176*31^(1/2)*atan((10*31^
(1/2)*x)/31 + (3*31^(1/2))/31))/18619375 + ((3230821*x)/3003125 + (9536857*
x^2)/6006250 + (4148364*x^3)/3003125 + 2554673/6006250)/((12*x)/25 + (29*x^
2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

3.37 $\int \frac{\left(2+3x+5x^2\right)^4}{3-x+2x^2}\,dx$

Optimal. Leaf size=84

$$\frac{122691x}{128}-\frac{28747x^2}{128}-\frac{21229x^3}{96}+\frac{6245x^4}{64}+\frac{1855x^5}{8}+\frac{3625x^6}{24}+\frac{625x^7}{14}+\frac{1156639\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}}+\frac{307461}{512}\log\left(3\right.$$

[Out] 122691/128*x-28747/128*x^2-21229/96*x^3+6245/64*x^4+1855/8*x^5+3625/24*x^6+625/14*x^7+307461/512*ln(2*x^2-x+3)+1156639/5888*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{1156639\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}}+\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

```
[Out] (122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*x
^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \int \left(\frac{122691}{128} - \frac{28747x}{64} - \frac{21229x^2}{32} + \frac{6245x^3}{16} + \frac{9275x^4}{8} + \frac{3625x^5}{4} + \frac{625x^6}{2} - \frac{14641}{128\,(3}\right.$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} - \frac{14641}{128}\int$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512}$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512}$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{1156639}{2}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 0.86

$$\frac{x(2576511-603687x-594412x^2+262290x^3+623280x^4+406000x^5+120000x^6)}{2688} - \frac{1156639\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{256\sqrt{23}} + \frac{307461}{512}\log\left(3-x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

```
[Out] (x*(2576511 - 603687*x - 594412*x^2 + 262290*x^3 + 623280*x^4 + 406000*x^5
+ 120000*x^6))/2688 - (1156639*ArcTan[(-1 + 4*x)/Sqrt[23]])/(256*Sqrt[23])
+ (307461*Log[3 - x + 2*x^2])/512
```

Maple [A]
time = 0.20, size = 64, normalized size = 0.76

method	result
default	$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\ln(2x^2-x+3)}{512}-\frac{1156639\sqrt{23}\,\mathrm{arct}}{58}$
risch	$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\ln(16x^2-8x+24)}{512}-\frac{1156639\sqrt{23}\,\mathrm{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3),x,method=_RETURNVERBOSE)

[Out] 625/14*x^7+3625/24*x^6+1855/8*x^5+6245/64*x^4-21229/96*x^3-28747/128*x^2+12 2691/128*x+307461/512*ln(2*x^2-x+3)-1156639/5888*23^(1/2)*arctan(1/23*(4*x- 1)*23^(1/2))

Maxima [A]
time = 0.50, size = 63, normalized size = 0.75

$$\frac{625}{14}\,x^7+\frac{3625}{24}\,x^6+\frac{1855}{8}\,x^5+\frac{6245}{64}\,x^4-\frac{21229}{96}\,x^3-\frac{28747}{128}\,x^2-\frac{1156639}{5888}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{122691}{128}\,x+\frac{307461}{512}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="maxima")

[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/ 128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12 8*x + 307461/512*log(2*x^2 - x + 3)

Fricas [A]
time = 1.72, size = 63, normalized size = 0.75

$$\frac{625}{14}\,x^7+\frac{3625}{24}\,x^6+\frac{1855}{8}\,x^5+\frac{6245}{64}\,x^4-\frac{21229}{96}\,x^3-\frac{28747}{128}\,x^2-\frac{1156639}{5888}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{122691}{128}\,x+\frac{307461}{512}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="fricas")

[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/ 128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12 8*x + 307461/512*log(2*x^2 - x + 3)

Sympy [A]
time = 0.06, size = 87, normalized size = 1.04

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{512}-\frac{1156639\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3),x)
```

```
[Out] 625*x**7/14 + 3625*x**6/24 + 1855*x**5/8 + 6245*x**4/64 - 21229*x**3/96 - 2
8747*x**2/128 + 122691*x/128 + 307461*log(x**2 - x/2 + 3/2)/512 - 1156639*s
qrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/5888
```

Giac [**A**]
time = 5.12, size = 63, normalized size = 0.75

$$\frac{625}{14}\,x^{7}+\frac{3625}{24}\,x^{6}+\frac{1855}{8}\,x^{5}+\frac{6245}{64}\,x^{4}-\frac{21229}{96}\,x^{3}-\frac{28747}{128}\,x^{2}-\frac{1156639}{5888}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{122691}{128}\,x+\frac{307461}{512}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/
128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12
8*x + 307461/512*log(2*x^2 - x + 3)
```

Mupad [**B**]
time = 3.44, size = 65, normalized size = 0.77

$$\frac{122691\,x}{128}+\frac{307461\,\ln\left(2\,x^2-x+3\right)}{512}-\frac{1156639\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{5888}-\frac{28747\,x^2}{128}-\frac{21229\,x^3}{96}+\frac{6245\,x^4}{64}+\frac{1855\,x^5}{8}+\frac{3625\,x^6}{24}+\frac{625\,x^7}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3),x)
```

```
[Out] (122691*x)/128 + (307461*log(2*x^2 - x + 3))/512 - (1156639*23^(1/2)*atan((
4*23^(1/2)*x)/23 - 23^(1/2)/23))/5888 - (28747*x^2)/128 - (21229*x^3)/96 +
(6245*x^4)/64 + (1855*x^5)/8 + (3625*x^6)/24 + (625*x^7)/14
```

3.38 $\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{1331}{128}\log\left(3-x+2x^2\right)$$

```
[Out] -4795/32*x-829/32*x^2+965/24*x^3+575/16*x^4+25/2*x^5+1331/128*ln(2*x^2-x+3)
-59895/1472*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1671, 648, 632, 210, 642}

$$-\frac{59895\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

```
[Out] (-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (5
9895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/
128
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx &= \int \left(-\frac{4795}{32}-\frac{829x}{16}+\frac{965x^2}{8}+\frac{575x^3}{4}+\frac{125x^4}{2}+\frac{1331(11+x)}{32\,(3-x+2x^2)}\right)dx\\ &= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{32}\int \frac{11+x}{3-x+2x^2}\,dx\\ &= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{128}\int \frac{-1+4x}{3-x+2x^2}\,dx+\frac{59895}{128}\int\\ &= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{128}\log\left(3-x+2x^2\right)-\frac{59895}{64}\text{Subs}\\ &= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{1331}{128}\log(3\end{aligned}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 0.90

$$\frac{59895\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{1}{384}\left(4x\left(-14385-2487x+3860x^2+3450x^3+1200x^4\right)+3993\log\left(3-x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

```
[Out] (59895*ArcTan[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (4*x*(-14385 - 2487*x +
 3860*x^2 + 3450*x^3 + 1200*x^4) + 3993*Log[3 - x + 2*x^2])/384
```

Maple [A]
time = 0.13, size = 54, normalized size = 0.77

method	result	size
default	$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\ln(2x^2-x+3)}{128}+\frac{59895\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{1472}$	54
risch	$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\ln(16x^2-8x+24)}{128}+\frac{59895\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{1472}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 25/2*x^5+575/16*x^4+965/24*x^3-829/32*x^2-4795/32*x+1331/128*ln(2*x^2-x+3)+
59895/1472*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.50, size = 53, normalized size = 0.76

$$\frac{25}{2}\,x^5+\frac{575}{16}\,x^4+\frac{965}{24}\,x^3-\frac{829}{32}\,x^2+\frac{59895}{1472}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{4795}{32}\,x+\frac{1331}{128}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

Fricas [A]
time = 2.62, size = 53, normalized size = 0.76

$$\frac{25}{2}\,x^5+\frac{575}{16}\,x^4+\frac{965}{24}\,x^3-\frac{829}{32}\,x^2+\frac{59895}{1472}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{4795}{32}\,x+\frac{1331}{128}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

Sympy [A]
time = 0.05, size = 73, normalized size = 1.04

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{59895\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3),x)
```

```
[Out] 25*x**5/2 + 575*x**4/16 + 965*x**3/24 - 829*x**2/32 - 4795*x/32 + 1331*log(
x**2 - x/2 + 3/2)/128 + 59895*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/
1472
```

Giac [A]
time = 4.70, size = 53, normalized size = 0.76

$$\frac{25}{2}\,x^{5}+\frac{575}{16}\,x^{4}+\frac{965}{24}\,x^{3}-\frac{829}{32}\,x^{2}+\frac{59895}{1472}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{4795}{32}\,x+\frac{1331}{128}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

Mupad [B]
time = 0.04, size = 55, normalized size = 0.79

$$\frac{1331\,\ln\left(2\,x^2-x+3\right)}{128}-\frac{4795\,x}{32}+\frac{59895\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}-\frac{829\,x^2}{32}+\frac{965\,x^3}{24}+\frac{575\,x^4}{16}+\frac{25\,x^5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3),x)
```

```
[Out] (1331*log(2*x^2 - x + 3))/128 - (4795*x)/32 + (59895*23^(1/2)*atan((4*23^(1
/2)*x)/23 - 23^(1/2)/23))/1472 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16
 + (25*x^5)/2
```

3.39 $\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx$

Optimal. Leaf size=56

$$\frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}-\frac{363}{32}\log\left(3-x+2x^2\right)$$

```
[Out] 51/8*x+85/8*x^2+25/6*x^3-363/32*ln(2*x^2-x+3)+847/368*arctan(1/23*(1-4*x)*2
3^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{847\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}+\frac{25x^3}{6}+\frac{85x^2}{8}-\frac{363}{32}\log\left(2x^2-x+3\right)+\frac{51x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] (51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*S
qrt[23]) - (363*Log[3 - x + 2*x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx &= \int \left(\frac{51}{8}+\frac{85x}{4}+\frac{25x^2}{2}-\frac{121(1+3x)}{8(3-x+2x^2)}\right)dx\\ &= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{121}{8}\int \frac{1+3x}{3-x+2x^2}\,dx\\ &= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{363}{32}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{847}{32}\int \frac{1}{3-x+2x^2}\,dx\\ &= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{363}{32}\log\left(3-x+2x^2\right)+\frac{847}{16}\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+\right.\\ &= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}-\frac{363}{32}\log\left(3-x+2x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.93

$$\frac{1}{24}x\left(153+255x+100x^2\right)-\frac{847\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{16\sqrt{23}}-\frac{363}{32}\log\left(3-x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] (x*(153 + 255*x + 100*x^2))/24 - (847*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt
[23]) - (363*Log[3 - x + 2*x^2])/32
```

Maple [A]
time = 0.14, size = 44, normalized size = 0.79

method	result	size
default	$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\ln(2x^2-x+3)}{32}-\frac{847\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{368}$	44
risch	$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\ln(16x^2-8x+24)}{32}-\frac{847\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{368}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 25/6*x^3+85/8*x^2+51/8*x-363/32*ln(2*x^2-x+3)-847/368*23^(1/2)*arctan(1/23*
(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.52, size = 43, normalized size = 0.77

$$\frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

Fricas [A]
time = 1.95, size = 43, normalized size = 0.77

$$\frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

Sympy [A]
time = 0.05, size = 60, normalized size = 1.07

$$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{847\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{368}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3),x)
```

```
[Out] 25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*log(x**2 - x/2 + 3/2)/32 - 847*sqrt(23
)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/368
```

Giac [A]
time = 2.13, size = 43, normalized size = 0.77

$$\frac{25}{6}\,x^{3}+\frac{85}{8}\,x^{2}-\frac{847}{368}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

Mupad [B]
time = 3.44, size = 45, normalized size = 0.80

$$\frac{51\,x}{8}-\frac{363\,\ln\left(2\,x^{2}-x+3\right)}{32}-\frac{847\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{368}+\frac{85\,x^{2}}{8}+\frac{25\,x^{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3),x)
```

```
[Out] (51*x)/8 - (363*log(2*x^2 - x + 3))/32 - (847*23^(1/2)*atan((4*23^(1/2)*x)/
23 - 23^(1/2)/23))/368 + (85*x^2)/8 + (25*x^3)/6
```

3.40 $\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx$

Optimal. Leaf size=42

$$\frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(3-x+2x^2\right)$$

```
[Out] 5/2*x+11/8*ln(2*x^2-x+3)+33/92*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{33\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

```
[Out] (5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2*
x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx &= \int \left(\frac{5}{2}-\frac{11(1-x)}{2\left(3-x+2x^2\right)}\right)dx \\
&= \frac{5x}{2}-\frac{11}{2}\int \frac{1-x}{3-x+2x^2}\,dx \\
&= \frac{5x}{2}+\frac{11}{8}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{33}{8}\int \frac{1}{3-x+2x^2}\,dx \\
&= \frac{5x}{2}+\frac{11}{8}\log\left(3-x+2x^2\right)+\frac{33}{4}\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right) \\
&= \frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(3-x+2x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$\frac{5x}{2}-\frac{33\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(3-x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

```
[Out] (5*x)/2 - (33*ArcTan[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2
*x^2])/8
```

Maple [A]
time = 0.11, size = 34, normalized size = 0.81

method	result	size

default	$\frac{5x}{2}+\frac{11\ln\left(2x^{2}-x+3\right)}{8}-\frac{33\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{92}$	34
risch	$\frac{5x}{2}+\frac{11\ln\left(16x^{2}-8x+24\right)}{8}-\frac{33\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{92}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*x+11/8*ln(2*x^2-x+3)-33/92*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.52, size = 33, normalized size = 0.79

$$-\frac{33}{92}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

Fricas [A]
time = 2.17, size = 33, normalized size = 0.79

$$-\frac{33}{92}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

Sympy [A]
time = 0.04, size = 46, normalized size = 1.10

$$\frac{5x}{2}+\frac{11\log\left(x^{2}-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{33\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3),x)
```

[Out] 5*x/2 + 11*log(x**2 - x/2 + 3/2)/8 - 33*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/92

Giac [A]
time = 3.81, size = 33, normalized size = 0.79

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="giac")

[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 - x + 3)

Mupad [B]
time = 0.04, size = 35, normalized size = 0.83

$$\frac{5\,x}{2}+\frac{11\,\ln\left(2\,x^2-x+3\right)}{8}-\frac{33\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3),x)

[Out] (5*x)/2 + (11*log(2*x^2 - x + 3))/8 - (33*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/92

3.41 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx$

Optimal. Leaf size=73

$$\frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}}-\frac{1}{44}\log\left(3-x+2x^2\right)+\frac{1}{44}\log\left(2+3x+5x^2\right)$$

```
[Out] -1/44*ln(2*x^2-x+3)+1/44*ln(5*x^2+3*x+2)+3/506*arctan(1/23*(1-4*x)*23^(1/2)
)*23^(1/2)+13/682*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {994, 648, 632, 210, 642}

$$\frac{3\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (3*ArcTan[(1 - 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 994

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2
)), x_Symbol] :> With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*
f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(c^2*d - b*c*e + b^2*f - a*c*f - (c^2
*e - b*c*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*e^2 - c*d*f - b
*e*f + a*f^2 + (c*e*f - b*f^2)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx &= \frac{1}{242}\int \frac{-11-22x}{3-x+2x^2}\,dx+\frac{1}{242}\int \frac{88+55x}{2+3x+5x^2}\,dx\\ &= -\left(\frac{1}{44}\int \frac{-1+4x}{3-x+2x^2}\,dx\right)+\frac{1}{44}\int \frac{3+10x}{2+3x+5x^2}\,dx-\frac{3}{44}\int \frac{1}{3-x+2\\ &= -\frac{1}{44}\log\left(3-x+2x^2\right)+\frac{1}{44}\log\left(2+3x+5x^2\right)+\frac{3}{22}\text{Subst}\left(\int \frac{1}{-23-\right.\\ &= \frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}}-\frac{1}{44}\log\left(3-x+2x^2\right)+\frac{1}{44}\text{lo}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 1.00

$$-\frac{3\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}}-\frac{1}{44}\log\left(3-x+2x^2\right)+\frac{1}{44}\log\left(2+3x+5x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt
[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

Maple [A]
time = 0.14, size = 60, normalized size = 0.82

method	result	size
default	$-\frac{\ln(2x^2-x+3)}{44}-\frac{3\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{506}+\frac{\ln(5x^2+3x+2)}{44}+\frac{13\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{682}$	60
risch	$\frac{13\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{682}+\frac{\ln(100x^2+60x+40)}{44}-\frac{\ln(16x^2-8x+24)}{44}-\frac{3\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{506}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/44*ln(2*x^2-x+3)-3/506*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))+1/44*ln(5*
x^2+3*x+2)+13/682*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.53, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{1}{44}\log\left(5\,x^2+3\,x+2\right)-\frac{1}{44}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

Fricas [A]
time = 3.07, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{1}{44}\log\left(5\,x^2+3\,x+2\right)-\frac{1}{44}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

Sympy [A]
time = 0.10, size = 83, normalized size = 1.14

$$-\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{44}+\frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{44}-\frac{3\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{506}+\frac{13\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{682}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2),x)
```

```
[Out] -log(x**2 - x/2 + 3/2)/44 + log(x**2 + 3*x/5 + 2/5)/44 - 3*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/506 + 13*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sq
rt(31)/31)/682
```

Giac [A]
time = 2.47, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{1}{44}\log\left(5\,x^2+3\,x+2\right)-\frac{1}{44}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

Mupad [B]
time = 0.19, size = 79, normalized size = 1.08

$$\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{44}+\frac{\sqrt{31}\,13\mathrm{i}}{1364}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{1}{44}+\frac{\sqrt{31}\,13\mathrm{i}}{1364}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 - 1/44) - log(x + (23^(1
/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 + 1/44) - log(x - (31^(1/2)*1i)/10 + 3
/10)*((31^(1/2)*13i)/1364 - 1/44) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(
1/2)*13i)/1364 + 1/44)
```

3.42 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=94

$$\frac{4+65x}{682(2+3x+5x^2)}+\frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}}+\frac{3}{968}\log\left(3-x+2x^2\right)-\frac{3}{968}\log\left(2+3x+5x^2\right)$$

```
[Out] 1/682*(4+65*x)/(5*x^2+3*x+2)+3/968*ln(2*x^2-x+3)-3/968*ln(5*x^2+3*x+2)+7/11
132*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+2891/465124*arctan(1/31*(3+10*x)
*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {988, 1086, 648, 632, 210, 642}

$$\frac{7\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}+\frac{65x+4}{682(5x^2+3x+2)}+\frac{3}{968}\log\left(2x^2-x+3\right)-\frac{3}{968}\log\left(5x^2+3x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + (7*ArcTan[(1 - 4*x)/Sqrt[23]])/(484*Sq
rt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{-1804+1397x-1430x^2}{(3-x+2x^2)(2+3x+5x^2)}\,dx}{7502}$$
$$= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{18755-22506x}{3-x+2x^2}\,dx}{1815484} - \frac{\int \frac{-158026+56265x}{2+3x+5x^2}\,dx}{1815484}$$
$$= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968}\int \frac{-1+4x}{3-x+2x^2}\,dx - \frac{3}{968}\int \frac{3+10x}{2+3x+5x^2}$$
$$= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968}\log\left(3-x+2x^2\right) - \frac{3}{968}\log\left(2+3x+5x\right.$$
$$= \frac{4+65x}{682(2+3x+5x^2)} + \frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{96}$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 1.00

$$\frac{4+65x}{682(2+3x+5x^2)} - \frac{7\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968}\log\left(3-x+2x^2\right) - \frac{3}{968}\log\left(2+3x+5x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) - (7*ArcTan[(-1 + 4*x)/Sqrt[23]])/(484*S
qrt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
 x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

Maple [A]
time = 0.14, size = 77, normalized size = 0.82

method	result
risch	$\frac{\frac{13x}{682}+\frac{2}{1705}}{x^2+\frac{3}{5}x+\frac{2}{5}} + \frac{3\ln(16x^2-8x+24)}{968} - \frac{7\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{11132} - \frac{3\ln(100x^2+60x+40)}{968} + \frac{2891\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right.}{465124}$
default	$\frac{3\ln(2x^2-x+3)}{968} - \frac{7\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{11132} - \frac{-\frac{286x}{31}-\frac{88}{155}}{484\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)} - \frac{3\ln(5x^2+3x+2)}{968} + \frac{2891\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)}{465124}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/968*ln(2*x^2-x+3)-7/11132*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))-1/484*(-
286/31*x-88/155)/(x^2+3/5*x+2/5)-3/968*ln(5*x^2+3*x+2)+2891/465124*arctan(1
/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.59, size = 78, normalized size = 0.83

$$\frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{65\,x+4}{682\,(5\,x^2+3\,x+2)}-\frac{3}{968}\log\left(5\,x^2+3\,x+2\right)+\frac{3}{968}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*
log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

Fricas [A]
time = 2.02, size = 117, normalized size = 1.24

$$\frac{132986\,\sqrt{31}\,(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-13454\,\sqrt{23}\,(5\,x^2+3\,x+2)\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-66309\,(5\,x^2+3\,x+2)\log\left(5\,x^2+3\,x+2\right)+66309\,(5\,x^2+3\,x+2)\log\left(2\,x^2-x+3\right)+2039180\,x+125488}{21395704\,(5\,x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/21395704*(132986*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x +
3)) - 13454*sqrt(23)*(5*x^2 + 3*x + 2)*arctan(1/23*sqrt(23)*(4*x - 1)) - 66
309*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 66309*(5*x^2 + 3*x + 2)*log(2*
x^2 - x + 3) + 2039180*x + 125488)/(5*x^2 + 3*x + 2)
```

Sympy [A]
time = 0.13, size = 102, normalized size = 1.09

$$\frac{65x+4}{3410x^2+2046x+1364}+\frac{3\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968}-\frac{3\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968}-\frac{7\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{11132}+\frac{2891\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{465124}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

```
[Out] (65*x + 4)/(3410*x**2 + 2046*x + 1364) + 3*log(x**2 - x/2 + 3/2)/968 - 3*lo
g(x**2 + 3*x/5 + 2/5)/968 - 7*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/
11132 + 2891*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/465124
```

Giac [A]
time = 3.38, size = 78, normalized size = 0.83

$$\frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{65\,x+4}{682\,(5\,x^2+3\,x+2)}-\frac{3}{968}\log\left(5\,x^2+3\,x+2\right)+\frac{3}{968}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*
log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

Mupad [B]
time = 3.57, size = 95, normalized size = 1.01

$$\frac{\frac{13x}{682}+\frac{2}{1705}}{x^2+\frac{3x}{5}+\frac{2}{5}}+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{3}{968}+\frac{\sqrt{31}\,2891\mathrm{i}}{930248}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{3}{968}+\frac{\sqrt{31}\,2891\mathrm{i}}{930248}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] ((13*x)/682 + 2/1705)/((3*x)/5 + x^2 + 2/5) + log(x - (23^(1/2)*1i)/4 - 1/4
)*((23^(1/2)*7i)/22264 + 3/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)
*7i)/22264 - 3/968) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)/93
0248 + 3/968) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)/930248 -
 3/968)
```

3.43 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=115

$$\frac{4+65x}{1364(2+3x+5x^2)^2}+\frac{7923+21605x}{465124(2+3x+5x^2)}-\frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{847793\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{10232728\sqrt{31}}-\frac{\log(3-x+2x^2)}{21296}$$

```
[Out] 1/1364*(4+65*x)/(5*x^2+3*x+2)^2+1/465124*(7923+21605*x)/(5*x^2+3*x+2)-1/212
96*ln(2*x^2-x+3)+1/21296*ln(5*x^2+3*x+2)-45/244904*arctan(1/23*(1-4*x)*23^(
1/2))*23^(1/2)+847793/317214568*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {988, 1074, 1086, 648, 632, 210, 642}

$$-\frac{45\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{847793\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{10232728\sqrt{31}}+\frac{65x+4}{1364(5x^2+3x+2)^2}+\frac{21605x+7923}{465124(5x^2+3x+2)}-\frac{\log(2x^2-x+3)}{21296}+\frac{\log(5x^2+3x+2)}{21296}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + (7923 + 21605*x)/(465124*(2 + 3*x +
 5*x^2)) - (45*ArcTan[(1 - 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (847793*ArcTa
n[(3 + 10*x)/Sqrt[31]])/(10232728*Sqrt[31]) - Log[3 - x + 2*x^2]/21296 + Lo
g[2 + 3*x + 5*x^2]/21296
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
```

```
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx &= \frac{4+65x}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5753+3509x-4290x^2}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx}{15004} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-14522420+3833038x-}{(3-x+2x^2)(2+3x}}{11256000} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-58838186+5116364x}{3-x+2x^2}}{27239521936} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-1+4x}{3-x+2x^2}\,dx}{21296} + \frac{\int \frac{}{2}}{} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\log(3-x+2x^2)}{21296} + \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 104, normalized size = 0.90

$$
\frac{45\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{1695586\sqrt{31}\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right) + 31\left(\frac{44(17210+89144x+104430x^2+108025x^3)}{(2+3x+5x^2)^2} - 961\log(3-x+2x^2) + 961\log(2+3x+5x^2)\right)}{634429136}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (45*ArcTan[(-1 + 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (1695586*Sqrt[31]*ArcTa
n[(3 + 10*x)/Sqrt[31]] + 31*((44*(17210 + 89144*x + 104430*x^2 + 108025*x^3
))/(2 + 3*x + 5*x^2)^2 - 961*Log[3 - x + 2*x^2] + 961*Log[2 + 3*x + 5*x^2])
)/634429136
```

Maple [A]
time = 0.14, size = 89, normalized size = 0.77

method	result
default	$-\frac{\ln(2x^2-x+3)}{21296}+\frac{45\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{244904}+\frac{\frac{108025}{465124}x^3+\frac{52215}{232562}x^2+\frac{2026}{10571}x+\frac{8605}{232562}}{(5x^2+3x+2)^2}+\frac{\ln(5x^2+3x+2)}{21296}+\frac{847793\,\mathrm{a}}{}$
risch	$\frac{\frac{108025}{465124}x^3+\frac{52215}{232562}x^2+\frac{2026}{10571}x+\frac{8605}{232562}}{(5x^2+3x+2)^2}+\frac{\ln(100x^2+60x+40)}{21296}+\frac{847793\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{317214568}-\frac{\ln(16x^2-8x+24)}{21296}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21296*ln(2*x^2-x+3)+45/244904*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))+25/
10648*(95062/961*x^3+459492/4805*x^2+1961168/24025*x+75724/4805)/(5*x^2+3*x
+2)^2+1/21296*ln(5*x^2+3*x+2)+847793/317214568*arctan(1/31*(3+10*x)*31^(1/2
))*31^(1/2)
```

Maxima [A]
time = 0.51, size = 98, normalized size = 0.85

$$\frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{108025x^3+104430x^2+89144x+17210}{465124(25x^4+30x^3+29x^2+12x+4)}+\frac{1}{21296}\log(5x^2+3x+2)-\frac{1}{21296}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 +
89144*x + 17210)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1/21296*log(5*x^2
+ 3*x + 2) - 1/21296*log(2*x^2 - x + 3)
```

Fricas [A]
time = 2.31, size = 177, normalized size = 1.54

$$\frac{3388960300x^3+38998478\sqrt{31}(25x^4+30x^3+29x^2+12x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+2681190\sqrt{23}(25x^4+30x^3+29x^2+12x+4)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+3276177960x^2+685193(25x^4+30x^3+29x^2+12x+4)\log(5x^2+3x+2)-685193(25x^4+30x^3+29x^2+12x+4)\log(2x^2-x+3)+2796625568x+539912120}{14591870128(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/14591870128*(3388960300*x^3 + 38998478*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2
 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2681190*sqrt(23)*(25*x^4 +
```

```
30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3276177960*x^
2 + 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 + 3*x + 2) - 685
193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(2*x^2 - x + 3) + 2796625568*x
 + 539912120)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [A]
time = 0.15, size = 119, normalized size = 1.03

$$\frac{108025x^3 + 104430x^2 + 89144x + 17210}{11628100x^4 + 13953720x^3 + 13488596x^2 + 5581488x + 1860496} - \frac{\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{21296} + \frac{\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{21296} + \frac{45\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{244904} + \frac{847793\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{317214568}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (108025*x**3 + 104430*x**2 + 89144*x + 17210)/(11628100*x**4 + 13953720*x**
3 + 13488596*x**2 + 5581488*x + 1860496) - log(x**2 - x/2 + 3/2)/21296 + lo
g(x**2 + 3*x/5 + 2/5)/21296 + 45*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/2
3)/244904 + 847793*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/31721456
8
```

Giac [A]
time = 2.25, size = 88, normalized size = 0.77

$$\frac{847793}{317214568}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x + 3)\right) + \frac{45}{244904}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) + \frac{108025\,x^3 + 104430\,x^2 + 89144\,x + 17210}{465124\,(5\,x^2 + 3\,x + 2)^2} + \frac{1}{21296}\,\log\left(5\,x^2 + 3\,x + 2\right) - \frac{1}{21296}\,\log\left(2\,x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 +
89144*x + 17210)/(5*x^2 + 3*x + 2)^2 + 1/21296*log(5*x^2 + 3*x + 2) - 1/212
96*log(2*x^2 - x + 3)
```

Mupad [B]
time = 0.18, size = 115, normalized size = 1.00

$$\frac{\frac{4321\,x^3}{465124} + \frac{10443\,x^2}{1162810} + \frac{2026\,x}{264275} + \frac{1721}{1162810}}{x^4 + \frac{6\,x^3}{5} + \frac{29\,x^2}{25} + \frac{12\,x}{25} + \frac{4}{25}} + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{21296} + \frac{\sqrt{23}\,45\mathrm{i}}{489808}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{21296} + \frac{\sqrt{31}\,847793\mathrm{i}}{634429136}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{1}{21296} + \frac{\sqrt{31}\,847793\mathrm{i}}{634429136}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{21296} + \frac{\sqrt{23}\,45\mathrm{i}}{489808}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 - 1/21296) - log(x -
(23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 + 1/21296) - log(x - (31^(1/2
)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 - 1/21296) + log(x + (31^(1/
2)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 + 1/21296) + ((2026*x)/2642
75 + (10443*x^2)/1162810 + (4321*x^3)/465124 + 1721/1162810)/((12*x)/25 + (
29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

3.44 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\,(3-x+2x^2)}-\frac{13292697\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}-\frac{30613}{128}\log($$

```
[Out] -89359/64*x-1185/8*x^2+9775/48*x^3+2125/16*x^4+125/4*x^5-14641/2944*(101+79
*x)/(2*x^2-x+3)-30613/128*ln(2*x^2-x+3)-13292697/33856*arctan(1/23*(1-4*x)*
23^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$-\frac{13292697\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}+\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{14641(79x+101)}{2944\,(2x^2-x+3)}-\frac{30613}{128}\log\left(2x^2-x+3\right)-\frac{89359x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) - (13292697*ArcTan[(1 - 4*x)/
Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^2}\,dx = -\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{\frac{832627}{64}-\frac{661181x}{64}-\frac{488267x^2}{32}+\frac{143635x^3}{16}+\frac{213325x^4}{8}+\frac{83375}{4}}{3-x+2x^2}$$

$$= -\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}+\frac{1}{23}\int \left(-\frac{2055257}{64}-\frac{27255x}{4}+\frac{224825x^2}{16}+\frac{48875x^3}{4}+143\right.$$

$$= -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}+\frac{1331}{736}\int$$

$$= -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}-\frac{30613}{128}\int$$

$$= -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}-\frac{30613}{128}\text{lo}$$

$$= -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}-\frac{1329269}{1}$$

Mathematica [A]

time = 0.04, size = 91, normalized size = 1.00

$$-\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)}+\frac{13292697\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{1472\sqrt{23}}-\frac{30613}{128}\log\left(3-x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (13292697*ArcTan[(-1 + 4*x)
/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

Maple [A]
time = 0.11, size = 71, normalized size = 0.78

method	result
risch	$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}+\frac{-\frac{1156639x}{5888}-\frac{1478741}{5888}}{x^2-\frac{1}{2}x+\frac{3}{2}}-\frac{30613\ln\left(16x^2-8x+24\right)}{128}+\frac{13292697\sqrt{23}\ \text{arc}}{33}$
default	$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}-\frac{1331\left(\frac{869x}{92}+\frac{1111}{92}\right)}{64\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}-\frac{30613\ln\left(2x^2-x+3\right)}{128}+\frac{13292697\sqrt{23}\ \arctan\left(\right.}{33856}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 125/4*x^5+2125/16*x^4+9775/48*x^3-1185/8*x^2-89359/64*x-1331/64*(869/92*x+1
111/92)/(x^2-1/2*x+3/2)-30613/128*ln(2*x^2-x+3)+13292697/33856*23^(1/2)*arc
tan(1/23*(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.50, size = 72, normalized size = 0.79

$$\frac{125}{4}\,x^5+\frac{2125}{16}\,x^4+\frac{9775}{48}\,x^3-\frac{1185}{8}\,x^2+\frac{13292697}{33856}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{89359}{64}\,x-\frac{14641\,(79\,x+101)}{2944\,(2\,x^2-x+3)}-\frac{30613}{128}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(23
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2
*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

Fricas [A]
time = 1.48, size = 98, normalized size = 1.08

$$\frac{12696000\,x^7+47610000\,x^6+74800600\,x^5-20609840\,x^4-413058012\,x^3+79756182\,\sqrt{23}\,(2\,x^2-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+193356906\,x^2-48582831\,(2\,x^2-x+3)\log\left(2\,x^2-x+3\right)-930684489\,x-102033129}{203136\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/203136*(12696000*x^7 + 47610000*x^6 + 74800600*x^5 - 20609840*x^4 - 41305
8012*x^3 + 79756182*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)
) + 193356906*x^2 - 48582831*(2*x^2 - x + 3)*log(2*x^2 - x + 3) - 930684489
*x - 102033129)/(2*x^2 - x + 3)
```

Sympy [A]
time = 0.07, size = 90, normalized size = 0.99

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}+\frac{-1156639x-1478741}{5888x^2-2944x+8832}-\frac{30613\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{13292697\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{33856}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**5/4 + 2125*x**4/16 + 9775*x**3/48 - 1185*x**2/8 - 89359*x/64 + (-115
6639*x - 1478741)/(5888*x**2 - 2944*x + 8832) - 30613*log(x**2 - x/2 + 3/2)
/128 + 13292697*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/33856
```

Giac [A]
time = 3.44, size = 72, normalized size = 0.79

$$\frac{125}{4}\,x^5+\frac{2125}{16}\,x^4+\frac{9775}{48}\,x^3-\frac{1185}{8}\,x^2+\frac{13292697}{33856}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{89359}{64}\,x-\frac{14641\,(79\,x+101)}{2944\,(2\,x^2-x+3)}-\frac{30613}{128}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(23
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2
*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

Mupad [B]
time = 3.46, size = 72, normalized size = 0.79

$$\frac{13292697\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{33856}-\frac{30613\,\ln\left(2\,x^2-x+3\right)}{128}-\frac{\frac{1156639\,x}{5888}+\frac{1478741}{5888}}{x^2-\frac{x}{2}+\frac{3}{2}}-\frac{89359\,x}{64}-\frac{1185\,x^2}{8}+\frac{9775\,x^3}{48}+\frac{2125\,x^4}{16}+\frac{125\,x^5}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^2,x)
```

```
[Out] (13292697*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/33856 - (30613*lo
g(2*x^2 - x + 3))/128 - ((1156639*x)/5888 + 1478741/5888)/(x^2 - x/2 + 3/2)
 - (89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/
4
```

3.45 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx$

Optimal. Leaf size=77

$$\frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\,(3-x+2x^2)}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log\left(3-x+2x^2\right)$$

```
[Out] 915/16*x+175/4*x^2+125/12*x^3-1331/736*(17-45*x)/(2*x^2-x+3)-2057/32*ln(2*x
^2-x+3)+223971/8464*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{223971\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}+\frac{125x^3}{12}+\frac{175x^2}{4}-\frac{1331(17-45x)}{736\,(2x^2-x+3)}-\frac{2057}{32}\log\left(2x^2-x+3\right)+\frac{915x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 - (1331*(17 - 45*x))/(736*(3 - x +
2*x^2)) + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[3
- x + 2*x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^2}\,dx &= -\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{-\frac{25195}{16}-\frac{19067x}{16}+\frac{22195x^2}{8}+\frac{13225x^3}{4}+\frac{2875x^4}{2}}{3-x+2x^2}\,dx\\
&= -\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}+\frac{1}{23}\int \left(\frac{21045}{16}+\frac{4025x}{2}+\frac{2875x^2}{4}-\frac{121(365+391x)}{8\left(3-x+2x^2\right)}\right)dx\\
&= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}-\frac{121}{184}\int \frac{365+391x}{3-x+2x^2}\,dx\\
&= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}-\frac{2057}{32}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{223971}{736}\int\\
&= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}-\frac{2057}{32}\log\left(3-x+2x^2\right)+\frac{223971}{368}\text{Su}\\
&= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log(
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 77, normalized size = 1.00

$$\frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}+\frac{1331(-17+45x)}{736\,(3-x+2x^2)}-\frac{223971\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log\left(3-x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 + (1331*(-17 + 45*x))/(736*(3 - x +
 2*x^2)) - (223971*ArcTan[(-1 + 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[
3 - x + 2*x^2])/32
```

Maple [A]
time = 0.14, size = 61, normalized size = 0.79

method	result	size
risch	$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}+\frac{\frac{59895x}{1472}-\frac{22627}{1472}}{x^2-\frac{1}{2}x+\frac{3}{2}}-\frac{2057\ln(16x^2-8x+24)}{32}-\frac{223971\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{8464}$	60
default	$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}-\frac{121\left(-\frac{495x}{92}+\frac{187}{92}\right)}{16\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}-\frac{2057\ln(2x^2-x+3)}{32}-\frac{223971\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{8464}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 125/12*x^3+175/4*x^2+915/16*x-121/16*(-495/92*x+187/92)/(x^2-1/2*x+3/2)-205
7/32*ln(2*x^2-x+3)-223971/8464*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.66, size = 62, normalized size = 0.81

$$\frac{125}{12}\,x^3+\frac{175}{4}\,x^2-\frac{223971}{8464}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{915}{16}\,x+\frac{1331\,(45\,x-17)}{736\,(2\,x^2-x+3)}-\frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 -
x + 3)
```

Fricas [A]
time = 2.26, size = 88, normalized size = 1.14

$$\frac{1058000\,x^5+3914600\,x^4+5173620\,x^3-1343826\,\sqrt{23}\,(2\,x^2-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+3761190\,x^2-3264459\,(2\,x^2-x+3)\log\left(2\,x^2-x+3\right)+12845385\,x-1561263}{50784\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/50784*(1058000*x^5 + 3914600*x^4 + 5173620*x^3 - 1343826*sqrt(23)*(2*x^2
- x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3761190*x^2 - 3264459*(2*x^2 - x
 + 3)*log(2*x^2 - x + 3) + 12845385*x - 1561263)/(2*x^2 - x + 3)
```

Sympy [A]
time = 0.07, size = 75, normalized size = 0.97

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} + \frac{59895x - 22627}{1472x^2 - 736x + 2208} - \frac{2057\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{223971\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x
+ 2208) - 2057*log(x**2 - x/2 + 3/2)/32 - 223971*sqrt(23)*atan(4*sqrt(23)*x
/23 - sqrt(23)/23)/8464
```

Giac [A]
time = 2.99, size = 62, normalized size = 0.81

$$\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{915}{16}x + \frac{1331(45x - 17)}{736(2x^2 - x + 3)} - \frac{2057}{32}\log\left(2x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 -
x + 3)
```

Mupad [B]
time = 3.42, size = 61, normalized size = 0.79

$$\frac{915x}{16} - \frac{2057\ln\left(2x^2 - x + 3\right)}{32} + \frac{\frac{59895x}{1472} - \frac{22627}{1472}}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{223971\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464} + \frac{175x^2}{4} + \frac{125x^3}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^2,x)
```

```
[Out] (915*x)/16 - (2057*log(2*x^2 - x + 3))/32 + ((59895*x)/1472 - 22627/1472)/(
x^2 - x/2 + 3/2) - (223971*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/
8464 + (175*x^2)/4 + (125*x^3)/12
```

3.46 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2} \, dx$

Optimal. Leaf size=63

$$\frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)$$

```
[Out] 25/4*x+121/184*(19-7*x)/(2*x^2-x+3)+55/8*ln(2*x^2-x+3)+1859/2116*arctan(1/2
3*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{1859\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{121(19-7x)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)+\frac{25x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] (25*x)/4 + (121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (1859*ArcTan[(1 - 4*x)/
Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^2}\,dx &= \frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{\frac{163}{4}+\frac{1955x}{4}+\frac{575x^2}{2}}{3-x+2x^2}\,dx \\
&= \frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1}{23}\int \left(\frac{575}{4}-\frac{11(71-115x)}{2\left(3-x+2x^2\right)}\right)\,dx \\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}-\frac{11}{46}\int \frac{71-115x}{3-x+2x^2}\,dx \\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{55}{8}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{1859}{184}\int \frac{1}{3-x+2x^2}\,dx \\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{55}{8}\log\left(3-x+2x^2\right)+\frac{1859}{92}\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,\right. \\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)
\end{aligned}
$$

Mathematica [A]

time = 0.02, size = 63, normalized size = 1.00

$$\frac{25x}{4}-\frac{121(-19+7x)}{184(3-x+2x^2)}-\frac{1859\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] (25*x)/4 - (121*(-19 + 7*x))/(184*(3 - x + 2*x^2)) - (1859*ArcTan[(-1 + 4*x
)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

Maple [A]
time = 0.15, size = 51, normalized size = 0.81

method	result	size
risch	$\frac{25x}{4}+\frac{-\frac{847x}{368}+\frac{2299}{368}}{x^2-\frac{1}{2}x+\frac{3}{2}}+\frac{55\ln(16x^2-8x+24)}{8}-\frac{1859\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2116}$	50
default	$\frac{25x}{4}+\frac{-\frac{847x}{368}+\frac{2299}{368}}{x^2-\frac{1}{2}x+\frac{3}{2}}+\frac{55\ln(2x^2-x+3)}{8}-\frac{1859\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2116}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 25/4*x+11/4*(-77/92*x+209/92)/(x^2-1/2*x+3/2)+55/8*ln(2*x^2-x+3)-1859/2116*
23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.51, size = 52, normalized size = 0.83

$$-\frac{1859}{2116}\sqrt{23}\,\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{25}{4}\,x-\frac{121\,(7\,x-19)}{184\,(2\,x^2-x+3)}+\frac{55}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] -1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*x
 - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

Fricas [A]
time = 0.94, size = 78, normalized size = 1.24

$$\frac{52900\,x^3-3718\,\sqrt{23}\,(2\,x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-26450\,x^2+29095\,(2\,x^2-x+3)\log\left(2\,x^2-x+3\right)+59869\,x+52877}{4232\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/4232*(52900*x^3 - 3718*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x
 - 1)) - 26450*x^2 + 29095*(2*x^2 - x + 3)*log(2*x^2 - x + 3) + 59869*x + 5
2877)/(2*x^2 - x + 3)
```

Sympy [A]
time = 0.07, size = 61, normalized size = 0.97

$$\frac{25x}{4} + \frac{2299 - 847x}{368x^2 - 184x + 552} + \frac{55\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{8} - \frac{1859\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)
```

```
[Out] 25*x/4 + (2299 - 847*x)/(368*x**2 - 184*x + 552) + 55*log(x**2 - x/2 + 3/2)
/8 - 1859*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/2116
```

Giac [A]
time = 1.55, size = 52, normalized size = 0.83

$$-\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x - 1)\right) + \frac{25}{4}\,x - \frac{121\,(7\,x - 19)}{184\,(2\,x^2 - x + 3)} + \frac{55}{8}\log\left(2\,x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] -1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*x
 - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

Mupad [B]
time = 3.40, size = 52, normalized size = 0.83

$$\frac{25\,x}{4} + \frac{55\,\ln\left(2\,x^2 - x + 3\right)}{8} - \frac{\frac{847\,x}{368} - \frac{2299}{368}}{x^2 - \frac{x}{2} + \frac{3}{2}} - \frac{1859\,\sqrt{23}\,\operatorname{atan}\left(\frac{4\,\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^2,x)
```

```
[Out] (25*x)/4 + (55*log(2*x^2 - x + 3))/8 - ((847*x)/368 - 2299/368)/(x^2 - x/2
+ 3/2) - (1859*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/2116
```

3.47 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{11(5+3x)}{46\left(3-x+2x^2\right)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

```
[Out] -11/46*(5+3*x)/(2*x^2-x+3)-82/529*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1674, 12, 632, 210}

$$-\frac{82\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*
Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx &= -\frac{11(5+3x)}{46\,(3-x+2x^2)}+\frac{1}{23}\int \frac{41}{3-x+2x^2}\,dx\\
&= -\frac{11(5+3x)}{46\,(3-x+2x^2)}+\frac{41}{23}\int \frac{1}{3-x+2x^2}\,dx\\
&= -\frac{11(5+3x)}{46\,(3-x+2x^2)}-\frac{82}{23}\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right)\\
&= -\frac{11(5+3x)}{46\,(3-x+2x^2)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.00

$$-\frac{11(5+3x)}{46\,(3-x+2x^2)}+\frac{82\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23
*Sqrt[23])
```

Maple [A]
time = 0.11, size = 34, normalized size = 0.79

method	result	size
default	$\frac{-\frac{33x}{92}-\frac{55}{92}}{x^2-\frac{1}{2}x+\frac{3}{2}}+\frac{82\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{529}$	34

risch	$\frac{-\frac{33x}{92}-\frac{55}{92}}{x^2-\frac{1}{2}x+\frac{3}{2}}+\frac{82\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{529}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-33/92*x-55/92)/(x^2-1/2*x+3/2)+82/529*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/
2))
```

Maxima [A]
time = 0.51, size = 36, normalized size = 0.84

$$\frac{82}{529}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{11\,(3\,x+5)}{46\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
x + 3)
```

Fricas [A]
time = 1.92, size = 45, normalized size = 1.05

$$\frac{164\,\sqrt{23}\,(2\,x^2-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-759\,x-1265}{1058\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1058*(164*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 759*
x - 1265)/(2*x^2 - x + 3)
```

Sympy [A]
time = 0.05, size = 42, normalized size = 0.98

$$\frac{-33x-55}{92x^2-46x+138}+\frac{82\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{529}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2,x)
```

```
[Out] (-33*x - 55)/(92*x**2 - 46*x + 138) + 82*sqrt(23)*atan(4*sqrt(23)*x/23 - sq
rt(23)/23)/529
```

Giac [A]
time = 1.92, size = 36, normalized size = 0.84

$$\frac{82}{529}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{11\,(3\,x+5)}{46\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
x + 3)
```

Mupad [B]
time = 0.04, size = 36, normalized size = 0.84

$$\frac{82\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{529}-\frac{\frac{33\,x}{92}+\frac{55}{92}}{x^2-\frac{x}{2}+\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^2,x)
```

```
[Out] (82*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/529 - ((33*x)/92 + 55/9
2)/(x^2 - x/2 + 3/2)
```

3.48 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx$

Optimal. Leaf size=94

$$\frac{13-6x}{506(3-x+2x^2)}+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}}-\frac{13}{968}\log\left(3-x+2x^2\right)+\frac{13}{968}\log\left(2+3x+5x^2\right)$$

```
[Out] 1/506*(13-6*x)/(2*x^2-x+3)-13/968*ln(2*x^2-x+3)+13/968*ln(5*x^2+3*x+2)+241/
256036*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+69/15004*arctan(1/31*(3+10*x)
*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {988, 1086, 648, 632, 210, 642}

$$\frac{241\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}+\frac{13-6x}{506(2x^2-x+3)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) + (241*ArcTan[(1 - 4*x)/Sqrt[23]])/(11132*
Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 - x
 + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{13-6x}{506(3-x+2x^2)} - \frac{\int \frac{-1892-1067x+330x^2}{(3-x+2x^2)(2+3x+5x^2)}\,dx}{5566}$$

$$= \frac{13-6x}{506(3-x+2x^2)} - \frac{\int \frac{-3509+72358x}{3-x+2x^2}\,dx}{1346972} - \frac{\int \frac{-150282-180895x}{2+3x+5x^2}\,dx}{1346972}$$

$$= \frac{13-6x}{506(3-x+2x^2)} - \frac{241\int \frac{1}{3-x+2x^2}\,dx}{22264} - \frac{13}{968}\int \frac{-1+4x}{3-x+2x^2}\,dx + \frac{1}{9}$$

$$= \frac{13-6x}{506(3-x+2x^2)} - \frac{13}{968}\log\left(3-x+2x^2\right) + \frac{13}{968}\log\left(2+3x+5x^2\right)$$

$$= \frac{13-6x}{506(3-x+2x^2)} + \frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{96}$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 1.00

$$\frac{13-6x}{506(3-x+2x^2)} - \frac{241\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968}\log\left(3-x+2x^2\right) + \frac{13}{968}\log\left(2+3x+5x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) - (241*ArcTan[(-1 + 4*x)/Sqrt[23]])/(11132
*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 -
x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

Maple [A]
time = 0.14, size = 77, normalized size = 0.82

method	result
risch	$\frac{-\frac{3x}{506}+\frac{13}{1012}}{x^2-\frac{1}{2}x+\frac{3}{2}} + \frac{69\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)\sqrt{31}}{15004} + \frac{13\ln(100x^2+60x+40)}{968} - \frac{13\ln(16x^2-8x+24)}{968} - \frac{241\sqrt{23}\,\arctan\left(\frac{(4x}{}\right.}{256036}$
default	$-\frac{\frac{66x}{23}-\frac{143}{23}}{484(x^2-\frac{1}{2}x+\frac{3}{2})} - \frac{13\ln(2x^2-x+3)}{968} - \frac{241\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{256036} + \frac{13\ln(5x^2+3x+2)}{968} + \frac{69\arctan\left(\frac{(3+10x)\sqrt{3}}{31}\right.}{15004}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/484*(66/23*x-143/23)/(x^2-1/2*x+3/2)-13/968*ln(2*x^2-x+3)-241/256036*23^
(1/2)*arctan(1/23*(4*x-1)*23^(1/2))+13/968*ln(5*x^2+3*x+2)+69/15004*arctan(
1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.51, size = 78, normalized size = 0.83

$$\frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{6\,x-13}{506\,(2\,x^2-x+3)}+\frac{13}{968}\log\left(5\,x^2+3\,x+2\right)-\frac{13}{968}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*l
og(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

Fricas [A]
time = 1.65, size = 117, normalized size = 1.24

$$\frac{73002\sqrt{31}\,(2\,x^2-x+3)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-14942\sqrt{23}\,(2\,x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+213187\,(2\,x^2-x+3)\log\left(5\,x^2+3\,x+2\right)-213187\,(2\,x^2-x+3)\log\left(2\,x^2-x+3\right)-188232\,x+407836}{15874232\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/15874232*(73002*sqrt(31)*(2*x^2 - x + 3)*arctan(1/31*sqrt(31)*(10*x + 3))
 - 14942*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 213187*
(2*x^2 - x + 3)*log(5*x^2 + 3*x + 2) - 213187*(2*x^2 - x + 3)*log(2*x^2 - x
 + 3) - 188232*x + 407836)/(2*x^2 - x + 3)
```

Sympy [A]
time = 0.13, size = 102, normalized size = 1.09

$$\frac{13-6x}{1012x^2-506x+1518}-\frac{13\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968}+\frac{13\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968}-\frac{241\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{256036}+\frac{69\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{15004}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

```
[Out] (13 - 6*x)/(1012*x**2 - 506*x + 1518) - 13*log(x**2 - x/2 + 3/2)/968 + 13*l
og(x**2 + 3*x/5 + 2/5)/968 - 241*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/2
3)/256036 + 69*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/15004
```

Giac [A]
time = 1.41, size = 78, normalized size = 0.83

$$\frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{6\,x-13}{506\,(2\,x^2-x+3)}+\frac{13}{968}\log\left(5\,x^2+3\,x+2\right)-\frac{13}{968}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*l
og(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

Mupad [B]
time = 3.58, size = 96, normalized size = 1.02

$$-\frac{\frac{3x}{506}-\frac{13}{1012}}{x^2-\frac{x}{2}+\frac{3}{2}}-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{13}{968}+\frac{\sqrt{31}\,69\mathrm{i}}{30008}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{13}{968}+\frac{\sqrt{31}\,69\mathrm{i}}{30008}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{13}{968}+\frac{\sqrt{23}\,241\mathrm{i}}{512072}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{13}{968}+\frac{\sqrt{23}\,241\mathrm{i}}{512072}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 + 13/968) - log(x -
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 - 13/968) - ((3*x)/506 - 13/
1012)/(x^2 - x/2 + 3/2) + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i)/5
12072 - 13/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i)/512072 +
13/968)
```

3.49 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=127

$$-\frac{25(117-137x)}{172546\,(2+3x+5x^2)}+\frac{13-6x}{506\,(3-x+2x^2)\,(2+3x+5x^2)}+\frac{2769\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}}+\frac{12643\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{165044\sqrt{31}}+$$

```
[Out] -25/172546*(117-137*x)/(5*x^2+3*x+2)+1/506*(13-6*x)/(2*x^2-x+3)/(5*x^2+3*x+
2)+19/10648*ln(2*x^2-x+3)-19/10648*ln(5*x^2+3*x+2)+2769/2816396*arctan(1/23
*(1-4*x)*23^(1/2))*23^(1/2)+12643/5116364*arctan(1/31*(3+10*x)*31^(1/2))*31
^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$,
Rules used = {988, 1074, 1086, 648, 632, 210, 642}

$$\frac{2769\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}}+\frac{12643\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{165044\sqrt{31}}-\frac{25(117-137x)}{172546\,(5x^2+3x+2)}+\frac{13-6x}{506\,(2x^2-x+3)\,(5x^2+3x+2)}+\frac{19\log\left(2x^2-x+3\right)}{10648}-\frac{19\log\left(5x^2+3x+2\right)}{10648}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-25*(117 - 137*x))/(172546*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(506*(3 - x + 2
*x^2)*(2 + 3*x + 5*x^2)) + (2769*ArcTan[(1 - 4*x)/Sqrt[23]])/(122452*Sqrt[2
3]) + (12643*ArcTan[(3 + 10*x)/Sqrt[31]])/(165044*Sqrt[31]) + (19*Log[3 - x
 + 2*x^2])/10648 - (19*Log[2 + 3*x + 5*x^2])/10648
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
```

```
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx = \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{-2321-2299x+990x^2}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx}{5566}$$

$$= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{-3\ldots}{\ldots}}{\ldots}$$

$$= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{132\ldots}{\ldots}}{\ldots}$$

$$= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{19\int \ldots}{\ldots}$$

$$= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{19\log \ldots}{\ldots}$$

$$= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{2769\ldots}{1\ldots}$$

Mathematica [A]
time = 0.04, size = 106, normalized size = 0.83

$$\frac{\frac{31372(-4342+11154x-9275x^2+6850x^3)}{6+7x+16x^2+x^3+10x^4} - 5322018\sqrt{23}\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right) + 13376294\sqrt{31}\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right) + 9659011\log(3-x+2x^2) - 9659011\log(2+3x+5x^2)}{5413113112}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((31372*(-4342 + 11154*x - 9275*x^2 + 6850*x^3))/(6 + 7*x + 16*x^2 + x^3 +
10*x^4) - 5322018*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 13376294*Sqrt[31]*
ArcTan[(3 + 10*x)/Sqrt[31]] + 9659011*Log[3 - x + 2*x^2] - 9659011*Log[2 +
3*x + 5*x^2])/5413113112
```

Maple [A]
time = 0.14, size = 94, normalized size = 0.74

method	result
default	$\frac{-\frac{77x}{23}-\frac{341}{46}}{5324x^2-2662x+7986}+\frac{19\ln(2x^2-x+3)}{10648}-\frac{2769\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2816396}-\frac{-\frac{759x}{31}+\frac{1078}{155}}{5324\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{19\ln(5x^2+3x+2)}{10648}$
risch	$\frac{\frac{3425}{86273}x^3-\frac{9275}{172546}x^2+\frac{507}{7843}x-\frac{2171}{86273}}{(2x^2-x+3)(5x^2+3x+2)}+\frac{19\ln(16x^2-8x+24)}{10648}-\frac{2769\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2816396}-\frac{19\ln(100x^2+60x+40)}{10648}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5324*(-77/23*x-341/46)/(x^2-1/2*x+3/2)+19/10648*ln(2*x^2-x+3)-2769/281639
6*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))-1/5324*(-759/31*x+1078/155)/(x^2+3
/5*x+2/5)-19/10648*ln(5*x^2+3*x+2)+12643/5116364*arctan(1/31*(3+10*x)*31^(1
/2))*31^(1/2)
```

Maxima [A]
time = 0.51, size = 96, normalized size = 0.76

$$\frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{6850x^3-9275x^2+11154x-4342}{172546(10x^4+x^3+16x^2+7x+6)}-\frac{19}{10648}\log(5x^2+3x+2)+\frac{19}{10648}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 1115
4*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x +
2) + 19/10648*log(2*x^2 - x + 3)
```

Fricas [A]
time = 1.45, size = 167, normalized size = 1.31

$$\frac{214898200x^3+13376294\sqrt{31}(10x^4+x^3+16x^2+7x+6)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-5322018\sqrt{23}(10x^4+x^3+16x^2+7x+6)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-290975300x^2-9659011(10x^4+x^3+16x^2+7x+6)\log(5x^2+3x+2)+9659011(10x^4+x^3+16x^2+7x+6)\log(2x^2-x+3)+349923288x-136217224}{5413113112(10x^4+x^3+16x^2+7x+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/5413113112*(214898200*x^3 + 13376294*sqrt(31)*(10*x^4 + x^3 + 16*x^2 + 7*
x + 6)*arctan(1/31*sqrt(31)*(10*x + 3)) - 5322018*sqrt(23)*(10*x^4 + x^3 +
```

```
16*x^2 + 7*x + 6)*arctan(1/23*sqrt(23)*(4*x - 1)) - 290975300*x^2 - 9659011
*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*log(5*x^2 + 3*x + 2) + 9659011*(10*x^4 +
 x^3 + 16*x^2 + 7*x + 6)*log(2*x^2 - x + 3) + 349923288*x - 136217224)/(10*
x^4 + x^3 + 16*x^2 + 7*x + 6)
```

Sympy [A]
time = 0.15, size = 122, normalized size = 0.96

$$\frac{6850x^3-9275x^2+11154x-4342}{1725460x^4+172546x^3+2760736x^2+1207822x+1035276}+\frac{19\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{10648}-\frac{19\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{10648}-\frac{2769\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{2816396}+\frac{12643\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{5116364}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] (6850*x**3 - 9275*x**2 + 11154*x - 4342)/(1725460*x**4 + 172546*x**3 + 2760
736*x**2 + 1207822*x + 1035276) + 19*log(x**2 - x/2 + 3/2)/10648 - 19*log(x
**2 + 3*x/5 + 2/5)/10648 - 2769*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/2816396 + 12643*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/5116364
```

Giac [A]
time = 1.65, size = 96, normalized size = 0.76

$$\frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{6850\,x^3-9275\,x^2+11154\,x-4342}{172546\,(10\,x^4+x^3+16\,x^2+7\,x+6)}-\frac{19}{10648}\log\left(5\,x^2+3\,x+2\right)+\frac{19}{10648}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 1115
4*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x +
2) + 19/10648*log(2*x^2 - x + 3)
```

Mupad [B]
time = 0.18, size = 115, normalized size = 0.91

$$\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{19}{10648}+\frac{\sqrt{31}\,12643\mathrm{i}}{10232728}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{19}{10648}+\frac{\sqrt{31}\,12643\mathrm{i}}{10232728}\right)+\frac{\frac{685\,x^3}{172546}-\frac{1855\,x^2}{345092}+\frac{507\,x}{78430}-\frac{2171}{862730}}{x^4+\frac{x^3}{10}+\frac{8\,x^2}{5}+\frac{7\,x}{10}+\frac{3}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 + 19/10648) - log(
x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 - 19/10648) - log(x -
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 + 19/10648) + log(x +
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 - 19/10648) + ((507*x)
/78430 - (1855*x^2)/345092 + (685*x^3)/172546 - 2171/862730)/((7*x)/10 + (8
*x^2)/5 + x^3/10 + x^4 + 3/5)
```

3.50 $\int \frac{1}{(3-x+2x^2)^2 (2+3x+5x^2)^3} \, dx$

<u>**Optimal**</u>. Leaf size=148

$$\frac{-9446+5765x}{690184\left(2+3x+5x^2\right)^2}+\frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2}+\frac{1765599+3996965x}{235352744\left(2+3x+5x^2\right)}-\frac{25557\tan^{-1}\left(\frac{1-}{\sqrt{2}}\right.}{5387888\sqrt{23}}$$

```
[Out] 1/690184*(-9446+5765*x)/(5*x^2+3*x+2)^2+1/506*(13-6*x)/(2*x^2-x+3)/(5*x^2+3
*x+2)^2+1/235352744*(1765599+3996965*x)/(5*x^2+3*x+2)+97/468512*ln(2*x^2-x+
3)-97/468512*ln(5*x^2+3*x+2)-25557/123921424*arctan(1/23*(1-4*x)*23^(1/2))*
23^(1/2)+4464079/6978720496*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {988, 1074, 1086, 648, 632, 210, 642}

$$-\frac{25557\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{5387888\sqrt{23}}+\frac{4464079\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{225120016\sqrt{31}}-\frac{9446-5765x}{690184\left(5x^2+3x+2\right)^2}+\frac{3996965x+1765599}{235352744\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{97\log\left(2x^2-x+3\right)}{468512}-\frac{97\log\left(5x^2+3x+2\right)}{468512}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -1/690184*(9446 - 5765*x)/(2 + 3*x + 5*x^2)^2 + (13 - 6*x)/(506*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (1765599 + 3996965*x)/(235352744*(2 + 3*x + 5*x
^2)) - (25557*ArcTan[(1 - 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*Arc
Tan[(3 + 10*x)/Sqrt[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/46
8512 - (97*Log[2 + 3*x + 5*x^2])/468512
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 632</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 642</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
```

```
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx = \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-2750-3531x+1650x^2}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx}{5566}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \int$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{}{2}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{}{2}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{}{2}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{}{2}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{}{2}$$

Mathematica [A]
time = 0.06, size = 136, normalized size = 0.92

$$\frac{-11+90x}{244904(3-x+2x^2)} + \frac{-98+345x}{30008(2+3x+5x^2)^2} + \frac{67573+164380x}{10232728(2+3x+5x^2)} + \frac{25557\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{5387888\sqrt{23}} + \frac{4464079\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{225120016\sqrt{31}} + \frac{97\log(3-x+2x^2)}{468512} - \frac{97\log(2+3x+5x^2)}{468512}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-11 + 90*x)/(244904*(3 - x + 2*x^2)) + (-98 + 345*x)/(30008*(2 + 3*x + 5*x
^2)^2) + (67573 + 164380*x)/(10232728*(2 + 3*x + 5*x^2)) + (25557*ArcTan[(-
1 + 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3 + 10*x)/Sqrt[31
]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/468512 - (97*Log[2 + 3*x
 + 5*x^2])/468512
```

Maple [A]
time = 0.16, size = 106, normalized size = 0.72

method	result
default	$\frac{\frac{990x}{23}-\frac{121}{23}}{234256x^2-117128x+351384}+\frac{97\ln(2x^2-x+3)}{468512}+\frac{25557\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{123921424}-\frac{25\left(-\frac{723272}{961}x^3-\frac{3656422}{4805}x^2-\frac{1428072}{24025}\right.}{234256(5x^2+3x+2)^2}$
risch	$\frac{\frac{19984825}{117676372}x^5+\frac{21652955}{235352744}x^4+\frac{69648769}{235352744}x^3+\frac{23910151}{117676372}x^2+\frac{5333615}{29419093}x+\frac{158567}{5348926}}{(2x^2-x+3)(5x^2+3x+2)^2}-\frac{97\ln(100x^2+60x+40)}{468512}+\frac{4464079\arctan\left(\frac{(3+10x)}{31}\right.}{697872049}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/234256*(990/23*x-121/23)/(x^2-1/2*x+3/2)+97/468512*ln(2*x^2-x+3)+25557/12
3921424*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))-25/234256*(-723272/961*x^3-3
656422/4805*x^2-14280728/24025*x-2238016/24025)/(5*x^2+3*x+2)^2-97/468512*l
n(5*x^2+3*x+2)+4464079/6978720496*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.50, size = 118, normalized size = 0.80

$$\frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{235352744(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)}-\frac{97}{468512}\log(5x^2+3x+2)+\frac{97}{468512}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921
424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 +
21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) - 97/468512*log(5*x^2 + 3
*x + 2) + 97/468512*log(2*x^2 - x + 3)
```

Fricas [A]
time = 2.31, size = 237, normalized size = 1.60

$$\frac{1253927859800x^5+679296504260x^4+2185021181068x^3+4722995582\sqrt{31}(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+1522737174\sqrt{23}(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+1500218514344x^2-1528665583(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)\log(5x^2+3x+2)+1528665583(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)\log(2x^2-x+3)+1338609358240x+218880812656}{7383486284768(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/7383486284768*(1253927859800*x^5 + 679296504260*x^4 + 2185021181068*x^3 +
 4722995582*sqrt(31)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x +
12)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1522737174*sqrt(23)*(50*x^6 + 35*x^5
 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*arctan(1/23*sqrt(23)*(4*x - 1)) +
 1500218514344*x^2 - 1528665583*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^
2 + 32*x + 12)*log(5*x^2 + 3*x + 2) + 1528665583*(50*x^6 + 35*x^5 + 103*x^4
 + 85*x^3 + 83*x^2 + 32*x + 12)*log(2*x^2 - x + 3) + 1338609358240*x + 2188
80812656)/(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

Sympy [A]
time = 0.17, size = 143, normalized size = 0.97

$$\frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{11767637200x^6 + 8237346040x^5 + 24241332632x^4 + 20004983240x^3 + 19534277752x^2 + 7531287808x + 2824232928} + \frac{97\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{468512} - \frac{97\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{468512} + \frac{25557\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{123921424} + \frac{4464079\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{6978720496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] (39969650*x**5 + 21652955*x**4 + 69648769*x**3 + 47820302*x**2 + 42668920*x
 + 6976948)/(11767637200*x**6 + 8237346040*x**5 + 24241332632*x**4 + 200049
83240*x**3 + 19534277752*x**2 + 7531287808*x + 2824232928) + 97*log(x**2 -
x/2 + 3/2)/468512 - 97*log(x**2 + 3*x/5 + 2/5)/468512 + 25557*sqrt(23)*atan
(4*sqrt(23)*x/23 - sqrt(23)/23)/123921424 + 4464079*sqrt(31)*atan(10*sqrt(3
1)*x/31 + 3*sqrt(31)/31)/6978720496
```

Giac [A]
time = 1.05, size = 110, normalized size = 0.74

$$\frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{235352744(5x^2+3x+2)^2(2x^2-x+3)} - \frac{97}{468512}\log\left(5x^2+3x+2\right) + \frac{97}{468512}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921
424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 +
21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/((5*x^2
+ 3*x + 2)^2*(2*x^2 - x + 3)) - 97/468512*log(5*x^2 + 3*x + 2) + 97/468512*
log(2*x^2 - x + 3)
```

Mupad [B]
time = 3.59, size = 135, normalized size = 0.91

$$\frac{\frac{799393x^5}{235352744} + \frac{4330591x^4}{2353527440} + \frac{69648769x^3}{11767637200} + \frac{23910151x^2}{5883818600} + \frac{1066723x}{294190930} + \frac{158567}{267446300}}{x^6 + \frac{7x^5}{10} + \frac{103x^4}{50} + \frac{17x^3}{10} + \frac{83x^2}{50} + \frac{16x}{25} + \frac{6}{25}} + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{97}{468512} + \frac{\sqrt{23}\,25557\mathrm{i}}{247842848}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{97}{468512} + \frac{\sqrt{23}\,25557\mathrm{i}}{247842848}\right) - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{97}{468512} + \frac{\sqrt{31}\,4464079\mathrm{i}}{13957440992}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{97}{468512} + \frac{\sqrt{31}\,4464079\mathrm{i}}{13957440992}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 + 97/468512) -
log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 - 97/468512) +
((1066723*x)/294190930 + (23910151*x^2)/5883818600 + (69648769*x^3)/1176763
7200 + (4330591*x^4)/2353527440 + (799393*x^5)/235352744 + 158567/267446300
)/((16*x)/25 + (83*x^2)/50 + (17*x^3)/10 + (103*x^4)/50 + (7*x^5)/10 + x^6
+ 6/25) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i)/13957440992
 + 97/468512) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i)/13957
440992 - 97/468512)
```

3.51 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx$

Optimal. Leaf size=98

$$\frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\,(3-x+2x^2)^2}+\frac{1331(5229+76420x)}{135424\,(3-x+2x^2)}+\frac{63799791\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16928\sqrt{23}}-\frac{13915}{64}\,\mathrm{lo}$$

```
[Out] 2725/8*x+4875/32*x^2+625/24*x^3-14641/5888*(101+79*x)/(2*x^2-x+3)^2+1331/13
5424*(5229+76420*x)/(2*x^2-x+3)-13915/64*ln(2*x^2-x+3)+63799791/389344*arct
an(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{63799791\,\mathrm{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{16928\sqrt{23}}+\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424\,(2x^2-x+3)}-\frac{14641(79x+101)}{5888\,(2x^2-x+3)^2}-\frac{13915}{64}\log\left(2x^2-x+3\right)+\frac{2725x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) + (637997
91*ArcTan[(1 - 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2]
)/64
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = -\frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1}{46}\int \frac{\frac{2173869}{128} - \frac{661181x}{32} - \frac{488267x^2}{16} + \frac{143635x^3}{8} + \frac{213325x^4}{4} + 8}{(3-x+2x^2)^2}$$

$$= -\frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)} + \frac{\int \frac{-\frac{5460539}{8} - \frac{626865x}{2} + \frac{5170975x^2}{8} + \frac{1124125x}{2}}{3-x+2x^2}}{1058}$$

$$= -\frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)} + \frac{\int \left(\frac{1441525}{4} + \frac{2578875x}{8} + \frac{330625x^2}{4} - \right.}{1058}$$

$$= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)} - \frac{121\int}{}$$

$$= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)} - \frac{1391}{64}$$

$$= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)} - \frac{1391}{64}$$

$$= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)} + \frac{63799}{}$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 1.00

$$\frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\,(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424\,(3-x+2x^2)} - \frac{63799791\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{16928\sqrt{23}} - \frac{13915}{64}\log\left(3-x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) - (637997
91*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2
])/64
```

Maple [A]
time = 0.11, size = 73, normalized size = 0.74

method	result
default	$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{2725x}{8} - \frac{121\left(-\frac{210155}{4232}x^3 + \frac{362791}{16928}x^2 - \frac{561121}{8464}x + \frac{54263}{16928}\right)}{4(2x^2-x+3)^2} - \frac{13915\ln(2x^2-x+3)}{64} - \frac{63799791\sqrt{23}\ \mathrm{arctan}}{38934}$

risch	$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}+\frac{\frac{25428755}{16928}x^3-\frac{43897711}{67712}x^2+\frac{67895641}{33856}x-\frac{6565823}{67712}}{(2x^2-x+3)^2}-\frac{13915\ln(16x^2-8x+24)}{64}-\frac{63799791\sqrt{23}\ \text{arct}}{3893}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 625/24*x^3+4875/32*x^2+2725/8*x-121/4*(-210155/4232*x^3+362791/16928*x^2-56
1121/8464*x+54263/16928)/(2*x^2-x+3)^2-13915/64*ln(2*x^2-x+3)-63799791/3893
44*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.50, size = 82, normalized size = 0.84

$$\frac{625}{24}\,x^3+\frac{4875}{32}\,x^2-\frac{63799791}{389344}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2725}{8}\,x+\frac{1331\,(76420\,x^3-32981\,x^2+102022\,x-4933)}{67712\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}-\frac{13915}{64}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4
*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/
(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 13915/64*log(2*x^2 - x + 3)
```

Fricas [A]
time = 2.17, size = 128, normalized size = 1.31

$$\frac{486680000\,x^7+2360398000\,x^6+5100406400\,x^5+2157209100\,x^4+24531516180\,x^3-765597492\,\sqrt{23}\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-6171678159\,x^2-1015822830\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(2\,x^2-x+3\right)+23692590858\,x-453041787}{4672128\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/4672128*(486680000*x^7 + 2360398000*x^6 + 5100406400*x^5 + 2157209100*x^4
 + 24531516180*x^3 - 765597492*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*
arctan(1/23*sqrt(23)*(4*x - 1)) - 6171678159*x^2 - 1015822830*(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 23692590858*x - 453041787)/(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [A]
time = 0.09, size = 95, normalized size = 0.97

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}+\frac{101715020x^3-43897711x^2+135791282x-6565823}{270848x^4-270848x^3+880256x^2-406272x+609408}-\frac{13915\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{64}-\frac{63799791\sqrt{23}\ \operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{389344}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**3,x)
```

```
[Out] 625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 + 1
35791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x +
 609408) - 13915*log(x**2 - x/2 + 3/2)/64 - 63799791*sqrt(23)*atan(4*sqrt(2
3)*x/23 - sqrt(23)/23)/389344
```

Giac [A]
time = 1.34, size = 72, normalized size = 0.73

$$\frac{625}{24}x^3+\frac{4875}{32}x^2-\frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{2725}{8}x+\frac{1331\,(76420\,x^3-32981\,x^2+102022\,x-4933)}{67712\,(2\,x^2-x+3)^2}-\frac{13915}{64}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4
*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/
(2*x^2 - x + 3)^2 - 13915/64*log(2*x^2 - x + 3)
```

Mupad [B]
time = 0.05, size = 81, normalized size = 0.83

$$\frac{2725\,x}{8}-\frac{13915\,\ln\left(2\,x^2-x+3\right)}{64}-\frac{63799791\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{389344}+\frac{4875\,x^2}{32}+\frac{625\,x^3}{24}+\frac{\frac{25428755\,x^3}{67712}-\frac{43897711\,x^2}{270848}+\frac{67895641\,x}{135424}-\frac{6565823}{270848}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^3,x)
```

```
[Out] (2725*x)/8 - (13915*log(2*x^2 - x + 3))/64 - (63799791*23^(1/2)*atan((4*23^
(1/2)*x)/23 - 23^(1/2)/23))/389344 + (4875*x^2)/32 + (625*x^3)/24 + ((67895
641*x)/135424 - (43897711*x^2)/270848 + (25428755*x^3)/67712 - 6565823/2708
48)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4)
```

3.52 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} \, dx$

Optimal. Leaf size=84

$$\frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}+\frac{825}{32}\log\left(3-x+2x^2\right)$$

```
[Out] 125/8*x-1331/1472*(17-45*x)/(2*x^2-x+3)^2+121/33856*(21193-12828*x)/(2*x^2-
x+3)+825/32*ln(2*x^2-x+3)+165099/194672*arctan(1/23*(1-4*x)*23^(1/2))*23^(1
/2)
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{165099\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}+\frac{121(21193-12828x)}{33856\left(2x^2-x+3\right)}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 - (1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + (121*(21193 - 128
28*x))/(33856*(3 - x + 2*x^2)) + (165099*ArcTan[(1 - 4*x)/Sqrt[23]])/(8464*
Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{1}{46}\int \frac{-\frac{40885}{32} - \frac{19067x}{8} + \frac{22195x^2}{4} + \frac{13225x^3}{2} + 2875x^4}{(3-x+2x^2)^2}\,dx$$
$$= -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{\int \frac{\frac{23997}{2}+92575x+\frac{66125x^2}{2}}{3-x+2x^2}\,dx}{1058}$$
$$= -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{\int \left(\frac{66125}{4} - \frac{33(4557-13225x)}{4(3-x+2x^2)}\right)dx}{1058}$$
$$= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} - \frac{33\int \frac{4557-13225x}{3-x+2x^2}\,dx}{4232}$$
$$= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} - \frac{165099\int \frac{1}{3-x+2x^2}\,dx}{16928} + \frac{825}{32}$$
$$= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{825}{32}\log\left(3-x+2x^2\right) + \frac{16}{}$$
$$= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{8}{3}$$

Mathematica [A]
time = 0.03, size = 84, normalized size = 1.00

$$\frac{125x}{8} + \frac{1331(-17+45x)}{1472(3-x+2x^2)^2} - \frac{121(-21193+12828x)}{33856(3-x+2x^2)} - \frac{165099\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{825}{32}\log\left(3-x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 + (1331*(-17 + 45*x))/(1472*(3 - x + 2*x^2)^2) - (121*(-21193 + 1
2828*x))/(33856*(3 - x + 2*x^2)) - (165099*ArcTan[(-1 + 4*x)/Sqrt[23]])/(84
64*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

Maple [A]
time = 0.12, size = 63, normalized size = 0.75

method	result	size
default	$\frac{125x}{8}+\frac{-\frac{388047}{4232}x^3+\frac{3340447}{16928}x^2-\frac{1460833}{8464}x+\frac{3586319}{16928}}{(2x^2-x+3)^2}+\frac{825\ln(2x^2-x+3)}{32}-\frac{165099\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{194672}$	63

risch	$\frac{125x}{8}+\frac{-\frac{388047}{4232}x^{3}+\frac{3340447}{16928}x^{2}-\frac{1460833}{8464}x+\frac{3586319}{16928}}{\left(2x^{2}-x+3\right)^{2}}+\frac{825\ln\left(16x^{2}-8x+24\right)}{32}-\frac{165099\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{194672}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 125/8*x+11/2*(-35277/2116*x^3+303677/8464*x^2-132803/4232*x+326029/8464)/(2
*x^2-x+3)^2+825/32*ln(2*x^2-x+3)-165099/194672*23^(1/2)*arctan(1/23*(4*x-1)
*23^(1/2))
```

Maxima [A]
time = 0.51, size = 72, normalized size = 0.86

$$-\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{125}{8}x-\frac{121\left(12828x^{3}-27607x^{2}+24146x-29639\right)}{16928\left(4x^{4}-4x^{3}+13x^{2}-6x+9\right)}+\frac{825}{32}\log\left(2x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/169
28*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x
+ 9) + 825/32*log(2*x^2 - x + 3)
```

Fricas [A]
time = 1.91, size = 118, normalized size = 1.40

$$\frac{24334000x^{5}-24334000x^{4}+43385176x^{3}-330198\sqrt{23}\left(4x^{4}-4x^{3}+13x^{2}-6x+9\right)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+40329281x^{2}+10037775\left(4x^{4}-4x^{3}+13x^{2}-6x+9\right)\log\left(2x^{2}-x+3\right)-12446818x+82485337}{389344\left(4x^{4}-4x^{3}+13x^{2}-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/389344*(24334000*x^5 - 24334000*x^4 + 43385176*x^3 - 330198*sqrt(23)*(4*x
^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) + 40329281*x
^2 + 10037775*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) - 12446
818*x + 82485337)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [A]
time = 0.09, size = 82, normalized size = 0.98

$$\frac{125x}{8}+\frac{-1552188x^{3}+3340447x^{2}-2921666x+3586319}{67712x^{4}-67712x^{3}+220064x^{2}-101568x+152352}+\frac{825\log\left(x^{2}-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{165099\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{194672}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**3,x)
```

```
[Out] 125*x/8 + (-1552188*x**3 + 3340447*x**2 - 2921666*x + 3586319)/(67712*x**4
- 67712*x**3 + 220064*x**2 - 101568*x + 152352) + 825*log(x**2 - x/2 + 3/2)
/32 - 165099*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/194672
```

Giac [A]
time = 2.00, size = 62, normalized size = 0.74

$$-\frac{165099}{194672}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{125}{8}\,x-\frac{121\,(12828\,x^3-27607\,x^2+24146\,x-29639)}{16928\,(2\,x^2-x+3)^2}+\frac{825}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/169
28*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(2*x^2 - x + 3)^2 + 825/32*log
(2*x^2 - x + 3)
```

Mupad [B]
time = 0.05, size = 72, normalized size = 0.86

$$\frac{125\,x}{8}+\frac{825\,\ln\left(2\,x^2-x+3\right)}{32}-\frac{165099\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{194672}-\frac{\frac{388047\,x^3}{16928}-\frac{3340447\,x^2}{67712}+\frac{1460833\,x}{33856}-\frac{3586319}{67712}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^3,x)
```

```
[Out] (125*x)/8 + (825*log(2*x^2 - x + 3))/32 - (165099*23^(1/2)*atan((4*23^(1/2)
*x)/23 - 23^(1/2)/23))/194672 - ((1460833*x)/33856 - (3340447*x^2)/67712 +
(388047*x^3)/16928 - 3586319/67712)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4
)
```

3.53 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} \, dx$

Optimal. Leaf size=64

$$\frac{121(19-7x)}{368(3-x+2x^2)^2}-\frac{55(975+332x)}{8464(3-x+2x^2)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

```
[Out] 121/368*(19-7*x)/(2*x^2-x+3)^2-55/8464*(975+332*x)/(2*x^2-x+3)-4330/12167*a
rctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1674, 12, 632, 210}

$$-\frac{4330\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}+\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{55(332x+975)}{8464(2x^2-x+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (55*(975 + 332*x))/(8464*(3 - x
+ 2*x^2)) - (4330*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx &= \frac{121(19-7x)}{368\,(3-x+2x^2)^2}+\frac{1}{46}\int \frac{-\frac{195}{8}+\frac{1955x}{2}+575x^2}{(3-x+2x^2)^2}\,dx \\
&= \frac{121(19-7x)}{368\,(3-x+2x^2)^2}-\frac{55(975+332x)}{8464\,(3-x+2x^2)}+\frac{\int \frac{4330}{3-x+2x^2}\,dx}{1058} \\
&= \frac{121(19-7x)}{368\,(3-x+2x^2)^2}-\frac{55(975+332x)}{8464\,(3-x+2x^2)}+\frac{2165}{529}\int \frac{1}{3-x+2x^2}\,dx \\
&= \frac{121(19-7x)}{368\,(3-x+2x^2)^2}-\frac{55(975+332x)}{8464\,(3-x+2x^2)}-\frac{4330}{529}\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx, x, -1+4x \\
&= \frac{121(19-7x)}{368\,(3-x+2x^2)^2}-\frac{55(975+332x)}{8464\,(3-x+2x^2)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.80

$$-\frac{11(4909+938x+4045x^2+1660x^3)}{4232\,(-3+x-2x^2)^2}+\frac{4330\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (-11*(4909 + 938*x + 4045*x^2 + 1660*x^3))/(4232*(-3 + x - 2*x^2)^2) + (433
0*ArcTan[(-1 + 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Maple [A]
time = 0.13, size = 47, normalized size = 0.73

method	result	size

default	$\frac{-\frac{4565}{1058}x^3-\frac{44495}{4232}x^2-\frac{5159}{2116}x-\frac{53999}{4232}}{(2x^2-x+3)^2}+\frac{4330\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}$	47
risch	$\frac{-\frac{4565}{1058}x^3-\frac{44495}{4232}x^2-\frac{5159}{2116}x-\frac{53999}{4232}}{(2x^2-x+3)^2}+\frac{4330\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(-4565/4232*x^3-44495/16928*x^2-5159/8464*x-53999/16928)/(2*x^2-x+3)^2+43
30/12167*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.50, size = 56, normalized size = 0.88

$$\frac{4330}{12167}\sqrt{23}\,\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{11\,(1660\,x^3+4045\,x^2+938\,x+4909)}{4232\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4
045*x^2 + 938*x + 4909)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Fricas [A]
time = 1.76, size = 75, normalized size = 1.17

$$-\frac{419980\,x^3-34640\,\sqrt{23}\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+1023385\,x^2+237314\,x+1241977}{97336\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] -1/97336*(419980*x^3 - 34640*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1023385*x^2 + 237314*x + 1241977)/(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [A]
time = 0.07, size = 63, normalized size = 0.98

$$\frac{-18260x^3-44495x^2-10318x-53999}{16928x^4-16928x^3+55016x^2-25392x+38088}+\frac{4330\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**3,x)
```

```
[Out] (-18260*x**3 - 44495*x**2 - 10318*x - 53999)/(16928*x**4 - 16928*x**3 + 550
16*x**2 - 25392*x + 38088) + 4330*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/
23)/12167
```

Giac [A]
time = 0.95, size = 46, normalized size = 0.72

$$\frac{4330}{12167}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{11\,(1660\,x^3+4045\,x^2+938\,x+4909)}{4232\,(2\,x^2-x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4
045*x^2 + 938*x + 4909)/(2*x^2 - x + 3)^2
```

Mupad [B]
time = 3.47, size = 56, normalized size = 0.88

$$\frac{4330\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}-\frac{\frac{4565\,x^3}{4232}+\frac{44495\,x^2}{16928}+\frac{5159\,x}{8464}+\frac{53999}{16928}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^3,x)
```

```
[Out] (4330*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 - ((5159*x)/846
4 + (44495*x^2)/16928 + (4565*x^3)/4232 + 53999/16928)/((13*x^2)/4 - (3*x)/
2 - x^3 + x^4 + 9/4)
```

3.54 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

```
[Out] -11/92*(5+3*x)/(2*x^2-x+3)^2-131/2116*(1-4*x)/(2*x^2-x+3)-262/12167*arctan(
1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1674, 12, 628, 632, 210}

$$-\frac{262\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}-\frac{131(1-4x)}{2116\left(2x^2-x+3\right)}-\frac{11(3x+5)}{92\left(2x^2-x+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] (-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) - (131*(1 - 4*x))/(2116*(3 - x + 2*x
^2)) - (262*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3}\,dx &= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{131}{2\left(3-x+2x^2\right)^2}\,dx\\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}+\frac{131}{92}\int \frac{1}{\left(3-x+2x^2\right)^2}\,dx\\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}+\frac{131}{529}\int \frac{1}{3-x+2x^2}\,dx\\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}-\frac{262}{529}\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right)\\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 51, normalized size = 0.80

$$
\frac{\frac{46\left(-829+472x-393x^2+524x^3\right)}{\left(-3+x-2x^2\right)^2}+1048\sqrt{23}\,\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{48668}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] ((46*(-829 + 472*x - 393*x^2 + 524*x^3))/(-3 + x - 2*x^2)^2 + 1048*Sqrt[23]
*ArcTan[(-1 + 4*x)/Sqrt[23]])/48668
```

Maple [A]
time = 0.11, size = 47, normalized size = 0.73

method	result	size
default	$\frac{\frac{262}{529}x^3-\frac{393}{1058}x^2+\frac{236}{529}x-\frac{829}{1058}}{(2x^2-x+3)^2}+\frac{262\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}$	47
risch	$\frac{\frac{262}{529}x^3-\frac{393}{1058}x^2+\frac{236}{529}x-\frac{829}{1058}}{(2x^2-x+3)^2}+\frac{262\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(131/1058*x^3-393/4232*x^2+59/529*x-829/4232)/(2*x^2-x+3)^2+262/12167*23^
(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

Maxima [A]
time = 0.51, size = 56, normalized size = 0.88

$$\frac{262}{12167}\sqrt{23}\,\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{524\,x^3-393\,x^2+472\,x-829}{1058\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*
x^2 + 472*x - 829)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Fricas [A]
time = 1.57, size = 75, normalized size = 1.17

$$\frac{12052\,x^3+524\,\sqrt{23}\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-9039\,x^2+10856\,x-19067}{24334\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/24334*(12052*x^3 + 524*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan
(1/23*sqrt(23)*(4*x - 1)) - 9039*x^2 + 10856*x - 19067)/(4*x^4 - 4*x^3 + 13
*x^2 - 6*x + 9)
```

Sympy [A]
time = 0.07, size = 61, normalized size = 0.95

$$\frac{524x^3 - 393x^2 + 472x - 829}{4232x^4 - 4232x^3 + 13754x^2 - 6348x + 9522} + \frac{262\sqrt{23}\ \operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**3,x)
```

```
[Out] (524*x**3 - 393*x**2 + 472*x - 829)/(4232*x**4 - 4232*x**3 + 13754*x**2 - 6
348*x + 9522) + 262*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/12167
```

Giac [A]
time = 1.43, size = 46, normalized size = 0.72

$$\frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right) + \frac{524\,x^3 - 393\,x^2 + 472\,x - 829}{1058\,(2\,x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*
x^2 + 472*x - 829)/(2*x^2 - x + 3)^2
```

Mupad [B]
time = 0.04, size = 55, normalized size = 0.86

$$\frac{262\,\sqrt{23}\ \operatorname{atan}\left(\frac{4\,\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{12167} + \frac{\frac{131\,x^3}{1058} - \frac{393\,x^2}{4232} + \frac{59\,x}{529} - \frac{829}{4232}}{x^4 - x^3 + \frac{13\,x^2}{4} - \frac{3\,x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^3,x)
```

```
[Out] (262*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 + ((59*x)/529 -
(393*x^2)/4232 + (131*x^3)/1058 - 829/4232)/((13*x^2)/4 - (3*x)/2 - x^3 + x
^4 + 9/4)
```

3.55 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx$

Optimal. Leaf size=115

$$\frac{13-6x}{1012(3-x+2x^2)^2}+\frac{3625-746x}{256036(3-x+2x^2)}-\frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}+\frac{247\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{10648\sqrt{31}}-\frac{119\log(3-x+}{21296}$$

```
[Out] 1/1012*(13-6*x)/(2*x^2-x+3)^2+1/256036*(3625-746*x)/(2*x^2-x+3)-119/21296*l
n(2*x^2-x+3)+119/21296*ln(5*x^2+3*x+2)-53403/129554216*arctan(1/23*(1-4*x)*
23^(1/2))*23^(1/2)+247/330088*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {988, 1074, 1086, 648, 632, 210, 642}

$$-\frac{53403\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}+\frac{247\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{10648\sqrt{31}}+\frac{3625-746x}{256036(2x^2-x+3)}+\frac{13-6x}{1012(2x^2-x+3)^2}-\frac{119\log(2x^2-x+3)}{21296}+\frac{119\log(5x^2+3x+2)}{21296}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + (3625 - 746*x)/(256036*(3 - x + 2*x^2
)) - (53403*ArcTan[(1 - 4*x)/Sqrt[23]])/(5632792*Sqrt[23]) + (247*ArcTan[(3
 + 10*x)/Sqrt[31]])/(10648*Sqrt[31]) - (119*Log[3 - x + 2*x^2])/21296 + (11
9*Log[2 + 3*x + 5*x^2])/21296
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
```

```
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3\,(2+3x+5x^2)}\,dx &= \frac{13-6x}{1012\,(3-x+2x^2)^2} - \frac{\int \frac{-3652-1936x+990x^2}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx}{11132} \\
&= \frac{13-6x}{1012\,(3-x+2x^2)^2} + \frac{3625-746x}{256036\,(3-x+2x^2)} - \frac{\int \frac{-6551908-7779574x+9\ldots}{(3-x+2x^2)(2+3x+5\ldots}}{61960712} \\
&= \frac{13-6x}{1012\,(3-x+2x^2)^2} + \frac{3625-746x}{256036\,(3-x+2x^2)} - \frac{\int \frac{-154867174+335151124\ldots}{3-x+2x^2}}{14994492304} \\
&= \frac{13-6x}{1012\,(3-x+2x^2)^2} + \frac{3625-746x}{256036\,(3-x+2x^2)} + \frac{53403\int \frac{1}{3-x+2x^2}\,dx}{11265584} \ldots \\
&= \frac{13-6x}{1012\,(3-x+2x^2)^2} + \frac{3625-746x}{256036\,(3-x+2x^2)} - \frac{119\log(3-x+2x^2\ldots}{21296} \\
&= \frac{13-6x}{1012\,(3-x+2x^2)^2} + \frac{3625-746x}{256036\,(3-x+2x^2)} - \frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.\ldots}{5632792\sqrt{23}}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 99, normalized size = 0.86

$$
\frac{3310986\sqrt{23}\,\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right) + 6010498\sqrt{31}\,\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right) + 713\left(-\frac{44(-14164+7381x-7996x^2+1492x^3)}{(-3+x-2x^2)^2} - 62951\log(3-x+2x^2) + 62951\log(2+3x+5x^2)\right)}{8032361392}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

[Out] (3310986*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 6010498*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 713*((-44*(-14164 + 7381*x - 7996*x^2 + 1492*x^3))/(-3 + x - 2*x^2)^2 - 62951*Log[3 - x + 2*x^2] + 62951*Log[2 + 3*x + 5*x^2]))/8032361392

Maple [A]
time = 0.16, size = 89, normalized size = 0.77

method	result
default	$-\frac{\frac{8206}{529}x^3-\frac{43978}{529}x^2+\frac{81191}{1058}x-\frac{77902}{529}}{2662(2x^2-x+3)^2}-\frac{119\ln(2x^2-x+3)}{21296}+\frac{53403\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{129554216}+\frac{119\ln(5x^2+3x+2)}{21296}+\frac{247}{}$
risch	$\frac{-\frac{373}{64009}x^3+\frac{1999}{64009}x^2-\frac{61}{2116}x+\frac{3541}{64009}}{(2x^2-x+3)^2}-\frac{119\ln(16x^2-8x+24)}{21296}+\frac{53403\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{129554216}+\frac{247\arctan\left(\frac{(3+10x)\sqrt{3}}{31}\right.}{330088}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)

[Out] -1/2662*(8206/529*x^3-43978/529*x^2+81191/1058*x-77902/529)/(2*x^2-x+3)^2-119/21296*ln(2*x^2-x+3)+53403/129554216*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))+119/21296*ln(5*x^2+3*x+2)+247/330088*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)

Maxima [A]
time = 0.52, size = 98, normalized size = 0.85

$$\frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1492x^3-7996x^2+7381x-14164}{256036(4x^4-4x^3+13x^2-6x+9)}+\frac{119}{21296}\log(5x^2+3x+2)-\frac{119}{21296}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")

[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381*x - 14164)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) + 119/21296*log(5*x^2 + 3*x + 2) - 119/21296*log(2*x^2 - x + 3)

Fricas [A]
time = 2.03, size = 177, normalized size = 1.54

$$-\frac{46807024x^3-6010498\sqrt{31}(4x^4-4x^3+13x^2-6x+9)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-3310986\sqrt{23}(4x^4-4x^3+13x^2-6x+9)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-250850512x^2-44884063(4x^4-4x^3+13x^2-6x+9)\log(5x^2+3x+2)+44884063(4x^4-4x^3+13x^2-6x+9)\log(2x^2-x+3)+231556732x-444353008}{8032361392(4x^4-4x^3+13x^2-6x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")

[Out] -1/8032361392*(46807024*x^3 - 6010498*sqrt(31)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3310986*sqrt(23)*(4*x^4 - 4*x^3 +

```
 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) - 250850512*x^2 - 448840
63*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(5*x^2 + 3*x + 2) + 44884063*(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 231556732*x - 444353008)
/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [A]
time = 0.15, size = 122, normalized size = 1.06

$$\frac{-1492x^3 + 7996x^2 - 7381x + 14164}{1024144x^4 - 1024144x^3 + 3328468x^2 - 1536216x + 2304324} - \frac{119\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{21296} + \frac{119\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{21296} + \frac{53403\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{129554216} + \frac{247\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{330088}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] (-1492*x**3 + 7996*x**2 - 7381*x + 14164)/(1024144*x**4 - 1024144*x**3 + 33
28468*x**2 - 1536216*x + 2304324) - 119*log(x**2 - x/2 + 3/2)/21296 + 119*l
og(x**2 + 3*x/5 + 2/5)/21296 + 53403*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(2
3)/23)/129554216 + 247*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3300
88
```

Giac [A]
time = 1.97, size = 88, normalized size = 0.77

$$\frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - \frac{1492x^3 - 7996x^2 + 7381x - 14164}{256036(2x^2 - x + 3)^2} + \frac{119}{21296}\log\left(5x^2 + 3x + 2\right) - \frac{119}{21296}\log\left(2x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381
*x - 14164)/(2*x^2 - x + 3)^2 + 119/21296*log(5*x^2 + 3*x + 2) - 119/21296*
log(2*x^2 - x + 3)
```

Mupad [B]
time = 3.58, size = 116, normalized size = 1.01

$$-\ln\left(x + \frac{3}{10} - \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{119}{21296} + \frac{\sqrt{31}\,247\mathrm{i}}{660176}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{119}{21296} + \frac{\sqrt{31}\,247\mathrm{i}}{660176}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{119}{21296} + \frac{\sqrt{23}\,53403\mathrm{i}}{259108432}\right) + \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{119}{21296} + \frac{\sqrt{23}\,53403\mathrm{i}}{259108432}\right) - \frac{\frac{373x^3}{256036} - \frac{1999x^2}{256036} + \frac{61x}{8464} - \frac{3541}{256036}}{x^4 - x^3 + \frac{13x^2}{4} - \frac{3x}{2} + \frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 + 119/21296) - log
(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 - 119/21296) - log(x
- (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 + 119/21296) + log(x
+ (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 - 119/21296) - ((61*x
)/8464 - (1999*x^2)/256036 + (373*x^3)/256036 - 3541/256036)/((13*x^2)/4 -
(3*x)/2 - x^3 + x^4 + 9/4)
```

3.56 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=160

$$\frac{-2328909-252815x}{174616552\,(2+3x+5x^2)}+\frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)}+\frac{9665-1446x}{512072\,(3-x+2x^2)\,(2+3x+5x^2)}+\frac{2038\cdots}{\cdots}$$

```
[Out] 1/174616552*(-2328909-252815*x)/(5*x^2+3*x+2)+1/1012*(13-6*x)/(2*x^2-x+3)^2
/(5*x^2+3*x+2)+1/512072*(9665-1446*x)/(2*x^2-x+3)/(5*x^2+3*x+2)+181/468512*
ln(2*x^2-x+3)-181/468512*ln(5*x^2+3*x+2)+2038497/2850192752*arctan(1/23*(1-
4*x)*23^(1/2))*23^(1/2)+246757/225120016*arctan(1/31*(3+10*x)*31^(1/2))*31^
(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {988, 1074, 1086, 648, 632, 210, 642}

$$\frac{2038497\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{123921424\sqrt{23}}+\frac{246757\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{7261936\sqrt{31}}+\frac{9665-1446x}{512072\,(2x^2-x+3)\,(5x^2+3x+2)}-\frac{252815x+2328909}{174616552\,(5x^2+3x+2)}+\frac{13-6x}{1012\,(2x^2-x+3)^2\,(5x^2+3x+2)}+\frac{181\log(2x^2-x+3)}{468512}-\frac{181\log(5x^2+3x+2)}{468512}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -1/174616552*(2328909 + 252815*x)/(2 + 3*x + 5*x^2) + (13 - 6*x)/(1012*(3 -
 x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + (9665 - 1446*x)/(512072*(3 - x + 2*x^2)*
(2 + 3*x + 5*x^2)) + (2038497*ArcTan[(1 - 4*x)/Sqrt[23]])/(123921424*Sqrt[2
3]) + (246757*ArcTan[(3 + 10*x)/Sqrt[31]])/(7261936*Sqrt[31]) + (181*Log[3
- x + 2*x^2])/468512 - (181*Log[2 + 3*x + 5*x^2])/468512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
```

```
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx = \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} - \frac{\int \frac{-4081-3168x+1650x^2}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx}{11132}$$

$$= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x}$$

$$= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} +$$

$$= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} +$$

$$= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} +$$

$$= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} +$$

$$= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} +$$

Mathematica [A]
time = 0.09, size = 136, normalized size = 0.85

$$\frac{-31-14x}{22264(3-x+2x^2)^2} + \frac{-1782-2923x}{1408198(3-x+2x^2)} + \frac{-1474+1235x}{330088(2+3x+5x^2)} - \frac{2038497\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{123921424\sqrt{23}} + \frac{246757\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{7261936\sqrt{31}} + \frac{181\log(3-x+2x^2)}{468512} - \frac{181\log(2+3x+5x^2)}{468512}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-31 - 14*x)/(22264*(3 - x + 2*x^2)^2) + (-1782 - 2923*x)/(1408198*(3 - x +
 2*x^2)) + (-1474 + 1235*x)/(330088*(2 + 3*x + 5*x^2)) - (2038497*ArcTan[(-
1 + 4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(7261936*Sqrt[31]) + (181*Log[3 - x + 2*x^2])/468512 - (181*Log[2 + 3*
x + 5*x^2])/468512
```

Maple [A]
time = 0.16, size = 106, normalized size = 0.66

method	result
default	$\frac{-\frac{128612}{529}x^3-\frac{14102}{529}x^2-\frac{173195}{529}x-\frac{321497}{1058}}{58564(2x^2-x+3)^2}+\frac{181\ln(2x^2-x+3)}{468512}-\frac{2038497\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2850192752}-\frac{-\frac{5434x}{31}+\frac{32428}{155}}{234256(x^2+\frac{3}{5}x+\frac{2}{5})}$
risch	$\frac{-\frac{252815}{43654138}x^5-\frac{1038047}{21827069}x^4+\frac{5042869}{174616552}x^3-\frac{21674311}{174616552}x^2+\frac{1471955}{43654138}x-\frac{200677}{3968558}}{(2x^2-x+3)^2(5x^2+3x+2)}+\frac{181\ln(16x^2-8x+24)}{468512}-\frac{2038497\sqrt{23}\,\arctan(}{285019275}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/58564*(-128612/529*x^3-14102/529*x^2-173195/529*x-321497/1058)/(2*x^2-x+3
)^2+181/468512*ln(2*x^2-x+3)-2038497/2850192752*23^(1/2)*arctan(1/23*(4*x-1
)*23^(1/2))-1/234256*(-5434/31*x+32428/155)/(x^2+3/5*x+2/5)-181/468512*ln(5
*x^2+3*x+2)+246757/225120016*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.52, size = 116, normalized size = 0.72

$$\frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1011260x^5+8304376x^4-5042869x^3+21674311x^2-5887820x+8829788}{174616552(20x^6-8x^5+61x^4+x^3+53x^2+15x+18)}-\frac{181}{468512}\log(5x^2+3x+2)+\frac{181}{468512}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/285019
2752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 +
8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/(20*x^6 - 8
*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) - 181/468512*log(5*x^2 + 3*x + 2)
 + 181/468512*log(2*x^2 - x + 3)
```

Fricas [A]
time = 2.39, size = 227, normalized size = 1.42

$$-\frac{31725248720x^5+260524883872x^4-158204886268x^3-6004584838\sqrt{31}(20x^6-8x^5+61x^4+x^3+53x^2+15x+18)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+3917991234\sqrt{23}(20x^6-8x^5+61x^4+x^3+53x^2+15x+18)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+679966484692x^2+2116340147(20x^6-8x^5+61x^4+x^3+53x^2+15x+18)\log(5x^2+3x+2)-2116340147(20x^6-8x^5+61x^4+x^3+53x^2+15x+18)\log(2x^2-x+3)-184712689040x+277008109136}{5478070469344(20x^6-8x^5+61x^4+x^3+53x^2+15x+18)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/5478070469344*(31725248720*x^5 + 260524883872*x^4 - 158204886268*x^3 - 6
004584838*sqrt(31)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*arc
tan(1/31*sqrt(31)*(10*x + 3)) + 3917991234*sqrt(23)*(20*x^6 - 8*x^5 + 61*x^
4 + x^3 + 53*x^2 + 15*x + 18)*arctan(1/23*sqrt(23)*(4*x - 1)) + 67996648469
2*x^2 + 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*log
(5*x^2 + 3*x + 2) - 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15
*x + 18)*log(2*x^2 - x + 3) - 184712689040*x + 277008109136)/(20*x^6 - 8*x^
5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

Sympy [A]
time = 0.17, size = 143, normalized size = 0.89

$$\frac{-1011260x^5 - 8304376x^4 + 5042869x^3 - 21674311x^2 + 5887820x - 8829788}{3492331040x^6 - 1396932416x^5 + 10651609672x^4 + 174616552x^3 + 9254677256x^2 + 2619248280x + 3143097936} + \frac{181\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{468512} - \frac{181\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{468512} - \frac{2038497\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{2850192752} + \frac{246757\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{225120016}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

```
[Out] (-1011260*x**5 - 8304376*x**4 + 5042869*x**3 - 21674311*x**2 + 5887820*x -
8829788)/(3492331040*x**6 - 1396932416*x**5 + 10651609672*x**4 + 174616552*
x**3 + 9254677256*x**2 + 2619248280*x + 3143097936) + 181*log(x**2 - x/2 +
3/2)/468512 - 181*log(x**2 + 3*x/5 + 2/5)/468512 - 2038497*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/2850192752 + 246757*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/225120016
```

Giac [A]
time = 1.03, size = 110, normalized size = 0.69

$$\frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10x+3)\right) - \frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right) - \frac{1011260x^5 + 8304376x^4 - 5042869x^3 + 21674311x^2 - 5887820x + 8829788}{174616552\,(5x^2+3x+2)(2x^2-x+3)^2} - \frac{181}{468512}\log\left(5x^2+3x+2\right) + \frac{181}{468512}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/285019
2752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 +
8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/((5*x^2 + 3
*x + 2)*(2*x^2 - x + 3)^2) - 181/468512*log(5*x^2 + 3*x + 2) + 181/468512*l
og(2*x^2 - x + 3)
```

Mupad [B]
time = 3.60, size = 136, normalized size = 0.85

$$-\frac{\frac{50563\,x^5}{174616552} + \frac{1038047\,x^4}{436541380} - \frac{5042869\,x^3}{3492331040} + \frac{21674311\,x^2}{3492331040} - \frac{294391\,x}{174616552} + \frac{200677}{79371160}}{x^6 - \frac{2\,x^5}{5} + \frac{61\,x^4}{20} + \frac{x^3}{20} + \frac{53\,x^2}{20} + \frac{3\,x}{4} + \frac{9}{10}} - \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{181}{468512} + \frac{\sqrt{31}\,246757\mathrm{i}}{450240032}\right) + \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{181}{468512} + \frac{\sqrt{31}\,246757\mathrm{i}}{450240032}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{181}{468512} + \frac{\sqrt{23}\,2038497\mathrm{i}}{5700385504}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{181}{468512} + \frac{\sqrt{23}\,2038497\mathrm{i}}{5700385504}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 - 181/468512
) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 + 181/46
8512) - ((21674311*x^2)/3492331040 - (294391*x)/174616552 - (5042869*x^3)/3
492331040 + (1038047*x^4)/436541380 + (50563*x^5)/174616552 + 200677/793711
60)/((3*x)/4 + (53*x^2)/20 + x^3/20 + (61*x^4)/20 - (2*x^5)/5 + x^6 + 9/10)
 + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/5700385504 + 181/468
512) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/5700385504 - 181
/468512)
```

3.57 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=181

$$-\frac{5(223707+77020x)}{87308276\,(2+3x+5x^2)^2}+\frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2}+\frac{5(302-35x)}{64009\,(3-x+2x^2)\,(2+3x+5x^2)^2}+\frac{1}{14}$$

```
[Out] -5/87308276*(223707+77020*x)/(5*x^2+3*x+2)^2+1/1012*(13-6*x)/(2*x^2-x+3)^2/
(5*x^2+3*x+2)^2+5/64009*(302-35*x)/(2*x^2-x+3)/(5*x^2+3*x+2)^2+15/148860610
58*(2618306+7140435*x)/(5*x^2+3*x+2)+405/1288408*ln(2*x^2-x+3)-405/1288408*
ln(5*x^2+3*x+2)-880575/7838030068*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+27
68835/19191481364*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {988, 1074, 1086, 648, 632, 210, 642}

$$-\frac{880575\text{ArcTan}\left(\frac{1-4x}{\sqrt{23}}\right)}{340783916\sqrt{23}}+\frac{2768835\text{ArcTan}\left(\frac{10x+3}{\sqrt{31}}\right)}{619080044\sqrt{31}}+\frac{5(302-35x)}{64009\,(2x^2-x+3)\,(5x^2+3x+2)^2}+\frac{15(7140435x+2618306)}{14886061058\,(5x^2+3x+2)}-\frac{5(77020x+223707)}{87308276\,(5x^2+3x+2)^2}+\frac{13-6x}{1012\,(2x^2-x+3)^2\,(5x^2+3x+2)^2}+\frac{405\log(2x^2-x+3)}{1288408}-\frac{405\log(5x^2+3x+2)}{1288408}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-5*(223707 + 77020*x))/(87308276*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(1012*(
3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*(302 - 35*x))/(64009*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (15*(2618306 + 7140435*x))/(14886061058*(2 + 3*
x + 5*x^2)) - (880575*ArcTan[(1 - 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2
768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x +
 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
```

```
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^3\,(2+3x+5x^2)^3}\,dx = \frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2} - \frac{\int \frac{-4510-4400x+2310x^2}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx}{11132}$$

$$= \frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009\,(3-x+2x^2)\,(2+3x}$$

$$= -\frac{5(223707+77020x)}{87308276\,(2+3x+5x^2)^2} + \frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2}$$

$$= -\frac{5(223707+77020x)}{87308276\,(2+3x+5x^2)^2} + \frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2}$$

$$= -\frac{5(223707+77020x)}{87308276\,(2+3x+5x^2)^2} + \frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2}$$

$$= -\frac{5(223707+77020x)}{87308276\,(2+3x+5x^2)^2} + \frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2}$$

$$= -\frac{5(223707+77020x)}{87308276\,(2+3x+5x^2)^2} + \frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2}$$

$$= -\frac{5(223707+77020x)}{87308276\,(2+3x+5x^2)^2} + \frac{13-6x}{1012\,(3-x+2x^2)^2\,(2+3x+5x^2)^2}$$

Mathematica [A]
time = 0.07, size = 151, normalized size = 0.83

$$\frac{-4342+11154x-9275x^2+6850x^3}{345092\left(6+7x+16x^2+x^3+10x^4\right)^2}+\frac{5\left(14085977+51156233x-5711469x^2+42842610x^3\right)}{14886061058\left(6+7x+16x^2+x^3+10x^4\right)}+\frac{880575\tan^{-1}\left(\frac{-1+4x}{\sqrt{23}}\right)}{340783916\sqrt{23}}+\frac{2768835\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{619080044\sqrt{31}}+\frac{405\log\left(3-x+2x^2\right)}{1288408}-\frac{405\log\left(2+3x+5x^2\right)}{1288408}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-4342 + 11154*x - 9275*x^2 + 6850*x^3)/(345092*(6 + 7*x + 16*x^2 + x^3 + 1
0*x^4)^2) + (5*(14085977 + 51156233*x - 5711469*x^2 + 42842610*x^3))/(14886
061058*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)) + (880575*ArcTan[(-1 + 4*x)/Sqrt[
23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(6190800
44*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2]
)/1288408
```

Maple [A]
time = 0.14, size = 118, normalized size = 0.65

method	result
default	$\frac{\frac{302907}{529}x^3-\frac{368291}{529}x^2+\frac{2501587}{2116}x-\frac{665819}{1058}}{644204\left(2x^2-x+3\right)^2}+\frac{405\ln\left(2x^2-x+3\right)}{1288408}+\frac{880575\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{7838030068}-\frac{25\left(-\frac{3013197}{961}x^3-\frac{145}{4}\right.}{25768}$
risch	$\frac{\frac{1071065250}{7443030529}x^7-\frac{35680200}{7443030529}x^6+\frac{5956663105}{14886061058}x^5+\frac{2002653845}{14886061058}x^4+\frac{5543790435}{14886061058}x^3+\frac{4691822415}{29772122116}x^2+\frac{1254420353}{7443030529}x+\frac{235280627}{14886061058}}{\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}-\frac{405\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/644204*(302907/529*x^3-368291/529*x^2+2501587/2116*x-665819/1058)/(2*x^2-
x+3)^2+405/1288408*ln(2*x^2-x+3)+880575/7838030068*23^(1/2)*arctan(1/23*(4*
x-1)*23^(1/2))-25/2576816*(-3013197/961*x^3-14516062/4805*x^2-51193868/2402
5*x-5423968/24025)/(5*x^2+3*x+2)^2-405/1288408*ln(5*x^2+3*x+2)+2768835/1919
1481364*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

Maxima [A]
time = 0.52, size = 138, normalized size = 0.76

$$\frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10x+3)\right)+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{4284261000x^7-142720800x^6+11913326210x^5+4005307690x^4+11087580870x^3+4691822415x^2+5017681412x+470561254}{29772122116\left(100x^8+20x^7+321x^6+172x^5+390x^4+236x^3+241x^2+84x+36\right)}-\frac{405}{1288408}\log\left(5x^2+3x+2\right)+\frac{405}{1288408}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838
030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000
```

```
*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 +
 4691822415*x^2 + 5017681412*x + 470561254)/(100*x^8 + 20*x^7 + 321*x^6 + 1
72*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) - 405/1288408*log(5*x^2 +
 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

Fricas [A]
time = 2.09, size = 297, normalized size = 1.64

$$\frac{67202918046000\,x^{7}-2238718468800\,x^{6}+186872434930060\,x^{5}+62827256425340\,x^{4}+173919793526820\,x^{3}+67376830890\,\sqrt{31}\,(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36)\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+52466419650\,\sqrt{23}\,(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36)\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+73595926401690\,x^{2}-146799174285\,(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36)\log\left(5\,x^{2}+3\,x+2\right)+146799174285\,(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36)\log\left(2\,x^{2}-x+3\right)+78707350628632\,x+7381223830244}{467005507511576\,(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/467005507511576*(67202918046000*x^7 - 2238718468800*x^6 + 186872434930060
*x^5 + 62827256425340*x^4 + 173919793526820*x^3 + 67376830890*sqrt(31)*(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 52466419650*sqrt(23)*(100*x^8 + 20*x^7
 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*arctan(1/23
*sqrt(23)*(4*x - 1)) + 73595926401690*x^2 - 146799174285*(100*x^8 + 20*x^7
+ 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(5*x^2 +
3*x + 2) + 146799174285*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 2
36*x^3 + 241*x^2 + 84*x + 36)*log(2*x^2 - x + 3) + 78707350628632*x + 73812
23830244)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x
^2 + 84*x + 36)
```

Sympy [A]
time = 0.20, size = 163, normalized size = 0.90

$$\frac{4284261000x^{7}-142720800x^{6}+11913326210x^{5}+4005307690x^{4}+11087580870x^{3}+4691822415x^{2}+5017681412x+470561254}{2977212211600x^{8}+595442442320x^{7}+9556851199236x^{6}+5120805003952x^{5}+11611127625240x^{4}+7026220819376x^{3}+7175081429956x^{2}+2500858257744x+1071796396176}+\frac{405\log\left(x^{2}-\frac{x}{2}+\frac{3}{2}\right)}{1288408}-\frac{405\log\left(x^{2}+\frac{3x}{5}+\frac{2}{5}\right)}{1288408}+\frac{880575\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{7838030068}+\frac{2768835\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{19191481364}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

```
[Out] (4284261000*x**7 - 142720800*x**6 + 11913326210*x**5 + 4005307690*x**4 + 11
087580870*x**3 + 4691822415*x**2 + 5017681412*x + 470561254)/(2977212211600
*x**8 + 595442442320*x**7 + 9556851199236*x**6 + 5120805003952*x**5 + 11611
127625240*x**4 + 7026220819376*x**3 + 7175081429956*x**2 + 2500858257744*x
+ 1071796396176) + 405*log(x**2 - x/2 + 3/2)/1288408 - 405*log(x**2 + 3*x/5
 + 2/5)/1288408 + 880575*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/78380
30068 + 2768835*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19191481364
```

Giac [A]
time = 0.77, size = 116, normalized size = 0.64

$$\frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{4284261000\,x^{7}-142720800\,x^{6}+11913326210\,x^{5}+4005307690\,x^{4}+11087580870\,x^{3}+4691822415\,x^{2}+5017681412\,x+470561254}{29772122116\,(10\,x^{4}+x^{3}+16\,x^{2}+7\,x+6)^{2}}-\frac{405}{1288408}\log\left(5\,x^{2}+3\,x+2\right)+\frac{405}{1288408}\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838
030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000
*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 +
 4691822415*x^2 + 5017681412*x + 470561254)/(10*x^4 + x^3 + 16*x^2 + 7*x +
6)^2 - 405/1288408*log(5*x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

Mupad [B]
time = 3.59, size = 155, normalized size = 0.86

$$\frac{\frac{21421305\,x^7}{14886061058}-\frac{356802\,x^6}{7443030529}+\frac{1191332621\,x^5}{297721221160}+\frac{400530769\,x^4}{297721221160}+\frac{1108758087\,x^3}{297721221160}+\frac{938364483\,x^2}{595442442320}+\frac{1254420353\,x}{744303052900}+\frac{235280627}{1488606105800}}{x^8+\frac{x^7}{5}+\frac{321\,x^6}{100}+\frac{43\,x^5}{25}+\frac{39\,x^4}{10}+\frac{59\,x^3}{25}+\frac{241\,x^2}{100}+\frac{21\,x}{25}+\frac{9}{25}}+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\,\left(\frac{405}{1288408}+\frac{\sqrt{23}\,880575\mathrm{i}}{15676060136}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\,\left(\frac{405}{1288408}+\frac{\sqrt{31}\,2768835\mathrm{i}}{38382962728}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\,\left(-\frac{405}{1288408}+\frac{\sqrt{31}\,2768835\mathrm{i}}{38382962728}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\,\left(-\frac{405}{1288408}+\frac{\sqrt{23}\,880575\mathrm{i}}{15676060136}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 + 405/128840
8) - log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 - 405/1
288408) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/38382962728
 + 405/1288408) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/383
82962728 - 405/1288408) + ((1254420353*x)/744303052900 + (938364483*x^2)/59
5442442320 + (1108758087*x^3)/297721221160 + (400530769*x^4)/297721221160 +
 (1191332621*x^5)/297721221160 - (356802*x^6)/7443030529 + (21421305*x^7)/1
4886061058 + 235280627/1488606105800)/((21*x)/25 + (241*x^2)/100 + (59*x^3)
/25 + (39*x^4)/10 + (43*x^5)/25 + (321*x^6)/100 + x^7/5 + x^8 + 9/25)
```

3.58 $\int \sqrt{3-x+2x^2}\,(2+3x+5x^2)^4\,dx$

Optimal. Leaf size=208

$$-\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541(3-x+2x^2)^{3/2}}{440401920}+\frac{804243809x(3-x+2x^2)^{3/2}}{36700160}-8394835$$

```
[Out] 27185733541/440401920*(2*x^2-x+3)^(3/2)+804243809/36700160*x*(2*x^2-x+3)^(3
/2)-83948353/2293760*x^2*(2*x^2-x+3)^(3/2)+8325631/1032192*x^3*(2*x^2-x+3)^
(3/2)+4796405/43008*x^4*(2*x^2-x+3)^(3/2)+233225/1536*x^5*(2*x^2-x+3)^(3/2)
+14125/144*x^6*(2*x^2-x+3)^(3/2)+125/4*x^7*(2*x^2-x+3)^(3/2)-8267844569/268
435456*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-359471503/67108864*(1-4*x)*(2
*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$-\frac{83948353(2x^2-x+3)^{3/2}x^2}{2293760}+\frac{804243809(2x^2-x+3)^{3/2}x}{36700160}+\frac{27185733541(2x^2-x+3)^{3/2}}{440401920}-\frac{359471503(1-4x)\sqrt{2x^2-x+3}}{67108864}+\frac{125}{4}(2x^2-x+3)^{3/2}x^7+\frac{14125}{144}(2x^2-x+3)^{3/2}x^6+\frac{233225(2x^2-x+3)^{3/2}x^5}{1536}+\frac{4796405(2x^2-x+3)^{3/2}x^4}{43008}+\frac{8325631(2x^2-x+3)^{3/2}x^3}{1032192}-\frac{8267844569\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{134217728\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-359471503*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/67108864 + (27185733541*(3 - x +
 2*x^2)^(3/2))/440401920 + (804243809*x*(3 - x + 2*x^2)^(3/2))/36700160 - (
83948353*x^2*(3 - x + 2*x^2)^(3/2))/2293760 + (8325631*x^3*(3 - x + 2*x^2)^
(3/2))/1032192 + (4796405*x^4*(3 - x + 2*x^2)^(3/2))/43008 + (233225*x^5*(3
 - x + 2*x^2)^(3/2))/1536 + (14125*x^6*(3 - x + 2*x^2)^(3/2))/144 + (125*x^
7*(3 - x + 2*x^2)^(3/2))/4 - (8267844569*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1342
17728*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx = \frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+\frac{1}{20}\int \sqrt{3-x+2x^2}\left(320+1920x+7520\right.$$

$$= \frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2}+\frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+\frac{1}{360}\int \sqrt{3-x}$$

$$= \frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2}+\frac{125}{4}x^7\left(3-\right.$$

$$= \frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008}+\frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{14125}{144}x^6$$

$$= \frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192}+\frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008}+\frac{233225x}{}$$

$$= -\frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760}+\frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192}+\frac{4796}{}$$

$$= \frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160}-\frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760}+\frac{83256}{}$$

$$= \frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160}-\frac{839}{}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}$$

Mathematica [A]
time = 0.81, size = 95, normalized size = 0.46

$$\frac{4\sqrt{3-x+2x^2}\,(3801512106459+537752185764x-174418077792x^2+2211683657856x^3+5354741991424x^4+7612808028160x^5+7725962035200x^6+6327795712000x^7+3486515200000x^8+1321205760000x^9)-2604371039235\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{84557168640}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3801512106459 + 537752185764*x - 174418077792*x^2 +
 2211683657856*x^3 + 5354741991424*x^4 + 7612808028160*x^5 + 7725962035200*
```

```
x^6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9) - 26043710
39235*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/84557168640
```

Maple [A]
time = 0.14, size = 166, normalized size = 0.80

method	result
risch	$\frac{(1321205760000x^9+3486515200000x^8+6327795712000x^7+7725962035200x^6+7612808028160x^5+5354741991424x^4+22116836578}{21139292160}$
trager	$\left(\frac{125}{2}x^9+\frac{11875}{72}x^8+\frac{689675}{2304}x^7+\frac{7859255}{21504}x^6+\frac{185859571}{516096}x^5+\frac{373517159}{1474560}x^4+\frac{5759592859}{55050240}x^3-\frac{259550711}{31457280}x^2+\right.$
default	$\frac{27185733541(2x^2-x+3)^{\frac{3}{2}}}{440401920}+\frac{359471503(4x-1)\sqrt{2x^2-x+3}}{67108864}+\frac{8267844569\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{268435456}+\frac{804243}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 27185733541/440401920*(2*x^2-x+3)^(3/2)+359471503/67108864*(4*x-1)*(2*x^2-x
+3)^(1/2)+8267844569/268435456*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+80424
3809/36700160*x*(2*x^2-x+3)^(3/2)-83948353/2293760*x^2*(2*x^2-x+3)^(3/2)+83
25631/1032192*x^3*(2*x^2-x+3)^(3/2)+4796405/43008*x^4*(2*x^2-x+3)^(3/2)+233
225/1536*x^5*(2*x^2-x+3)^(3/2)+14125/144*x^6*(2*x^2-x+3)^(3/2)+125/4*x^7*(2
*x^2-x+3)^(3/2)
```

Maxima [A]
time = 0.52, size = 177, normalized size = 0.85

$$\frac{125}{4}\,{\left(2\,x^2-x+3\right)}^{\frac{3}{2}}x^7+\frac{14125}{144}\,{\left(2\,x^2-x+3\right)}^{\frac{3}{2}}x^6+\frac{233225}{1536}\,{\left(2\,x^2-x+3\right)}^{\frac{3}{2}}x^5+\frac{4796405}{43008}\,{\left(2\,x^2-x+3\right)}^{\frac{3}{2}}x^4+\frac{8325631}{1032192}\,{\left(2\,x^2-x+3\right)}^{\frac{3}{2}}x^3-\frac{83948353}{2293760}\,{\left(2\,x^2-x+3\right)}^{\frac{3}{2}}x^2+\frac{804243809}{36700160}\,{\left(2\,x^2-x+3\right)}^{\frac{3}{2}}x+\frac{27185733541}{440401920}\,{\left(2\,x^2-x+3\right)}^{\frac{3}{2}}+\frac{359471503}{16777216}\,\sqrt{2\,x^2-x+3}\,x+\frac{8267844569}{268435456}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{359471503}{67108864}\,\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/4*(2*x^2 - x + 3)^(3/2)*x^7 + 14125/144*(2*x^2 - x + 3)^(3/2)*x^6 + 233
225/1536*(2*x^2 - x + 3)^(3/2)*x^5 + 4796405/43008*(2*x^2 - x + 3)^(3/2)*x^
4 + 8325631/1032192*(2*x^2 - x + 3)^(3/2)*x^3 - 83948353/2293760*(2*x^2 - x
 + 3)^(3/2)*x^2 + 804243809/36700160*(2*x^2 - x + 3)^(3/2)*x + 27185733541/
440401920*(2*x^2 - x + 3)^(3/2) + 359471503/16777216*sqrt(2*x^2 - x + 3)*x
+ 8267844569/268435456*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 359471503
/67108864*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 1.80, size = 98, normalized size = 0.47

$$\frac{1}{21139292160}\,\left(1321205760000\,x^9+3486515200000\,x^8+6327795712000\,x^7+7725962035200\,x^6+7612808028160\,x^5+5354741991424\,x^4+2211683657856\,x^3-174418077792\,x^2+537752185764\,x+3801512106459\right)\sqrt{2\,x^2-x+3}+\frac{8267844569}{536870912}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21139292160*(1321205760000*x^9 + 3486515200000*x^8 + 6327795712000*x^7 +
7725962035200*x^6 + 7612808028160*x^5 + 5354741991424*x^4 + 2211683657856*x
^3 - 174418077792*x^2 + 537752185764*x + 3801512106459)*sqrt(2*x^2 - x + 3)
 + 8267844569/536870912*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3}\,\left(5x^2 + 3x + 2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**4, x)
```

Giac [A]
time = 1.44, size = 93, normalized size = 0.45

$$\frac{1}{21139292160}\,(4\,(8\,(4\,(16\,(20\,(40\,(140\,(160\,(36\,x+95)x+27587)x+4715553)x+185859571)x+2614620113)x+17278778577)x-5450564931)x+134438046441)x+3801512106459)\sqrt{2\,x^2-x+3}-\frac{8267844569}{268435456}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36*x + 95)*x + 27587)*x + 471
5553)*x + 185859571)*x + 2614620113)*x + 17278778577)*x - 5450564931)*x + 1
34438046441)*x + 3801512106459)*sqrt(2*x^2 - x + 3) - 8267844569/268435456*
sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B]
time = 5.03, size = 221, normalized size = 1.06

$$\frac{8325631\,x^3\,{\left(2\,x^2-x+3\right)}^{3/2}}{1032192}-\frac{83948353\,x^2\,{\left(2\,x^2-x+3\right)}^{3/2}}{2293760}+\frac{4796405\,x^4\,{\left(2\,x^2-x+3\right)}^{3/2}}{43008}+\frac{233225\,x^5\,{\left(2\,x^2-x+3\right)}^{3/2}}{1536}+\frac{14125\,x^6\,{\left(2\,x^2-x+3\right)}^{3/2}}{144}+\frac{125\,x^7\,{\left(2\,x^2-x+3\right)}^{3/2}}{4}-\frac{41987163941\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{1174405120}-\frac{1825528867\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{36700160}+\frac{27185733541\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{7046430720}+\frac{804243809\,x\,{\left(2\,x^2-x+3\right)}^{3/2}}{36700160}+\frac{625271871443\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{9395240960}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] (8325631*x^3*(2*x^2 - x + 3)^(3/2))/1032192 - (83948353*x^2*(2*x^2 - x + 3)
^(3/2))/2293760 + (4796405*x^4*(2*x^2 - x + 3)^(3/2))/43008 + (233225*x^5*(
2*x^2 - x + 3)^(3/2))/1536 + (14125*x^6*(2*x^2 - x + 3)^(3/2))/144 + (125*x
^7*(2*x^2 - x + 3)^(3/2))/4 - (41987163941*2^(1/2)*log((2*x^2 - x + 3)^(1/2
) + (2^(1/2)*(2*x - 1/2))/2))/1174405120 - (1825528867*(x/2 - 1/8)*(2*x^2 -
 x + 3)^(1/2))/36700160 + (27185733541*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x
+ 45))/7046430720 + (804243809*x*(2*x^2 - x + 3)^(3/2))/36700160 + (6252718
71443*2^(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/9395240
960
```

3.59 $\int \sqrt{3-x+2x^2}\,(2+3x+5x^2)^3\,dx$

Optimal. Leaf size=166

$$-\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{22548119(3-x+2x^2)^{3/2}}{4587520}-\frac{9627393x(3-x+2x^2)^{3/2}}{1146880}+\frac{531681x^2(3-x}{7168}$$

```
[Out] -22548119/4587520*(2*x^2-x+3)^(3/2)-9627393/1146880*x*(2*x^2-x+3)^(3/2)+531
681/71680*x^2*(2*x^2-x+3)^(3/2)+247435/10752*x^3*(2*x^2-x+3)^(3/2)+8825/448
*x^4*(2*x^2-x+3)^(3/2)+125/16*x^5*(2*x^2-x+3)^(3/2)-155620231/8388608*arcsi
nh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-6766097/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{531681(2x^2-x+3)^{3/2}x^2}{71680}-\frac{9627393(2x^2-x+3)^{3/2}x}{1146880}-\frac{22548119(2x^2-x+3)^{3/2}}{4587520}-\frac{6766097(1-4x)\sqrt{2x^2-x+3}}{2097152}+\frac{125}{16}(2x^2-x+3)^{3/2}x^5+\frac{8825}{448}(2x^2-x+3)^{3/2}x^4+\frac{247435(2x^2-x+3)^{3/2}x^3}{10752}-\frac{155620231\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-6766097*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (22548119*(3 - x + 2*x^2
)^(3/2))/4587520 - (9627393*x*(3 - x + 2*x^2)^(3/2))/1146880 + (531681*x^2*
(3 - x + 2*x^2)^(3/2))/71680 + (247435*x^3*(3 - x + 2*x^2)^(3/2))/10752 + (
8825*x^4*(3 - x + 2*x^2)^(3/2))/448 + (125*x^5*(3 - x + 2*x^2)^(3/2))/16 -
(155620231*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx = \frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}\int \sqrt{3-x+2x^2}\left(128+576x+1824x\right.$$

$$= \frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2}+\frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2}+\frac{1}{224}\int \sqrt{3-x}$$

$$= \frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752}+\frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2}+\frac{125}{16}x^5\left(3-x\right.$$

$$= \frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680}+\frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752}+\frac{8825}{448}x^4\left(3\right.$$

$$= -\frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880}+\frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680}+\frac{247435x}{}$$

$$= -\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880}+\frac{531681x}{}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{96}{}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{96}{}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{96}{}$$

Mathematica [A]
time = 0.58, size = 85, normalized size = 0.51

$$\frac{4\sqrt{3-x+2x^2}\left(-3957369321-1621307916x+4583812128x^2+9872163456x^3+11212171264x^4+10958233600x^5+6955008000x^6+3440640000x^7\right)-16340124255\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{880803840}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-3957369321 - 1621307916*x + 4583812128*x^2 + 98721
63456*x^3 + 11212171264*x^4 + 10958233600*x^5 + 6955008000*x^6 + 3440640000
*x^7) - 16340124255*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/8808038
40
```

Maple [A]
time = 0.13, size = 132, normalized size = 0.80

method	result
risch	$\frac{(3440640000x^7+6955008000x^6+10958233600x^5+11212171264x^4+9872163456x^3+4583812128x^2-1621307916x-3957369321)\sqrt{2}}{220200960}$
trager	$\left(\frac{125}{8}x^7+\frac{7075}{224}x^6+\frac{267535}{5376}x^5+\frac{782099}{15360}x^4+\frac{25708759}{573440}x^3+\frac{6821149}{327680}x^2-\frac{135108993}{18350080}x-\frac{1319123107}{73400320}\right)\sqrt{2x^2-}$
default	$-\frac{22548119(2x^2-x+3)^{\frac{3}{2}}}{4587520}+\frac{6766097(4x-1)\sqrt{2x^2-x+3}}{2097152}+\frac{155620231\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}-\frac{9627393x(2}{114}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -22548119/4587520*(2*x^2-x+3)^(3/2)+6766097/2097152*(4*x-1)*(2*x^2-x+3)^(1/
2)+155620231/8388608*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-9627393/1146880
*x*(2*x^2-x+3)^(3/2)+531681/71680*x^2*(2*x^2-x+3)^(3/2)+247435/10752*x^3*(2
*x^2-x+3)^(3/2)+8825/448*x^4*(2*x^2-x+3)^(3/2)+125/16*x^5*(2*x^2-x+3)^(3/2)
```

Maxima [A]
time = 0.52, size = 143, normalized size = 0.86

$$\frac{125}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^5+\frac{8825}{448}\left(2x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{247435}{10752}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{531681}{71680}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{9627393}{1146880}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{22548119}{4587520}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{6766097}{524288}\sqrt{2x^2-x+3}\,x+\frac{155620231}{8388608}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{6766097}{2097152}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/16*(2*x^2 - x + 3)^(3/2)*x^5 + 8825/448*(2*x^2 - x + 3)^(3/2)*x^4 + 247
435/10752*(2*x^2 - x + 3)^(3/2)*x^3 + 531681/71680*(2*x^2 - x + 3)^(3/2)*x^
2 - 9627393/1146880*(2*x^2 - x + 3)^(3/2)*x - 22548119/4587520*(2*x^2 - x +
```

```
 3)^(3/2) + 6766097/524288*sqrt(2*x^2 - x + 3)*x + 155620231/8388608*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6766097/2097152*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 1.85, size = 88, normalized size = 0.53

$$\frac{1}{220200960}\left(3440640000\,x^{7}+6955008000\,x^{6}+10958233600\,x^{5}+11212171264\,x^{4}+9872163456\,x^{3}+4583812128\,x^{2}-1621307916\,x-3957369321\right)\sqrt{2\,x^{2}-x+3}+\frac{155620231}{16777216}\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/220200960*(3440640000*x^7 + 6955008000*x^6 + 10958233600*x^5 + 1121217126
4*x^4 + 9872163456*x^3 + 4583812128*x^2 - 1621307916*x - 3957369321)*sqrt(2
*x^2 - x + 3) + 155620231/16777216*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2x^{2}-x+3}\,\left(5x^{2}+3x+2\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3, x)
```

Giac [A]
time = 0.96, size = 83, normalized size = 0.50

$$\frac{1}{220200960}\left(4\left(8\left(4\left(16\left(100\left(120\left(140\,x+283\right)x+53507\right)x+5474693\right)x+77126277\right)x+143244129\right)x-405326979\right)x-3957369321\right)\sqrt{2\,x^{2}-x+3}-\frac{155620231}{8388608}\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^{2}-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/220200960*(4*(8*(4*(16*(100*(120*(140*x + 283)*x + 53507)*x + 5474693)*x
+ 77126277)*x + 143244129)*x - 405326979)*x - 3957369321)*sqrt(2*x^2 - x +
3) - 155620231/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
 3)) + 1)
```

Mupad [B]
time = 4.69, size = 187, normalized size = 1.13

$$\frac{531681\,x^{2}\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{71680}+\frac{247435\,x^{3}\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{10752}+\frac{8825\,x^{4}\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{448}+\frac{125\,x^{5}\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{16}+\frac{875316037\,\sqrt{2}\,\ln\left(\sqrt{2\,x^{2}-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{36700160}+\frac{38057219\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^{2}-x+3}}{1146880}-\frac{22548119\,\sqrt{2\,x^{2}-x+3}\,\left(32\,x^{2}-4\,x+45\right)}{73400320}-\frac{9627393\,x\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{1146880}-\frac{1555820211\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^{2}-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{293601280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (531681*x^2*(2*x^2 - x + 3)^(3/2))/71680 + (247435*x^3*(2*x^2 - x + 3)^(3/2
))/10752 + (8825*x^4*(2*x^2 - x + 3)^(3/2))/448 + (125*x^5*(2*x^2 - x + 3)^
(3/2))/16 + (875316037*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x -
1/2))/2))/36700160 + (38057219*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1146880 -
 (22548119*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/73400320 - (9627393*x
*(2*x^2 - x + 3)^(3/2))/1146880 - (1555820211*2^(1/2)*log(2*(2*x^2 - x + 3)
^(1/2) + (2^(1/2)*(4*x - 1))/2))/293601280
```

3.60 $\int \sqrt{3-x+2x^2}\,(2+3x+5x^2)^2\,dx$

Optimal. Leaf size=124

$$\frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}-\frac{2107(3-x+2x^2)^{3/2}}{3072}+\frac{769}{256}x(3-x+2x^2)^{3/2}+\frac{63}{16}x^2(3-x+2x^2)^{3/2}+\frac{25}{12}x^3($$

```
[Out] -2107/3072*(2*x^2-x+3)^(3/2)+769/256*x*(2*x^2-x+3)^(3/2)+63/16*x^2*(2*x^2-x
+3)^(3/2)+25/12*x^3*(2*x^2-x+3)^(3/2)+284533/65536*arcsinh(1/23*(1-4*x)*23^
(1/2))*2^(1/2)+12371/16384*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{63}{16}\left(2x^2-x+3\right)^{3/2}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{3/2}x-\frac{2107\left(2x^2-x+3\right)^{3/2}}{3072}+\frac{12371(1-4x)\sqrt{2x^2-x+3}}{16384}+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{284533\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (12371*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 - (2107*(3 - x + 2*x^2)^(3/2))/
3072 + (769*x*(3 - x + 2*x^2)^(3/2))/256 + (63*x^2*(3 - x + 2*x^2)^(3/2))/1
6 + (25*x^3*(3 - x + 2*x^2)^(3/2))/12 + (284533*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/(32768*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx = \frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{12}\int \sqrt{3-x+2x^2}\left(48+144x+123x^2\right.$$

$$= \frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2}+\frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{120}\int \sqrt{3-x+}$$

$$= \frac{769}{256}x\left(3-x+2x^2\right)^{3/2}+\frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2}+\frac{25}{12}x^3\left(3-x+2x\right.$$

$$= -\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}+\frac{769}{256}x\left(3-x+2x^2\right)^{3/2}+\frac{63}{16}x^2\left(3-x+2\right.$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}-\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}+\frac{769}{256}x\left(3-\right.$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}-\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}+\frac{769}{256}x\left(3-\right.$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}-\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072}+\frac{769}{256}x\left(3-\right.$$

Mathematica [A]
time = 0.36, size = 75, normalized size = 0.60

$$\frac{4\sqrt{3-x+2x^2}\left(-64023+328204x+365536x^2+408960x^3+284672x^4+204800x^5\right)+853599\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{196608}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-64023 + 328204*x + 365536*x^2 + 408960*x^3 + 28467
2*x^4 + 204800*x^5) + 853599*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]
)/196608
```

Maple [A]
time = 0.12, size = 98, normalized size = 0.79

method	result
risch	$\frac{(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023)\sqrt{2x^2-x+3}}{49152} - \frac{284533\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{65536}$
trager	$\left(\frac{25}{6}x^5+\frac{139}{24}x^4+\frac{1065}{128}x^3+\frac{11423}{1536}x^2+\frac{82051}{12288}x-\frac{21341}{16384}\right)\sqrt{2x^2-x+3} + \frac{284533\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{Root}\right.}{}$
default	$\frac{25x^3(2x^2-x+3)^{\frac{3}{2}}}{12}+\frac{63x^2(2x^2-x+3)^{\frac{3}{2}}}{16}+\frac{769x(2x^2-x+3)^{\frac{3}{2}}}{256}-\frac{2107(2x^2-x+3)^{\frac{3}{2}}}{3072}-\frac{12371(4x-1)\sqrt{2x^2-x+3}}{16384}-\frac{28}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/12*x^3*(2*x^2-x+3)^(3/2)+63/16*x^2*(2*x^2-x+3)^(3/2)+769/256*x*(2*x^2-x+
3)^(3/2)-2107/3072*(2*x^2-x+3)^(3/2)-12371/16384*(4*x-1)*(2*x^2-x+3)^(1/2)-
284533/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.50, size = 109, normalized size = 0.88

$$\frac{25}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{2107}{3072}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{12371}{4096}\sqrt{2x^2-x+3}\,x-\frac{284533}{65536}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{12371}{16384}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 25/12*(2*x^2 - x + 3)^(3/2)*x^3 + 63/16*(2*x^2 - x + 3)^(3/2)*x^2 + 769/256
*(2*x^2 - x + 3)^(3/2)*x - 2107/3072*(2*x^2 - x + 3)^(3/2) - 12371/4096*sqr
t(2*x^2 - x + 3)*x - 284533/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
+ 12371/16384*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 2.19, size = 78, normalized size = 0.63

$$\frac{1}{49152}\left(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023\right)\sqrt{2x^2-x+3}+\frac{284533}{131072}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(204800*x^5 + 284672*x^4 + 408960*x^3 + 365536*x^2 + 328204*x - 640
23)*sqrt(2*x^2 - x + 3) + 284533/131072*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3}\,\left(5x^2 + 3x + 2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2, x)
```

Giac [A]
time = 1.67, size = 73, normalized size = 0.59

$$\frac{1}{49152}\left(4\left(8\left(4\left(16\left(100\,x+139\right)x+3195\right)x+11423\right)x+82051\right)x-64023\right)\sqrt{2\,x^2-x+3}+\frac{284533}{65536}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/49152*(4*(8*(4*(16*(100*x + 139)*x + 3195)*x + 11423)*x + 82051)*x - 6402
3)*sqrt(2*x^2 - x + 3) + 284533/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 1)
```

Mupad [B]
time = 4.19, size = 153, normalized size = 1.23

$$\frac{63\,x^2\,{\left(2\,x^2-x+3\right)}^{3/2}}{16}+\frac{25\,x^3\,{\left(2\,x^2-x+3\right)}^{3/2}}{12}-\frac{29509\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{8192}-\frac{1283\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{256}-\frac{2107\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{49152}+\frac{769\,x\,{\left(2\,x^2-x+3\right)}^{3/2}}{256}-\frac{48461\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{65536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (63*x^2*(2*x^2 - x + 3)^(3/2))/16 + (25*x^3*(2*x^2 - x + 3)^(3/2))/12 - (29
509*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/8192 - (1
283*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/256 - (2107*(2*x^2 - x + 3)^(1/2)*(3
2*x^2 - 4*x + 45))/49152 + (769*x*(2*x^2 - x + 3)^(3/2))/256 - (48461*2^(1/
2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/65536
```

3.61 $\int \sqrt{3-x+2x^2}\,(2+3x+5x^2)\,dx$

Optimal. Leaf size=82

$$-\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}-\frac{1863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

```
[Out] 73/96*(2*x^2-x+3)^(3/2)+5/8*x*(2*x^2-x+3)^(3/2)-1863/2048*arcsinh(1/23*(1-4
*x)*23^(1/2))*2^(1/2)-81/512*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}+\frac{73}{96}\left(2x^2-x+3\right)^{3/2}-\frac{81}{512}(1-4x)\sqrt{2x^2-x+3}-\frac{1863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-81*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/512 + (73*(3 - x + 2*x^2)^(3/2))/96 + (
5*x*(3 - x + 2*x^2)^(3/2))/8 - (1863*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqr
t[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx = \frac{5}{8}x\left(3-x+2x^2\right)^{3/2} + \frac{1}{8}\int\left(1+\frac{73x}{2}\right)\sqrt{3-x+2x^2}\,dx$$

$$= \frac{73}{96}\left(3-x+2x^2\right)^{3/2} + \frac{5}{8}x\left(3-x+2x^2\right)^{3/2} + \frac{81}{64}\int\sqrt{3-x+2x^2}\,dx$$

$$= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96}\left(3-x+2x^2\right)^{3/2} + \frac{5}{8}x\left(3-x+2\right.$$

$$= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96}\left(3-x+2x^2\right)^{3/2} + \frac{5}{8}x\left(3-x+2\right.$$

$$= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2} + \frac{73}{96}\left(3-x+2x^2\right)^{3/2} + \frac{5}{8}x\left(3-x+2\right.$$

Mathematica [A]
time = 0.24, size = 65, normalized size = 0.79

$$\frac{4\sqrt{3-x+2x^2}\left(3261+2684x+1376x^2+1920x^3\right)-5589\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{6144}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3261 + 2684*x + 1376*x^2 + 1920*x^3) - 5589*Sqrt[2]
*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/6144
```

Maple [A]
time = 0.11, size = 64, normalized size = 0.78

method	result
risch	$\frac{(1920x^3+1376x^2+2684x+3261)\sqrt{2x^2-x+3}}{1536}+\frac{1863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{2048}$
default	$\frac{5x(2x^2-x+3)^{\frac{3}{2}}}{8}+\frac{73(2x^2-x+3)^{\frac{3}{2}}}{96}+\frac{81(4x-1)\sqrt{2x^2-x+3}}{512}+\frac{1863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{2048}$
trager	$\left(\frac{5}{4}x^3+\frac{43}{48}x^2+\frac{671}{384}x+\frac{1087}{512}\right)\sqrt{2x^2-x+3}-\frac{1863\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+}\right.}{2048}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 5/8*x*(2*x^2-x+3)^(3/2)+73/96*(2*x^2-x+3)^(3/2)+81/512*(4*x-1)*(2*x^2-x+3)^
(1/2)+1863/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.53, size = 75, normalized size = 0.91

$$\frac{5}{8}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{73}{96}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{81}{128}\sqrt{2x^2-x+3}\,x+\frac{1863}{2048}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{81}{512}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
[Out] 5/8*(2*x^2 - x + 3)^(3/2)*x + 73/96*(2*x^2 - x + 3)^(3/2) + 81/128*sqrt(2*x
^2 - x + 3)*x + 1863/2048*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 81/512
*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 2.46, size = 68, normalized size = 0.83

$$\frac{1}{1536}\left(1920x^3+1376x^2+2684x+3261\right)\sqrt{2x^2-x+3}+\frac{1863}{4096}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
[Out] 1/1536*(1920*x^3 + 1376*x^2 + 2684*x + 3261)*sqrt(2*x^2 - x + 3) + 1863/409
6*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2x^2-x+3}\cdot\left(5x^2+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2), x)
```

Giac [A]
time = 1.53, size = 63, normalized size = 0.77

$$\frac{1}{1536}\left(4\left(8\left(60\,x+43\right)x+671\right)x+3261\right)\sqrt{2\,x^{2}-x+3}-\frac{1863}{2048}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\,\left(\sqrt{2}\,x-\sqrt{2\,x^{2}-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1536*(4*(8*(60*x + 43)*x + 671)*x + 3261)*sqrt(2*x^2 - x + 3) - 1863/2048
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B]
time = 3.84, size = 119, normalized size = 1.45

$$\frac{23\,\sqrt{2}\,\ln\left(\sqrt{2\,x^{2}-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{256}+\frac{\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^{2}-x+3}}{8}+\frac{73\,\sqrt{2\,x^{2}-x+3}\,\left(32\,x^{2}-4\,x+45\right)}{1536}+\frac{5\,x\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{8}+\frac{1679\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^{2}-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] (23*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/256 + ((x
/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/8 + (73*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*
x + 45))/1536 + (5*x*(2*x^2 - x + 3)^(3/2))/8 + (1679*2^(1/2)*log(2*(2*x^2
- x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/2048
```

3.62 $\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=174

$$-\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)+\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(6+7\sqrt{2}+\left(20+13\sqrt{2}\right)\right.}{\sqrt{3-x+2x^2}}\right.$$

```
[Out] -1/5*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/155*arctanh(1/62*(6+x*(20-13*
2^(1/2))-7*2^(1/2))*682^(1/2)/(-13+10*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-4
433+3410*2^(1/2))^(1/2)+1/155*arctan(1/62*(6+7*2^(1/2)+x*(20+13*2^(1/2)))*6
82^(1/2)/(13+10*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(4433+3410*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$,
Rules used = {1003, 633, 221, 1049, 1043, 212, 210}

$$\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{\frac{11}{31}\left(10\sqrt{2}-13\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(10\sqrt{2}-13\right)}}\left(\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -1/5*(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]) + (Sqrt[(11*(13 + 10*Sqrt[2]))/3
1]*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2
])*x))/Sqrt[3 - x + 2*x^2]])/5 - (Sqrt[(11*(-13 + 10*Sqrt[2]))/31]*ArcTanh[
(Sqrt[11/(62*(-13 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sq
rt[3 - x + 2*x^2]])/5
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1003

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = -\left(\frac{1}{5}\int \frac{-11+11x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx\right) + \frac{2}{5}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1}{5}\sqrt{\frac{2}{23}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right) + \frac{\int \frac{121\left(2+\sqrt{2}\right)-121\sqrt{2}\,x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx}{110\sqrt{2}} - \frac{\int}{}$$

$$= -\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) - \frac{1}{5}\left(1331\left(20-13\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-907742\left(13-10\sqrt{}\right.}\right.$$

$$= -\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\,(6}{\sqrt{3-}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.20, size = 206, normalized size = 1.18

$$\frac{1}{5}\left(-\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)+11\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{-2\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+2\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] (-(Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]) + 11*RootSum[-56 - 26*Sq
rt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2*Log[-(Sqrt[2]*x) + Sqr
t[3 - x + 2*x^2] - #1] + 2*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1]*#1 + Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] +
 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2064 vs. 2(125) = 250.
time = 0.69, size = 2065, normalized size = 11.87

method	result

trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}+\mathrm{RootOf}\left(\_Z^2-2\right)\right)}{5}+\mathrm{RootOf}\left(24025\_Z^4+4\right.$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-1/52855*(8*(2^(1/2)-1+x)^2/(2^(1
/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(
1/2)*(285*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549
362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^
(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+
1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+386*(-8866+6820*2^(1/2))
^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*
(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2
^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^
(1/2))^(1/2)-274846*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/
2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^
(1/2))*2^(1/2)-1543366*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^
(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2
))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(
1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+1/21142*(
8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2
+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(
1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-
1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/
(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*
(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+2
18*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1
/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))
^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82
*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/
2)))*(-775687+549362*2^(1/2))^(1/2)+401698*arctanh(31/2*(8*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2
```

```
)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-63426*arctanh(31/2*(8*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/
2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/
2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1
-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(
1/2))^(1/2)+3/21142*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1
+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(1/2)*(-8866+6820*2
^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^
(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(
1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+54
9362*2^(1/2))^(1/2)+520*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-7756
87+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1
-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1
0368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)
^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^
(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+465124*arctanh(31/
2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x
)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arctanh(3
1/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1
-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2
^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^
(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2016 vs. $2(125) = 250$.
time = 3.97, size = 2016, normalized size = 11.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/1550*6050^(1/4)*sqrt(31)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/8
9125*(460*sqrt(5)*(4*6050^(3/4)*sqrt(31)*(4702*x^7 - 19541*x^6 + 40352*x^5
- 68777*x^4 + 35480*x^3 - 19080*x^2 - sqrt(2)*(4028*x^7 - 14488*x^6 + 30919
*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 17280) - 34560*x + 2764
8) + 5*6050^(1/4)*sqrt(31)*(22836*x^7 - 355266*x^6 + 1914360*x^5 - 4475096*
x^4 + 5840640*x^3 - 4011840*x^2 - sqrt(2)*(18463*x^7 - 280047*x^6 + 1453472
*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 3068928*x + 1990656) - 398
1312*x + 3068928))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) + 253000*sqrt(
31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^
4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 -
783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1
154304*x - 456192) - 2*sqrt(10)*(sqrt(5)*(4*6050^(3/4)*sqrt(31)*(15454*x^7
- 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - sqrt(2)*(1543
8*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + 44928*x^2 - 44928*x
) - 13824*x) + 5*6050^(1/4)*sqrt(31)*(77254*x^7 - 1000024*x^6 + 3868360*x^5
 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(69479*x^7 - 898236*x^
6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - 4478976*x) - 24
05376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) + 550*sqrt(31)*sqrt(2)*
(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*
x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x
^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25
*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087819
20*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517
*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*
sqrt(-(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)
*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*sqrt(2)*(2*x^2 - x + 3) + 755*x - 10
00)/x^2) + 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835
344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348
*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 +
4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 1419
1920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 247726
08*x + 18579456)) + 1/1550*6050^(1/4)*sqrt(31)*sqrt(5)*sqrt(2)*sqrt(13*sqrt
(2) + 20)*arctan(1/89125*(460*sqrt(5)*(4*6050^(3/4)*sqrt(31)*(4702*x^7 - 19
541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - sqrt(2)*(4028*x^7
 - 14488*x^6 + 30919*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 172
80) - 34560*x + 27648) + 5*6050^(1/4)*sqrt(31)*(22836*x^7 - 355266*x^6 + 19
14360*x^5 - 4475096*x^4 + 5840640*x^3 - 4011840*x^2 - sqrt(2)*(18463*x^7 -
280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 306892
8*x + 1990656) - 3981312*x + 3068928))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2)
+ 20) - 253000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385
256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335
*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 54
6048*x - 539136) + 1154304*x - 456192) - 2*sqrt(10)*(sqrt(5)*(4*6050^(3/4)*
sqrt(31)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824
*x^2 - sqrt(2)*(15438*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 +
```

```
 44928*x^2 - 44928*x) - 13824*x) + 5*6050^(1/4)*sqrt(31)*(77254*x^7 - 10000
24*x^6 + 3868360*x^5 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(6
9479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x
^2 - 4478976*x) - 2405376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) - 5
50*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt((6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(2)*(2*x^2 -
x + 3) - 755*x + 1000)/x^2) - 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53
385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 -
 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 556
8*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 -
 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34
615296*x^2 - 24772608*x + 18579456)) - 1/6200*6050^(1/4)*sqrt(5)*sqrt(13*sq
rt(2) + 20)*(13*sqrt(2) - 20)*log(40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(
2)*(2*x^2 - x + 3) - 755*x + 1000)/x^2) + 1/6200*6050^(1/4)*sqrt(5)*sqrt(13
*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-40*(6050^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2),x)
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2), x)
```

3.63 $\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=188

$$\frac{(3+10x)\sqrt{3-x+2x^2}}{31\,(2+3x+5x^2)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(419+277\right.}{\sqrt{3-x+2x^2}}\right.$$

```
[Out] 1/31*(3+10*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/42284*arctanh(1/31*(419+x*(
973-696*2^(1/2))-277*2^(1/2))*341^(1/2)/(-70517+49942*2^(1/2))^(1/2)/(2*x^2
-x+3)^(1/2))*(-48092594+34060444*2^(1/2))^(1/2)+1/42284*arctan(1/31*(419+27
7*2^(1/2)+x*(973+696*2^(1/2)))*341^(1/2)/(70517+49942*2^(1/2))^(1/2)/(2*x^2
-x+3)^(1/2))*(48092594+34060444*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {985, 1049, 1043, 212, 210}

$$\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\sqrt{2}+419\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\sqrt{2x^2-x+3}\,(10x+3)}{31\,(5x^2+3x+2)}-\frac{1}{62}\sqrt{\frac{1}{682}\left(49942\sqrt{2}-70517\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(49942\sqrt{2}-70517\right)}}\left(\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 49
942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))]*(419 + 277*
Sqrt[2] + (973 + 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62 - (Sqrt[(-70517
+ 49942*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))]*(419
- 277*Sqrt[2] + (973 - 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 985

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{31\,(2+3x+5x^2)} - \frac{1}{31}\int \frac{-\frac{63}{2}+11x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\,(2+3x+5x^2)} - \frac{\int \frac{\frac{11}{2}\left(85-63\sqrt{2}\right)-\frac{11}{2}\left(41-22\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx}{682\sqrt{2}} + \frac{\int \frac{\frac{11}{2}\left(85+63\sqrt{2}\right)-\frac{1}{2}\cdots}{\sqrt{3-x+2x\cdots}}}{682\cdots}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\,(2+3x+5x^2)} - \frac{1}{248}\left(11\left(99884-70517\sqrt{2}\right)\right)\mathrm{Subst}\left(\int \frac{\cdots}{-\frac{3751}{4}\left(70517\cdots\right.}\right.$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\,(2+3x+5x^2)} + \frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{\cdots}{31\left(70517+\cdots\right.}}}{\cdots}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.47, size = 419, normalized size = 2.23

$$\frac{\frac{50(3+10x)\sqrt{3-x+2x^2}}{2+3x+5x^2} - 6151\,\mathrm{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right] + 124\,\mathrm{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{49\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+10\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right] - 10\,\mathrm{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{191\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+55\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{13\sqrt{2}-17\#1-9\sqrt{2}\#1^2+10\#1^3}\&\right]}{1550}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((50*(3 + 10*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) - 6151*RootSum[-56 -
 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1]/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) &
 ] + 124*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 &
, (49*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[-(Sqrt[
2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2
 - 10*#1^3) & ] - 10*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3
 - 5*#1^4 & , (191*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1
+ 55*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(13*Sqrt[2] - 17*#1
 - 9*Sqrt[2]*#1^2 + 10*#1^3) & ])/1550
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 16356 vs. 2(140) = 280.
time = 0.95, size = 16357, normalized size = 87.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2102 vs. $2(140) = 280$.
time = 5.98, size = 2102, normalized size = 11.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/186703822445536*(88412*4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(2)*(5
*x^2 + 3*x + 2)*sqrt(70517*sqrt(2) + 99884)*arctan(1/106689264574623029 23*(
3096404*sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(537184*x^7 - 2047820*x^
6 + 4310846*x^5 - 6853210*x^4 + 3421536*x^3 - 1589328*x^2 - sqrt(2)*(370014
*x^7 - 1438653*x^6 + 3014868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2
- 2633472*x + 1893888) - 3787776*x + 2633472) + 774101*4988406728^(1/4)*sqr
t(341)*(40625*x^7 - 622509*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 -
5992992*x^2 - sqrt(2)*(28204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 +
 6800832*x^3 - 4341024*x^2 - 4810752*x + 3442176) - 6884352*x + 4810752))*s
qrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) + 302859847824736341 04*sqrt(
31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^
4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 -
783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1
154304*x - 456192) - 2*sqrt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(3
41)*(84604*x^7 - 121310*x^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624
*x^2 - sqrt(2)*(57082*x^7 - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^
3 + 110592*x^2 - 110592*x) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*
(6379*x^7 - 82508*x^6 + 318020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 -
 sqrt(2)*(4365*x^7 - 56468*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207
360*x^2 - 207360*x) - 331776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 9
9884) + 425261673562*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^
6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^
8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209
600*x^2 - 1036800*x) + 3276288*x) + 19330076071*sqrt(31)*(254591*x^8 - 4815
126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689
```

```
56928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 -
 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(4988406728^(1/4)*sqrt
(24971)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x -
 7)*sqrt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^
2 - x + 3) + 3623566781*x - 4799426200)/x^2) + 344158917982654933*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 88412
*4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(7051
7*sqrt(2) + 99884)*arctan(1/10668926457462302923*(3096404*sqrt(24971)*(11*4
988406728^(3/4)*sqrt(341)*(537184*x^7 - 2047820*x^6 + 4310846*x^5 - 6853210
*x^4 + 3421536*x^3 - 1589328*x^2 - sqrt(2)*(370014*x^7 - 1438653*x^6 + 3014
868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2 - 2633472*x + 1893888) -
3787776*x + 2633472) + 774101*4988406728^(1/4)*sqrt(341)*(40625*x^7 - 62250
9*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 - 5992992*x^2 - sqrt(2)*(28
204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 + 6800832*x^3 - 4341024*x^
2 - 4810752*x + 3442176) - 6884352*x + 4810752))*sqrt(2*x^2 - x + 3)*sqrt(7
0517*sqrt(2) + 99884) - 30285984782473634104*sqrt(31)*sqrt(2)*(28180*x^8 -
254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^
2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4
- 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sq
rt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(84604*x^7 - 121310*x
^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624*x^2 - sqrt(2)*(57082*x^7
 - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^3 + 110592*x^2 - 110592*x
) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*(6379*x^7 - 82508*x^6 + 3
18020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 - sqrt(2)*(4365*x^7 - 5646
8*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207360*x^2 - 207360*x) - 331
776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) - 425261673562*sqrt
(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*
x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x
^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32
76288*x) - 19330076071*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 -
90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)
*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 19
44*x) + 144820224*x))*sqrt((4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(31)
*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sqrt(70517*sqrt(2) + 9
9884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x + 3) - 3623566781*x
+ 4799426200)/x^2) - 344158917982654933*sqrt(31)*(2828123*x^8 - 9696916*x^7
 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*
x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}}{\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}}{{\left(5\,x^2+3\,x+2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2, x)
```

3.64 $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3} \, dx$

Optimal. Leaf size=223

$$\frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2}+\frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)}+\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\ \tan}{}$$

[Out] 1/62*(3+10*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2+1/84568*(3464+13665*x)*(2*x
^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/115350752*arctanh(1/31*(509587+x*(1235163-872
375*2^(1/2))-362788*2^(1/2))*341^(1/2)/(-112285869463+79399380740*2^(1/2))^
(1/2)/(2*x^2-x+3)^(1/2))*(-76578962973766+54150377664680*2^(1/2))^(1/2)+1/1
15350752*arctan(1/31*(509587+362788*2^(1/2)+x*(1235163+872375*2^(1/2)))*341
^(1/2)/(112285869463+79399380740*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(7657896
2973766+54150377664680*2^(1/2))^(1/2)

Rubi [A]

time = 0.30, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {985, 1074, 1049, 1043, 212, 210}

$$\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\ \text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(112285869463+79399380740\sqrt{2}\right)}}\left(\left(1235163+872375\sqrt{2}\right)x+362788\sqrt{2}+509587\right)}{\sqrt{2x^2-x+3}}\right)}{169136}+\frac{\sqrt{2x^2-x+3}\,(10x+3)}{62\left(5x^2+3x+2\right)^2}+\frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568\left(5x^2+3x+2\right)}-\frac{\sqrt{\frac{1}{682}\left(79399380740\sqrt{2}-112285869463\right)}\ \tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(79399380740\sqrt{2}-112285869463\right)}}\left(\left(1235163-872375\sqrt{2}\right)x-362788\sqrt{2}+509587\right)}{\sqrt{2x^2-x+3}}\right)}{169136}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]

[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + ((3464 + 13665*
x)*Sqrt[3 - x + 2*x^2])/(84568*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 + 7
9399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*Sq
rt[2]))]*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/169136 - (Sqrt[(-112285869463 + 79399380740*Sqrt[2])/682]*Arc
Tanh[(Sqrt[11/(31*(-112285869463 + 79399380740*Sqrt[2]))]*(509587 - 362788*
Sqrt[2] + (1235163 - 872375*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/169136

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 985

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
```

```
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{62\,(2+3x+5x^2)^2} - \frac{1}{62}\int \frac{-\frac{183}{2}+31x-40x^2}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\,(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\,(2+3x+5x^2)} - \frac{\int \frac{-213004+\frac{358655x}{4}}{\sqrt{3-x+2x^2}\,(2+3}}{465124}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\,(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\,(2+3x+5x^2)} - \frac{\int \frac{\frac{121}{4}\left(110061-77456\sqrt{2}\right.}{\sqrt{3-x+}}}{1023}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\,(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\,(2+3x+5x^2)} - \frac{\left(11\left(158798761480\right.\right.}{}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\,(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\,(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682}\left(1122858694\right.}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.59, size = 392, normalized size = 1.76

$$\frac{\frac{661250\sqrt{3-x+2x^2}\,(11020+51362x+58315x^2+68325x^3)}{(2+3x+5x^2)^2}+\text{RootSum}\left[-56-26\sqrt{2}\,\#1+17\#1^2+6\sqrt{2}\,\#1^3-5\#1^4\&,\frac{-537295920831\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)+120146195680\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1-45923442075\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\,\#1^2-10\#1^3}\&\right]-248\text{RootSum}\left[-56-26\sqrt{2}\,\#1+17\#1^2+6\sqrt{2}\,\#1^3-5\#1^4\&,\frac{-2139373897\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)+277937160\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1-228643025\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\,\#1^2-10\#1^3}\&\right]}{55920590000}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((661250*Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3))/(2
+ 3*x + 5*x^2)^2 + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 -
 5*#1^4 & , (-537295920831*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 1
20146195680*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 45923
442075*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17
*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 248*RootSum[-56 - 26*Sqrt[2]*#1 + 17*
#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2139373897*Log[-(Sqrt[2]*x) + Sqrt[3 -
 x + 2*x^2] - #1] + 277937160*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1]*#1 - 228643025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(
-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/55920590000
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44342 vs. $2(171) = 342$.
time = 1.00, size = 44343, normalized size = 198.85

method	result
trager	Expression too large to display
risch	$\frac{(68325x^3+58315x^2+51362x+11020)\sqrt{2x^2-x+3}}{84568(5x^2+3x+2)^2}+\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2182 vs. $2(171) = 342$.
time = 5.79, size = 2182, normalized size = 9.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/6505215189695292642599671424O*(14205421276*788032707736935368450^(1/4)*sq
rt(39699690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqr
t(112285869463*sqrt(2) + 158798761480)*arctan(1/861047662213971287591057659
551879544939625*(2461380802940*sqrt(39699690370)*(22*788032707736935368450^
(3/4)*sqrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 86714302
12*x^4 + 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195
871*x^6 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2
 - 3352976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*7
88032707736935368450^(1/4)*sqrt(341)*(50730703*x^7 - 778833417*x^6 + 411636
7112*x^5 - 9392273180*x^4 + 12133646496*x^3 - 7660912032*x^2 - sqrt(2)*(359
38543*x^7 - 551546778*x^6 + 2913578540*x^5 - 6643469608*x^4 + 8580088800*x^
3 - 5403919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216
))*sqrt(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) + 24442643
314461120421939701303408193533770000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^
7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(
2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*
x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(396996
90370/160673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*
(104024992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*
x^3 + 214417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5
 - 133050960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152
*x) + 615345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102
233612*x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2
- 4*sqrt(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116
274240*x^3 + 70118784*x^2 - 70118784*x) - 391744512*x))*sqrt(2*x^2 - x + 3)
*sqrt(112285869463*sqrt(2) + 158798761480) + 43175912524323866211143695850*
sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396
480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2440
47*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
+ 3276288*x) + 196254147837835755505198617 5*sqrt(31)*(254591*x^8 - 4815126*
x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 16895692
```

```
8*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 361
8*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(788032707736935368450^(1/
4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(12053
*x + 5138) - 17191*x - 6915)*sqrt(112285869463*sqrt(2) + 158798761480) - 15
0182556985858180945*x^2 - 134857806273015509420*sqrt(2)*(2*x^2 - x + 3) + 4
6280747152784868005 5*x - 612990028513706861000)/x^2) + 2777573103916036411 5
840569662963856288375*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 -
142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)
*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*
x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7
+ 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 -
24772608*x + 18579456)) + 14205421276*788032707736935368450^(1/4)*sqrt(3969
9690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(11228
5869463*sqrt(2) + 158798761480)*arctan(1/8610476622139712875910576595518795
44939625*(2461380802940*sqrt(39699690370)*(22*788032707736935368450^(3/4)*s
qrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 8671430212*x^4
+ 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195871*x^6
 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2 - 3352
976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*78803270
7736935368450^(1/4)*sqrt(341)*(50730703*x^7 - 778833417*x^6 + 4116367112*x^
5 - 9392273180*x^4 + 12133646496*x^3 - 7660912032*x^2 - sqrt(2)*(35938543*x
^7 - 551546778*x^6 + 2913578540*x^5 - 6643469608*x^4 + 8580088800*x^3 - 540
3919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216))*sqrt
(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) - 244426433144611
20421939701303408193533770 00*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3
96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(39699690370/1
60673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*(104024
992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*x^3 + 2
14417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5 - 1330
50960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152*x) + 6
15345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102233612*
x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2 - 4*sqr
t(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116274240*
x^3 + 70118784*x^2 - 70118784*x) - 391744512*x)...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^2-x+3}}{{\left(5\,x^2+3\,x+2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3, x)
```

3.65 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx$

Optimal. Leaf size=231

$$-\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+2x^2\right)^{3/2}}{134217728}+\frac{2124689283(3-x+2x^2)^{5/2}}{146800640}$$

```
[Out] -382121949/134217728*(1-4*x)*(2*x^2-x+3)^(3/2)+2124689283/146800640*(2*x^2-
x+3)^(5/2)+48669967/22020096*x*(2*x^2-x+3)^(5/2)-56422489/8257536*x^2*(2*x^
2-x+3)^(5/2)+10444117/294912*x^3*(2*x^2-x+3)^(5/2)+941905/9216*x^4*(2*x^2-x
+3)^(5/2)+95165/768*x^5*(2*x^2-x+3)^(5/2)+7625/96*x^6*(2*x^2-x+3)^(5/2)+625
/24*x^7*(2*x^2-x+3)^(5/2)-606427533063/8589934592*arcsinh(1/23*(1-4*x)*23^(
1/2))*2^(1/2)-26366414481/2147483648*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$-\frac{56422489(2x^2-x+3)^{5/2}x^2}{8257536}+\frac{48669967(2x^2-x+3)^{5/2}x}{22020096}+\frac{2124689283(2x^2-x+3)^{5/2}}{146800640}-\frac{382121949(1-4x)\left(2x^2-x+3\right)^{3/2}}{134217728}-\frac{26366414481(1-4x)\sqrt{2x^2-x+3}}{2147483648}+\frac{625}{24}(2x^2-x+3)^{5/2}x^7+\frac{7625}{96}(2x^2-x+3)^{5/2}x^6+\frac{95165}{768}(2x^2-x+3)^{5/2}x^5+\frac{941905(2x^2-x+3)^{5/2}x^4}{9216}+\frac{10444117(2x^2-x+3)^{5/2}x^3}{294912}-\frac{606427533063\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4294967296\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-26366414481*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2147483648 - (382121949*(1 - 4
*x)*(3 - x + 2*x^2)^(3/2))/134217728 + (2124689283*(3 - x + 2*x^2)^(5/2))/1
46800640 + (48669967*x*(3 - x + 2*x^2)^(5/2))/22020096 - (56422489*x^2*(3 -
 x + 2*x^2)^(5/2))/8257536 + (10444117*x^3*(3 - x + 2*x^2)^(5/2))/294912 +
(941905*x^4*(3 - x + 2*x^2)^(5/2))/9216 + (95165*x^5*(3 - x + 2*x^2)^(5/2))
/768 + (7625*x^6*(3 - x + 2*x^2)^(5/2))/96 + (625*x^7*(3 - x + 2*x^2)^(5/2)
)/24 - (606427533063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4294967296*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^4\,dx = \frac{625}{24}x^7\left(3-x+2x^2\right)^{5/2}+\frac{1}{24}\int \left(3-x+2x^2\right)^{3/2}\Big(384+2304x$$

$$= \frac{7625}{96}x^6\left(3-x+2x^2\right)^{5/2}+\frac{625}{24}x^7\left(3-x+2x^2\right)^{5/2}+\frac{1}{528}\int (3$$

$$= \frac{95165}{768}x^5\left(3-x+2x^2\right)^{5/2}+\frac{7625}{96}x^6\left(3-x+2x^2\right)^{5/2}+\frac{625}{24}x^7(3$$

$$= \frac{941905x^4\left(3-x+2x^2\right)^{5/2}}{9216}+\frac{95165}{768}x^5\left(3-x+2x^2\right)^{5/2}+\frac{7625}{96}x^6$$

$$= \frac{10444117x^3\left(3-x+2x^2\right)^{5/2}}{294912}+\frac{941905x^4\left(3-x+2x^2\right)^{5/2}}{9216}+$$

$$= -\frac{56422489x^2\left(3-x+2x^2\right)^{5/2}}{8257536}+\frac{10444117x^3\left(3-x+2x^2\right)^{5/2}}{294912}+$$

$$= \frac{48669967x\left(3-x+2x^2\right)^{5/2}}{22020096}-\frac{56422489x^2\left(3-x+2x^2\right)^{5/2}}{8257536}+$$

$$= \frac{2124689283\left(3-x+2x^2\right)^{5/2}}{146800640}+\frac{48669967x\left(3-x+2x^2\right)^{5/2}}{22020096}-$$

$$= -\frac{382121949(1-4x)\left(3-x+2x^2\right)^{3/2}}{134217728}+\frac{2124689283(3-x+2x}{146800640}$$

$$= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\,(3}{134217728}$$

$$= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\,(3}{134217728}$$

$$= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\,(3}{134217728}$$

Mathematica [A]
time = 1.02, size = 105, normalized size = 0.45

$$\frac{4\sqrt{3-x+2x^2}\,(74032009514181+12971175524316x+65151998063712x^2+239021184223104x^3+451581382260736x^4+675479464714240x^5+765087080448000x^6+745133229998080x^7+534038708224000x^8+349379651174400x^9+144451829760000x^{10}+70464307200000x^{11})-191024672914845\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{2705829396480}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(74032009514181 + 12971175524316*x + 65151998063712*
x^2 + 239021184223104*x^3 + 451581382260736*x^4 + 675479464714240*x^5 + 765
087080448000*x^6 + 745133229998080*x^7 + 534038708224000*x^8 + 349379651174
400*x^9 + 144451829760000*x^10 + 70464307200000*x^11) - 191024672914845*Sqr
t[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/2705829396480
```

Maple [A]
time = 0.14, size = 185, normalized size = 0.80

method	result
risch	$\frac{(70464307200000x^{11}+144451829760000x^{10}+349379651174400x^{9}+534038708224000x^{8}+745133229998080x^{7}+765087080448000x^{6}+\cdots}{67\cdots}$
trager	$\left(\frac{625}{6}x^{11}+\frac{5125}{24}x^{10}+\frac{33055}{64}x^{9}+\frac{1818925}{2304}x^{8}+\frac{81213077}{73728}x^{7}+\frac{778286825}{688128}x^{6}+\frac{16491197869}{16515072}x^{5}+\frac{31499817401}{47185920}x^{4}+\cdots\right.$
default	$\frac{26366414481(4x-1)\sqrt{2x^2-x+3}}{2147483648}+\frac{606427533063\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{8589934592}+\frac{382121949(4x-1)\left(2x^2-x+3\right)^{\frac{3}{2}}}{134217728}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 26366414481/2147483648*(4*x-1)*(2*x^2-x+3)^(1/2)+606427533063/8589934592*2^
(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+382121949/134217728*(4*x-1)*(2*x^2-x+3
)^(3/2)+48669967/22020096*x*(2*x^2-x+3)^(5/2)-56422489/8257536*x^2*(2*x^2-x
+3)^(5/2)+10444117/294912*x^3*(2*x^2-x+3)^(5/2)+941905/9216*x^4*(2*x^2-x+3)
^(5/2)+95165/768*x^5*(2*x^2-x+3)^(5/2)+7625/96*x^6*(2*x^2-x+3)^(5/2)+625/24
*x^7*(2*x^2-x+3)^(5/2)+2124689283/146800640*(2*x^2-x+3)^(5/2)
```

Maxima [A]
time = 0.51, size = 206, normalized size = 0.89

$$\frac{625}{24}\left(2x^2-x+3\right)^{\frac{5}{2}}x^7+\frac{7625}{96}\left(2x^2-x+3\right)^{\frac{5}{2}}x^6+\frac{95165}{768}\left(2x^2-x+3\right)^{\frac{5}{2}}x^5+\frac{941905}{9216}\left(2x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{10444117}{294912}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3-\frac{56422489}{8257536}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{48669967}{22020096}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{2124689283}{146800640}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{382121949}{33554432}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{382121949}{134217728}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{26366414481}{536870912}\sqrt{2x^2-x+3}\,x+\frac{606427533063}{8589934592}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{26366414481}{2147483648}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/24*(2*x^2 - x + 3)^(5/2)*x^7 + 7625/96*(2*x^2 - x + 3)^(5/2)*x^6 + 9516
5/768*(2*x^2 - x + 3)^(5/2)*x^5 + 941905/9216*(2*x^2 - x + 3)^(5/2)*x^4 + 1
0444117/294912*(2*x^2 - x + 3)^(5/2)*x^3 - 56422489/8257536*(2*x^2 - x + 3)
^(5/2)*x^2 + 48669967/22020096*(2*x^2 - x + 3)^(5/2)*x + 2124689283/1468006
40*(2*x^2 - x + 3)^(5/2) + 382121949/33554432*(2*x^2 - x + 3)^(3/2)*x - 382
121949/134217728*(2*x^2 - x + 3)^(3/2) + 26366414481/536870912*sqrt(2*x^2 -
 x + 3)*x + 606427533063/8589934592*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)
) - 26366414481/2147483648*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 5.62, size = 108, normalized size = 0.47

$$\frac{1}{676457349120}\left(70464307200000\,x^{11}+144451829760000\,x^{10}+349379651174400\,x^{9}+534038708224000\,x^{8}+745133229998080\,x^{7}+765087080448000\,x^{6}+675479464714240\,x^{5}+451581382260736\,x^{4}+239021184223104\,x^{3}+65151998063712\,x^{2}+12971175524316\,x+74032009514181\right)\sqrt{2\,x^{2}-x+3}+\frac{606427533063}{17179869184}\,\sqrt{2}\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/676457349120*(70464307200000*x^11 + 144451829760000*x^10 + 34937965117440
0*x^9 + 534038708224000*x^8 + 745133229998080*x^7 + 765087080448000*x^6 + 6
75479464714240*x^5 + 451581382260736*x^4 + 239021184223104*x^3 + 6515199806
3712*x^2 + 12971175524316*x + 74032009514181)*sqrt(2*x^2 - x + 3) + 6064275
33063/17179869184*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32
*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{3}{2}} \left(5x^2+3x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**4, x)
```

Giac [A]
time = 5.19, size = 103, normalized size = 0.45

$$\frac{1}{676457349120}\left(4\left(8\left(4\left(16\left(20\left(8\left(28\left(160\left(12\left(200\left(20\,x+41\right)x+19833\right)x+363785\right)x+81213077\right)x+2334860475\right)x+16491197869\right)x+220498721807\right)x+1867353001743\right)x+2035999939491\right)x+3242793881079\right)x+74032009514181\right)\sqrt{2\,x^{2}-x+3}-\frac{606427533063}{8589934592}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^{2}-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/676457349120*(4*(8*(4*(16*(20*(8*(28*(160*(12*(200*(20*x + 41)*x + 19833)
*x + 363785)*x + 81213077)*x + 2334860475)*x + 16491197869)*x + 22049872180
7)*x + 1867353001743)*x + 2035999939491)*x + 3242793881079)*x + 74032009514
181)*sqrt(2*x^2 - x + 3) - 606427533063/8589934592*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(2\,x^2-x+3\right)}^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4, x)
```

3.66 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \, dx$

Optimal. Leaf size=189

$$-\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}-\frac{4625907(3-x+2x^2)^{5/2}}{2293760}-\frac{81685x(3$$

```
[Out] -667795/2097152*(1-4*x)*(2*x^2-x+3)^(3/2)-4625907/2293760*(2*x^2-x+3)^(5/2)
-81685/114688*x*(2*x^2-x+3)^(5/2)+384739/43008*x^2*(2*x^2-x+3)^(5/2)+27785/
1536*x^3*(2*x^2-x+3)^(5/2)+725/48*x^4*(2*x^2-x+3)^(5/2)+25/4*x^5*(2*x^2-x+3
)^(5/2)-1059790665/134217728*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4607785
5/33554432*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{384739(2x^2-x+3)^{5/2}x^2}{43008}-\frac{81685(2x^2-x+3)^{5/2}x}{114688}-\frac{4625907(2x^2-x+3)^{5/2}}{2293760}-\frac{667795(1-4x)\left(2x^2-x+3\right)^{3/2}}{2097152}-\frac{46077855(1-4x)\sqrt{2x^2-x+3}}{33554432}+\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5+\frac{725}{48}\left(2x^2-x+3\right)^{5/2}x^4+\frac{27785(2x^2-x+3)^{5/2}x^3}{1536}-\frac{1059790665\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{67108864\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-46077855*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/33554432 - (667795*(1 - 4*x)*(3 -
 x + 2*x^2)^(3/2))/2097152 - (4625907*(3 - x + 2*x^2)^(5/2))/2293760 - (816
85*x*(3 - x + 2*x^2)^(5/2))/114688 + (384739*x^2*(3 - x + 2*x^2)^(5/2))/430
08 + (27785*x^3*(3 - x + 2*x^2)^(5/2))/1536 + (725*x^4*(3 - x + 2*x^2)^(5/2
))/48 + (25*x^5*(3 - x + 2*x^2)^(5/2))/4 - (1059790665*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(67108864*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx = \frac{25}{4}x^5\left(3-x+2x^2\right)^{5/2}+\frac{1}{20}\int \left(3-x+2x^2\right)^{3/2}\Big(160+720x+22$$

$$= \frac{725}{48}x^4\left(3-x+2x^2\right)^{5/2}+\frac{25}{4}x^5\left(3-x+2x^2\right)^{5/2}+\frac{1}{360}\int \left(3-x+\right.$$

$$= \frac{27785x^3\left(3-x+2x^2\right)^{5/2}}{1536}+\frac{725}{48}x^4\left(3-x+2x^2\right)^{5/2}+\frac{25}{4}x^5\left(3-x\right.$$

$$= \frac{384739x^2\left(3-x+2x^2\right)^{5/2}}{43008}+\frac{27785x^3\left(3-x+2x^2\right)^{5/2}}{1536}+\frac{725}{48}x^4\left(3\right.$$

$$= -\frac{81685x\left(3-x+2x^2\right)^{5/2}}{114688}+\frac{384739x^2\left(3-x+2x^2\right)^{5/2}}{43008}+\frac{27785x^3}{}$$

$$= -\frac{4625907\left(3-x+2x^2\right)^{5/2}}{2293760}-\frac{81685x\left(3-x+2x^2\right)^{5/2}}{114688}+\frac{384739x^2}{}$$

$$= -\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}-\frac{4625907\left(3-x+2x^2\right)^{5/2}}{2293760}-$$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x\right.}{2097152}$$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x\right.}{2097152}$$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x\right.}{2097152}$$

Mathematica [A]
time = 0.78, size = 95, normalized size = 0.50

$$\frac{4\sqrt{3-x+2x^2}\left(-72152399943+53985432012x+199615064544x^2+389257196928x^3+487891884032x^4+571298324480x^5+430820229120x^6+328328806400x^7+124780544000x^8+88080384000x^9\right)-111278019825\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{14092861440}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-72152399943 + 53985432012*x + 199615064544*x^2 + 3
89257196928*x^3 + 487891884032*x^4 + 571298324480*x^5 + 430820229120*x^6 +
328328806400*x^7 + 124780544000*x^8 + 88080384000*x^9) - 111278019825*Sqrt[
2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/14092861440
```

Maple [A]
time = 0.14, size = 151, normalized size = 0.80

method	result
risch	$\frac{(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+487891884032x^4+389257196928x^3+199}{3523215360}$
trager	$\left(25x^9+\frac{425}{12}x^8+\frac{35785}{384}x^7+\frac{438253}{3584}x^6+\frac{13947713}{86016}x^5+\frac{34032637}{245760}x^4+\frac{1013690617}{9175040}x^3+\frac{297046227}{5242880}x^2+\frac{449878}{293601}\right.$
default	$\frac{46077855(4x-1)\sqrt{2x^2-x+3}}{33554432}+\frac{1059790665\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{134217728}+\frac{667795(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{2097152}-\frac{81685x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 46077855/33554432*(4*x-1)*(2*x^2-x+3)^(1/2)+1059790665/134217728*2^(1/2)*ar
csinh(4/23*23^(1/2)*(x-1/4))+667795/2097152*(4*x-1)*(2*x^2-x+3)^(3/2)-81685
/114688*x*(2*x^2-x+3)^(5/2)+384739/43008*x^2*(2*x^2-x+3)^(5/2)+27785/1536*x
^3*(2*x^2-x+3)^(5/2)+725/48*x^4*(2*x^2-x+3)^(5/2)+25/4*x^5*(2*x^2-x+3)^(5/2
)-4625907/2293760*(2*x^2-x+3)^(5/2)
```

Maxima [A]
time = 0.51, size = 172, normalized size = 0.91

$$\frac{25}{4}\,(2\,x^2-x+3)^{\frac{5}{2}}x^5+\frac{725}{48}\,(2\,x^2-x+3)^{\frac{5}{2}}x^4+\frac{27785}{1536}\,(2\,x^2-x+3)^{\frac{5}{2}}x^3+\frac{384739}{43008}\,(2\,x^2-x+3)^{\frac{5}{2}}x^2-\frac{81685}{114688}\,(2\,x^2-x+3)^{\frac{5}{2}}x-\frac{4625907}{2293760}\,(2\,x^2-x+3)^{\frac{5}{2}}+\frac{667795}{524288}\,(2\,x^2-x+3)^{\frac{3}{2}}x-\frac{667795}{2097152}\,(2\,x^2-x+3)^{\frac{3}{2}}+\frac{46077855}{8388608}\,\sqrt{2\,x^2-x+3}\,x+\frac{1059790665}{134217728}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{46077855}{33554432}\,\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 25/4*(2*x^2 - x + 3)^(5/2)*x^5 + 725/48*(2*x^2 - x + 3)^(5/2)*x^4 + 27785/1
536*(2*x^2 - x + 3)^(5/2)*x^3 + 384739/43008*(2*x^2 - x + 3)^(5/2)*x^2 - 81
685/114688*(2*x^2 - x + 3)^(5/2)*x - 4625907/2293760*(2*x^2 - x + 3)^(5/2)
+ 667795/524288*(2*x^2 - x + 3)^(3/2)*x - 667795/2097152*(2*x^2 - x + 3)^(3
/2) + 46077855/8388608*sqrt(2*x^2 - x + 3)*x + 1059790665/134217728*sqrt(2)
*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46077855/33554432*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 4.44, size = 98, normalized size = 0.52

$$\frac{1}{3523215360}\,(88080384000\,x^9+124780544000\,x^8+328328806400\,x^7+430820229120\,x^6+571298324480\,x^5+487891884032\,x^4+389257196928\,x^3+199615064544\,x^2+53985432012\,x-72152399943)\,\sqrt{2\,x^2-x+3}+\frac{1059790665}{268435456}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/3523215360*(88080384000*x^9 + 124780544000*x^8 + 328328806400*x^7 + 43082
0229120*x^6 + 571298324480*x^5 + 487891884032*x^4 + 389257196928*x^3 + 1996
```

```
15064544*x^2 + 53985432012*x - 72152399943)*sqrt(2*x^2 - x + 3) + 105979066
5/268435456*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2 - x + 3\right)^{\frac{3}{2}} \left(5x^2 + 3x + 2\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3, x)
```

Giac [A]
time = 4.64, size = 93, normalized size = 0.49

$$\frac{1}{3523215360} \left(4 \left(8 \left(4 \left(16 \left(20 \left(8 \left(140 \left(160 \left(12 x + 17\right)x + 7157\right)x + 1314759\right)x + 13947713\right)x + 238228459\right)x + 3041071851\right)x + 6237970767\right)x + 13496358003\right)x - 72152399943\right) \sqrt{2 x^2 - x + 3} - \frac{1059790665}{134217728} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2} x - \sqrt{2 x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/3523215360*(4*(8*(4*(16*(20*(8*(140*(160*(12*x + 17)*x + 7157)*x + 131475
9)*x + 13947713)*x + 238228459)*x + 3041071851)*x + 6237970767)*x + 1349635
8003)*x - 72152399943)*sqrt(2*x^2 - x + 3) - 1059790665/134217728*sqrt(2)*l
og(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3, x)
```

3.67 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 \, dx$

Optimal. Leaf size=147

$$\frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576}+\frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}+\frac{73861(3-x+2x^2)^{5/2}}{215040}+\frac{24499x(3-x+}{10752}$$

```
[Out] 24293/196608*(1-4*x)*(2*x^2-x+3)^(3/2)+73861/215040*(2*x^2-x+3)^(5/2)+24499
/10752*x*(2*x^2-x+3)^(5/2)+1235/448*x^2*(2*x^2-x+3)^(5/2)+25/16*x^3*(2*x^2-
x+3)^(5/2)+12850997/4194304*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+558739/1
048576*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{1235}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{24499(2x^2-x+3)^{5/2}x}{10752}+\frac{73861(2x^2-x+3)^{5/2}}{215040}+\frac{24293(1-4x)\left(2x^2-x+3\right)^{3/2}}{196608}+\frac{558739(1-4x)\sqrt{2x^2-x+3}}{1048576}+\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{12850997\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (558739*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 + (24293*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/196608 + (73861*(3 - x + 2*x^2)^(5/2))/215040 + (24499*x*(3 -
 x + 2*x^2)^(5/2))/10752 + (1235*x^2*(3 - x + 2*x^2)^(5/2))/448 + (25*x^3*(
3 - x + 2*x^2)^(5/2))/16 + (12850997*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*
Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2\,dx = \frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{16}\int \left(3-x+2x^2\right)^{3/2}\Big(64+192x+239$$

$$= \frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{224}\int \left(3-x\right.$$

$$= \frac{24499x\left(3-x+2x^2\right)^{5/2}}{10752}+\frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{25}{16}x^3\left(3-x\right.$$

$$= \frac{73861\left(3-x+2x^2\right)^{5/2}}{215040}+\frac{24499x\left(3-x+2x^2\right)^{5/2}}{10752}+\frac{1235}{448}x^2\left(3-x\right.$$

$$= \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}+\frac{73861\left(3-x+2x^2\right)^{5/2}}{215040}+\frac{24499}{}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576}+\frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576}+\frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576}+\frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}$$

Mathematica [A]
time = 0.57, size = 85, normalized size = 0.58

$$\frac{4\sqrt{3-x+2x^2}\,(439831323+1619403428x+1799647136x^2+2728413312x^3+2061273088x^4+2025840640x^5+525926400x^6+688128000x^7)+1349354685\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{440401920}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(439831323 + 1619403428*x + 1799647136*x^2 + 2728413
312*x^3 + 2061273088*x^4 + 2025840640*x^5 + 525926400*x^6 + 688128000*x^7)
+ 1349354685*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/440401920
```

Maple [A]
time = 0.12, size = 117, normalized size = 0.80

method	result
risch	$\frac{(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323)\sqrt{2x^2-}}{110100480}$
trager	$\left(\frac{25}{4}x^7+\frac{535}{112}x^6+\frac{49459}{2688}x^5+\frac{143783}{7680}x^4+\frac{7105243}{286720}x^3+\frac{8034139}{491520}x^2+\frac{404850857}{27525120}x+\frac{146610441}{36700160}\right)\sqrt{2x^2-x+}$
default	$-\frac{558739(4x-1)\sqrt{2x^2-x+3}}{1048576}-\frac{12850997\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{4194304}-\frac{24293(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{196608}+\frac{24499x(2x}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -558739/1048576*(4*x-1)*(2*x^2-x+3)^(1/2)-12850997/4194304*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-24293/196608*(4*x-1)*(2*x^2-x+3)^(3/2)+24499/10752*x
*(2*x^2-x+3)^(5/2)+1235/448*x^2*(2*x^2-x+3)^(5/2)+25/16*x^3*(2*x^2-x+3)^(5/
2)+73861/215040*(2*x^2-x+3)^(5/2)
```

Maxima [A]
time = 0.51, size = 138, normalized size = 0.94

$$\frac{25}{16}\,(2\,x^2-x+3)^{\frac{5}{2}}x^3+\frac{1235}{448}\,(2\,x^2-x+3)^{\frac{5}{2}}x^2+\frac{24499}{10752}\,(2\,x^2-x+3)^{\frac{5}{2}}x+\frac{73861}{215040}\,(2\,x^2-x+3)^{\frac{5}{2}}-\frac{24293}{49152}\,(2\,x^2-x+3)^{\frac{3}{2}}x+\frac{24293}{196608}\,(2\,x^2-x+3)^{\frac{3}{2}}-\frac{558739}{262144}\,\sqrt{2\,x^2-x+3}\,x-\frac{12850997}{4194304}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{558739}{1048576}\,\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 25/16*(2*x^2 - x + 3)^(5/2)*x^3 + 1235/448*(2*x^2 - x + 3)^(5/2)*x^2 + 2449
9/10752*(2*x^2 - x + 3)^(5/2)*x + 73861/215040*(2*x^2 - x + 3)^(5/2) - 2429
3/49152*(2*x^2 - x + 3)^(3/2)*x + 24293/196608*(2*x^2 - x + 3)^(3/2) - 5587
```

```
39/262144*sqrt(2*x^2 - x + 3)*x - 12850997/4194304*sqrt(2)*arcsinh(1/23*sqr
t(23)*(4*x - 1)) + 558739/1048576*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 4.10, size = 88, normalized size = 0.60

$$\frac{1}{110100480}\left(688128000\,x^{7}+525926400\,x^{6}+2025840640\,x^{5}+2061273088\,x^{4}+2728413312\,x^{3}+1799647136\,x^{2}+1619403428\,x+439831323\right)\sqrt{2\,x^{2}-x+3}+\frac{12850997}{8388608}\,\sqrt{2}\,\log\left(4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/110100480*(688128000*x^7 + 525926400*x^6 + 2025840640*x^5 + 2061273088*x^
4 + 2728413312*x^3 + 1799647136*x^2 + 1619403428*x + 439831323)*sqrt(2*x^2
- x + 3) + 12850997/8388608*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2}-x+3\right)^{\frac{3}{2}}\left(5x^{2}+3x+2\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2, x)
```

Giac [A]
time = 2.98, size = 83, normalized size = 0.56

$$\frac{1}{110100480}\left(4\left(8\left(4\left(16\left(20\left(120\left(140\,x+107\right)x+49459\right)x+1006481\right)x+21315729\right)x+56238973\right)x+404850857\right)x+439831323\right)\sqrt{2\,x^{2}-x+3}+\frac{12850997}{4194304}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^{2}-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/110100480*(4*(8*(4*(16*(20*(120*(140*x + 107)*x + 49459)*x + 1006481)*x +
 21315729)*x + 56238973)*x + 404850857)*x + 439831323)*sqrt(2*x^2 - x + 3)
+ 12850997/4194304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
 + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2, x)
```

3.68 $\int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2) \, dx$

Optimal. Leaf size=105

$$-\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536}+\frac{107}{240}(3-x+2x^2)^{5/2}+\frac{5}{12}x(3-x+2x^2)^{5/2}$$

```
[Out] -179/1536*(1-4*x)*(2*x^2-x+3)^(3/2)+107/240*(2*x^2-x+3)^(5/2)+5/12*x*(2*x^2
-x+3)^(5/2)-94691/32768*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4117/8192*(1
-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{5}{12}x(2x^2-x+3)^{5/2}+\frac{107}{240}(2x^2-x+3)^{5/2}-\frac{179(1-4x)(2x^2-x+3)^{3/2}}{1536}-\frac{4117(1-4x)\sqrt{2x^2-x+3}}{8192}-\frac{94691\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-4117*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/8192 - (179*(1 - 4*x)*(3 - x + 2*x^2)
^(3/2))/1536 + (107*(3 - x + 2*x^2)^(5/2))/240 + (5*x*(3 - x + 2*x^2)^(5/2)
)/12 - (94691*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)\,dx &= \frac{5}{12}x\left(3-x+2x^2\right)^{5/2}+\frac{1}{12}\int \left(9+\frac{107x}{2}\right)\left(3-x+2x^2\right)^{3/2}\,dx\\ &= \frac{107}{240}\left(3-x+2x^2\right)^{5/2}+\frac{5}{12}x\left(3-x+2x^2\right)^{5/2}+\frac{179}{96}\int \left(3-x+2x\right.\\ &= -\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x+2x^2\right)^{5/2}+\frac{5}{12}x\left(3-\right.\\ &= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{1}{2}\\ &= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{1}{2}\\ &= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{1}{2}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 75, normalized size = 0.71

$$\frac{4\sqrt{3-x+2x^2}\left(388341+565276x+319072x^2+561024x^3+14336x^4+204800x^5\right)-1420365\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{491520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(388341 + 565276*x + 319072*x^2 + 561024*x^3 + 14336
*x^4 + 204800*x^5) - 1420365*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]
)/491520
```

Maple [A]
time = 0.10, size = 83, normalized size = 0.79

method	result
risch	$\frac{(204800x^5+14336x^4+561024x^3+319072x^2+565276x+388341)\sqrt{2x^2-x+3}}{122880} + \frac{94691\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{32768}$
trager	$\left(\frac{5}{3}x^5 + \frac{7}{60}x^4 + \frac{1461}{320}x^3 + \frac{9971}{3840}x^2 + \frac{141319}{30720}x + \frac{129447}{40960}\right)\sqrt{2x^2-x+3} - \frac{94691\,\mathrm{RootOf}(\_Z^2-2)\ln\left(-4\,\mathrm{Root}\right.}{}$
default	$\frac{5x(2x^2-x+3)^{\frac{5}{2}}}{12} + \frac{107(2x^2-x+3)^{\frac{5}{2}}}{240} + \frac{179(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{1536} + \frac{4117(4x-1)\sqrt{2x^2-x+3}}{8192} + \frac{94691\sqrt{2}\ \mathrm{arcsinh}\left(\right.}{32768}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/12*x*(2*x^2-x+3)^(5/2)+107/240*(2*x^2-x+3)^(5/2)+179/1536*(4*x-1)*(2*x^2-
x+3)^(3/2)+4117/8192*(4*x-1)*(2*x^2-x+3)^(1/2)+94691/32768*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.54, size = 104, normalized size = 0.99

$$\frac{5}{12}\left(2x^2-x+3\right)^{\frac{5}{2}}x + \frac{107}{240}\left(2x^2-x+3\right)^{\frac{5}{2}} + \frac{179}{384}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{179}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{4117}{2048}\sqrt{2x^2-x+3}\,x + \frac{94691}{32768}\sqrt{2}\ \mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right) - \frac{4117}{8192}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(5/2)*x + 107/240*(2*x^2 - x + 3)^(5/2) + 179/384*(2*x
^2 - x + 3)^(3/2)*x - 179/1536*(2*x^2 - x + 3)^(3/2) + 4117/2048*sqrt(2*x^2
 - x + 3)*x + 94691/32768*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 4117/8
192*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 2.18, size = 78, normalized size = 0.74

$$\frac{1}{122880}\left(204800x^5 + 14336x^4 + 561024x^3 + 319072x^2 + 565276x + 388341\right)\sqrt{2x^2-x+3} + \frac{94691}{65536}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/122880*(204800*x^5 + 14336*x^4 + 561024*x^3 + 319072*x^2 + 565276*x + 388
341)*sqrt(2*x^2 - x + 3) + 94691/65536*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2 - x + 3\right)^{\frac{3}{2}} \cdot \left(5x^2 + 3x + 2\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2), x)
```

Giac [A]
time = 3.89, size = 73, normalized size = 0.70

$$\frac{1}{122880} \left(4 \left(8 \left(4 \left(16 \left(100 x + 7\right)x + 4383\right)x + 9971\right)x + 141319\right)x + 388341\right)\sqrt{2 x^2 - x + 3} - \frac{94691}{32768} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2} x - \sqrt{2 x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 1/122880*(4*(8*(4*(16*(100*x + 7)*x + 4383)*x + 9971)*x + 141319)*x + 38834
1)*sqrt(2*x^2 - x + 3) - 94691/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq
rt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2 - x + 3\right)}^{3/2}\,\left(5\,x^2 + 3\,x + 2\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2), x)
```

3.69 $\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=197

$$-\frac{1}{100}(49-20x)\sqrt{3-x+2x^2}-\frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}}+\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(247+5\right.}}}{}\right.$$

```
[Out] -2203/2000*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/100*(49-20*x)*(2*x^2-x+
3)^(1/2)-11/3875*arctanh(1/62*(8+x*(130-69*2^(1/2))-61*2^(1/2))*682^(1/2)/(
-247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-84227+170500*2^(1/2))^(1/2)+11
/3875*arctan(1/62*(8+61*2^(1/2)+x*(130+69*2^(1/2)))*682^(1/2)/(247+500*2^(1
/2))^(1/2)/(2*x^2-x+3)^(1/2))*(84227+170500*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {991, 1090, 633, 221, 1049, 1043, 212, 210}

$$\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{100}\sqrt{2x^2-x+3}\,(49-20x)-\frac{11}{125}\sqrt{\frac{11}{31}\left(500\sqrt{2}-247\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(500\sqrt{2}-247\right)}}\left(\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right)-\frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] -1/100*((49 - 20*x)*Sqrt[3 - x + 2*x^2]) - (2203*ArcSinh[(1 - 4*x)/Sqrt[23]
])/(1000*Sqrt[2]) + (11*Sqrt[(11*(247 + 500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(
62*(247 + 500*Sqrt[2]))]*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x))/Sqrt[3 -
x + 2*x^2]])/125 - (11*Sqrt[(11*(-247 + 500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/
(62*(-247 + 500*Sqrt[2]))]*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x))/Sqrt[3
- x + 2*x^2]])/125
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 991

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
```

```
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx = -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{50}\int \frac{-\frac{731}{2}+\frac{1195x}{4}-\frac{2203x^2}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{250}\int \frac{-726+3146x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx + \cdots$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} + \frac{\int \frac{2662\left(16+3\sqrt{2}\right)+2662\left(10-13\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx}{5500\sqrt{2}} - \cdots$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} - \frac{1}{125}\left(322102\left(1000-\cdots\right.\right.$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125}\sqrt{\frac{11}{31}}\left(247+500\cdots\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.34, size = 228, normalized size = 1.16

$$\frac{20(-49+20x)\sqrt{3-x+2x^2} - 2203\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right) + 1936\,\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{-36\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+6\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+13\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]}{2000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (20*(-49 + 20*x)*Sqrt[3 - x + 2*x^2] - 2203*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6
- 2*x + 4*x^2]] + 1936*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1
^3 - 5*#1^4 & , (-36*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 6*Sqrt[
2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 13*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*
#1^3) & ])/2000
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3459 vs. $2(144) = 288$.
time = 0.83, size = 3460, normalized size = 17.56 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x*(2*x^2-x+3)^(1/2)-49/100*(2*x^2-x+3)^(1/2)+2203/2000*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-2/1321375*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/
2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(4245*2^(1/2)
*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2
)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(
1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)
))*(-775687+549362*2^(1/2))^(1/2)+6154*(-8866+6820*2^(1/2))^(1/2)*arctan(1/
11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(2
3*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2
^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+123
25786*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)
-359414*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-
1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*
(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8
-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1
/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-2/264275*(8*(2^(1/2)-1+x
)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^
(1/2)*2^(1/2)*(2365*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-
775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)
^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-
1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)
/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+3338*(-8866+68
20*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+
```

```
3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485
*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x
)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-77568
7+549362*2^(1/2))^(1/2)+3192442*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-
x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+68
20*2^(1/2))^(1/2))*2^(1/2)-5264358*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866
+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/
2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(
1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/
2)-13/105710*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(285*2^(1/2)*(-8866+6820*2^(1/2))
^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*
(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2
^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^
(1/2))^(1/2)+386*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+5493
62*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+2
4*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(
1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1
-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)-274846*arctanh(31/2*(8*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3
*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-1543366*arctanh(31/2*(8
*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2
)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)
-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))
/(-8866+6820*2^(1/2))^(1/2)+3/10571*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^
(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*2^(1/
2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1
/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))
^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82
*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/
2)))*(-775687+549362*2^(1/2))^(1/2)+218*(-8866+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")

[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2027 vs. $2(144) = 288$.
time = 4.39, size = 2027, normalized size = 10.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")

[Out] 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arc
tan(1/10605875*(230*sqrt(10)*(2*24200^(3/4)*sqrt(31)*(20846*x^7 - 109153*x^
6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - sqrt(2)*(28854*x^7
- 90639*x^6 + 200187*x^5 - 262838*x^4 + 117544*x^3 - 23472*x^2 - 186624*x +
86400) - 172800*x + 186624) + 5*24200^(1/4)*sqrt(31)*(112238*x^7 - 1817988
*x^6 + 10351960*x^5 - 25791248*x^4 + 34522560*x^3 - 28368000*x^2 - sqrt(2)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - 19989504*x + 10533888) - 21067776*x + 19989504))*sqrt(2*x^2 - x
+ 3)*sqrt(247*sqrt(2) + 1000) + 30107000*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(5/
119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)*(46522*x^7 - 71117*x^6 + 257247*x^5
- 273360*x^4 + 484920*x^3 - 269568*x^2 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 +
33771*x^5 - 5576*x^4 - 576*x^3 + 32184*x^2 - 32184*x) + 269568*x) + 5*2420
0^(1/4)*sqrt(31)*(309512*x^7 - 4017952*x^6 + 15741280*x^5 - 22625280*x^4 +
37693440*x^3 - 13519872*x^2 - sqrt(2)*(516957*x^7 - 6676948*x^6 + 25569820*
x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 46199808*x) + 13519872*x
))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) + 130900*sqrt(31)*sqrt(2)*(
123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x
^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^
5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 595
0*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781
920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 51
7*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))
*sqrt((24200^(1/4)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x -
76)*sqrt(247*sqrt(2) + 1000) + 58310*x^2 + 52360*sqrt(2)*(2*x^2 - x + 3) -
179690*x + 238000)/x^2) + 342125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 5338
5560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7
744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*
x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4
661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3461
5296*x^2 - 24772608*x + 18579456)) + 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)

```
*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arctan(1/10605875*(230*sqrt(10)*(2*24200^
(3/4)*sqrt(31)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x
^3 - 156600*x^2 - sqrt(2)*(28854*x^7 - 90639*x^6 + 200187*x^5 - 262838*x^4
+ 117544*x^3 - 23472*x^2 - 186624*x + 86400) - 172800*x + 186624) + 5*24200
^(1/4)*sqrt(31)*(112238*x^7 - 1817988*x^6 + 10351960*x^5 - 25791248*x^4 + 3
4522560*x^3 - 28368000*x^2 - sqrt(2)*(125839*x^7 - 1864281*x^6 + 9323336*x^
5 - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - 19989504*x + 10533888) - 2
1067776*x + 19989504))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) - 30107
000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1
549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 3961
04*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53
9136) + 1154304*x - 456192) - sqrt(5/119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)
*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2
 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 + 33771*x^5 - 5576*x^4 - 576*x^3 + 3218
4*x^2 - 32184*x) + 269568*x) + 5*24200^(1/4)*sqrt(31)*(309512*x^7 - 4017952
*x^6 + 15741280*x^5 - 22625280*x^4 + 37693440*x^3 - 13519872*x^2 - sqrt(2)*
(516957*x^7 - 6676948*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46
199808*x^2 - 46199808*x) + 13519872*x))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2
) + 1000) - 130900*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6
- 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8
- 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 120960
0*x^2 - 1036800*x) + 3276288*x) - 5950*sqrt(31)*(254591*x^8 - 4815126*x^7 +
 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(24200^(1/4)*sqrt(10)*sqrt(2*x^
2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2) + 1000) - 58310*
x^2 - 52360*sqrt(2)*(2*x^2 - x + 3) + 179690*x - 238000)/x^2) - 342125*sqrt
(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*
x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 97
89*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223
064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^
5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) +
11/36890000*24200^(1/4)*sqrt(10)*sqrt(247*sqrt(2) + 1000)*(247*sqrt(2) - 10
00)*log(1512500/119*(24200^(1/4)*sqrt(10)*sqrt(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{5x^2 + 3x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2), x)
```

3.70 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=232

$$\frac{4}{155}(4-5x)\sqrt{3-x+2x^2}+\frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)}-\frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)+\frac{\sqrt{\frac{11}{31}\left(3169333+2265\ldots\right.}}{\ldots}$$

```
[Out] 1/31*(3+10*x)*(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)-2/25*arcsinh(1/23*(1-4*x)*23^
(1/2))*2^(1/2)+4/155*(4-5*x)*(2*x^2-x+3)^(1/2)-1/48050*arctanh(1/62*(3514+x
*(9440-6477*2^(1/2))-2963*2^(1/2))*682^(1/2)/(-3169333+2265350*2^(1/2))^(1/
2)/(2*x^2-x+3)^(1/2))*(-1080742553+772484350*2^(1/2))^(1/2)+1/48050*arctan(
1/62*(3514+2963*2^(1/2)+x*(9440+6477*2^(1/2)))*682^(1/2)/(3169333+2265350*2
^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(1080742553+772484350*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {985, 1080, 1090, 633, 221, 1049, 1043, 212, 210}

$$\frac{\sqrt{\frac{11}{31}\left(3169333+2265350\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{62\left(3169333+2265350\sqrt{2}\right)}}\left(\left(9440+6477\sqrt{2}\right)x+2963\sqrt{2}+3514\right)}{\sqrt{2x^2-x+3}}\right)}{1550}+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\sqrt{2x^2-x+3}-\frac{\sqrt{\frac{11}{31}\left(2265350\sqrt{2}-3169333\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(2265350\sqrt{2}-3169333\right)}}\left(\left(9440-6477\sqrt{2}\right)x-2963\sqrt{2}+3514\right)}{\sqrt{2x^2-x+3}}\right)}{1550}-\frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*(4 - 5*x)*Sqrt[3 - x + 2*x^2])/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/
(31*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/25 + (Sqrt
[(11*(3169333 + 2265350*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(3169333 + 226535
0*Sqrt[2]))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3 - x +
2*x^2]])/1550 - (Sqrt[(11*(-3169333 + 2265350*Sqrt[2]))/31]*ArcTanh[(Sqrt[1
1/(62*(-3169333 + 2265350*Sqrt[2]))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/1550
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 985

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx = \frac{(3+10x)\,(3-x+2x^2)^{3/2}}{31\,(2+3x+5x^2)} - \frac{1}{31}\int \frac{\sqrt{3-x+2x^2}\,\left(-\frac{69}{2}+13x+40x^2\right)}{2+3x+5x^2}\,dx$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\,(3-x+2x^2)^{3/2}}{31\,(2+3x+5x^2)} + \frac{\int \frac{13070-5750x+2480x^2}{\sqrt{3-x+2x^2}\,(2+3x+}}{3100}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\,(3-x+2x^2)^{3/2}}{31\,(2+3x+5x^2)} + \frac{\int \frac{60390-36190x}{\sqrt{3-x+2x^2}\,(2+3x+}}{15500}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\,(3-x+2x^2)^{3/2}}{31\,(2+3x+5x^2)} + \frac{1}{25}\left(2\sqrt{\frac{2}{23}}\right)\text{Subst}\Bigg($$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\,(3-x+2x^2)^{3/2}}{31\,(2+3x+5x^2)} - \frac{2}{25}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\,(3-x+2x^2)^{3/2}}{31\,(2+3x+5x^2)} - \frac{2}{25}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.52, size = 416, normalized size = 1.79

$$\frac{50\left(\frac{55(7+13x)\sqrt{3-x+2x^2}}{2+3x+5x^2}-62\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)\right)+682\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{999\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+310\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+100\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]+11\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{-72888\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+8230\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+2025\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]}{38750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (50*((55*(7 + 13*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) - 62*Sqrt[2]*Log
[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]) + 682*RootSum[-56 - 26*Sqrt[2]*#1 + 17
*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (999*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*
x^2] - #1] + 310*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 +
100*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1
 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 11*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2
 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-72888*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^
2] - #1] + 8230*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 2
```

```
025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1
 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/38750
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28184 vs. $2(175) = 350$.
time = 0.98, size = 28185, normalized size = 121.49

method	result
trager	Expression too large to display
risch	$\frac{11(7+13x)\sqrt{2x^2-x+3}}{155(5x^2+3x+2)}+\frac{2\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{25}+\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\,\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maple [F]

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2150 vs. $2(175) = 350$.
time = 2.79, size = 2150, normalized size = 9.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/90746855745853600*(10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(2)*(5*x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/172758074198
807633719789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118
*x^7 - 9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x
^2 - sqrt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 113
67152*x^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) +
1404517*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x
^5 - 70476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 423
2733*x^6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 4669
7472*x + 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(31693
33*sqrt(2) + 4530700) + 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^
8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 9849
6*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*
x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
sqrt(45307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7
- 12210811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2
- sqrt(2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 121140
72*x^3 + 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(
1/4)*sqrt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 1
12374720*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 5039126
0*x^5 - 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304
*x))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) + 7590571938849196
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
 + 3276288*x) + 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 323035
80*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 1548
8*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268
*x^2 - 1944*x) + 144820224*x))*sqrt(-(1987037073032^(1/4)*sqrt(45307)*sqrt(
62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*s
qrt(3169333*sqrt(2) + 4530700) - 11567627293306*x^2 - 10387257161336*sqrt(2
)*(2*x^2 - x + 3) + 35647177985494*x - 47214805278800)/x^2) + 5572841103187
343023219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^
5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 -
 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x
 - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x
^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x +
 18579456)) + 10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(2)*(5*
x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/1727580741988076337
19789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118*x^7 -
9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x^2 - sq
rt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 11367152*x
^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) + 1404517
```

```
*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x^5 - 70
476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 4232733*x^
6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 46697472*x
+ 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt
(2) + 4530700) - 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
 sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(45
307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7 - 12210
811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2 - sqrt(
2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 12114072*x^3
+ 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(1/4)*sq
rt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 11237472
0*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 50391260*x^5 -
 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304*x))*sq
rt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) - 7590571938849196*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) - 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6
- 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(
2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 -
1944*x) + 144820224*x))*sqrt((1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 142...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
```

```
ding error%%%{174900625,[8]%%%}+%%%{%%{[-419761500,0]:[1,0,-2]%%},[7]%%%}+%
%%{-68
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}}{\left(5\,x^2+3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2,x)
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2, x)
```

3.71 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} \, dx$

Optimal. Leaf size=223

$$\frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2}+\frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)}+\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\tan^{-1}}{}$$

[Out] 1/62*(3+10*x)*(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2+3/3844*(277+696*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-3/5243216*arctanh(1/31*(29367+x*(70517-49942*2^(1/2))-20575*2^(1/2))*341^(1/2)/(-366990269+259509026*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-250287363458+176985155732*2^(1/2))^(1/2)+3/5243216*arctan(1/31*(29367+20575*2^(1/2)+x*(70517+49942*2^(1/2)))*341^(1/2)/(366990269+259509026*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(250287363458+176985155732*2^(1/2))^(1/2)

Rubi [A]
time = 0.28, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {985, 1027, 1049, 1043, 212, 210}

$$\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(366990269+259509026\sqrt{2}\right)}}\left(\left(70517+49942\sqrt{2}\right)x+20575\sqrt{2}+29367\right)}{\sqrt{2x^2-x+3}}\right)}{7688}+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(696x+277)\sqrt{2x^2-x+3}}{3844\left(5x^2+3x+2\right)}-\frac{3\sqrt{\frac{1}{682}\left(259509026\sqrt{2}-366990269\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(259509026\sqrt{2}-366990269\right)}}\left(\left(70517-49942\sqrt{2}\right)x-20575\sqrt{2}+29367\right)}{\sqrt{2x^2-x+3}}\right)}{7688}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]

[Out] ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(277 + 696*x)*Sqrt[3 - x + 2*x^2])/(3844*(2 + 3*x + 5*x^2)) + (3*Sqrt[(366990269 + 259509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))]*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/7688 - (3*Sqrt[(-366990269 + 259509026*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-366990269 + 259509026*Sqrt[2]))]*(29367 - 20575*Sqrt[2] + (70517 - 49942*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/7688

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 985

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1027

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(g*b - 2*a*h - (b*h - 2*g*
c)*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1)
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} - \frac{1}{62}\int \frac{\left(-\frac{189}{2}+33x\right)\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx$$

$$= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\int \frac{\frac{13359}{4}-1353x}{\sqrt{3-x+2x^2}\,(2-}}{1922}$$

$$= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\int \frac{-\frac{33}{4}\left(6257-4453\sqrt{2}\right.}{\sqrt{3-x+}}}{42}$$

$$= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\left(99\left(519018052-\right.\right.}{}$$

$$= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{3\sqrt{\frac{1}{682}\left(36699026\right.}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.79, size = 572, normalized size = 2.57

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((3306250*Sqrt[3 - x + 2*x^2]*(2220 + 8343*x + 10171*x^2 + 11680*x^3))/(2 +
 3*x + 5*x^2)^2 - 42578694225*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqr
t[2]*#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]/(-13*Sq
rt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 406695200*RootSum[-56 - 26*
Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (93*Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
```

```
] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 14*RootS
um[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (4926449381*
Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 2660991465*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[
2]*#1^2 - 10*#1^3) & ] - 186*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt
[2]*#1^3 - 5*#1^4 & , (155209944*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*
x^2] - #1]*#1 - 248390285*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2
)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/12709225000
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81551 vs. $2(171) = 342$.
time = 0.97, size = 81552, normalized size = 365.70 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(171) = 342$.
time = 3.27, size = 2183, normalized size = 9.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/857730714176979241096966*(189113268*134689869150937352^(1/4)*sqrt(1297545
13)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(366990269*
sqrt(2) + 519018052)*arctan(1/10672590923431936755592676225454473*(160895596
12*sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)*(38305160*x^7 - 1
47261352*x^6 + 309398878*x^5 - 495410374*x^4 + 248212864*x^3 - 117285552*x^
2 - sqrt(2)*(26988622*x^7 - 104036813*x^6 + 218448200*x^5 - 350579241*x^4 +
 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585792) - 271171584*x + 19
1303424) + 4022389903*134689869150937352^(1/4)*sqrt(341)*(2906601*x^7 - 446
04657*x^6 + 235604928*x^5 - 537156764*x^4 + 693706464*x^3 - 436717728*x^2 -
```

sqrt(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 - 379661892*x^4 + 4905
00864*x^3 - 309827808*x^2 - 348696576*x + 246965760) - 493931520*x + 348696
576))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) + 30296387137
48420756426308089806504*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x
^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8
 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144
*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(259509026/713)*(sq
rt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)*(5980372*x^7 - 8582986
*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 + 12517632*x^2 - sqrt(2)*
(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*x^4 + 9050328*x^3 + 864
0000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134689869150937352^(1/4)*s
qrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 - 29282112*x^4 + 37610208
*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*x^6 + 15927060*x^5 - 20
630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*x) - 22726656*x))*sqrt(
2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) + 8186887989068712800954
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
 + 3276288*x) + 372131272230396036407*sqrt(31)*(254591*x^8 - 4815126*x^7 +
32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
- 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
+ 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(134689869150937352^(1/4)*sqrt(1
29754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 9
73*x - 419)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39
11902897144396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 177813768052
01800)/x^2) + 34427712656232054050298955565983*sqrt(31)*(2828123*x^8 - 9696
916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37
981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 155
69*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2
585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249
088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 189113268*13468986915093
7352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12
*x + 4)*sqrt(366990269*sqrt(2) + 519018052)*arctan(1/1067259092343193675559
267622545473*(16089559612*sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt
(341)*(38305160*x^7 - 147261352*x^6 + 309398878*x^5 - 495410374*x^4 + 24821
2864*x^3 - 117285552*x^2 - sqrt(2)*(26988622*x^7 - 104036813*x^6 + 21844820
0*x^5 - 350579241*x^4 + 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585
792) - 271171584*x + 191303424) + 4022389903*134689869150937352^(1/4)*sqrt(
341)*(2906601*x^7 - 44604657*x^6 + 235604928*x^5 - 537156764*x^4 + 69370646
4*x^3 - 436717728*x^2 - sqrt(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5
 - 379661892*x^4 + 490500864*x^3 - 309827808*x^2 - 348696576*x + 246965760)
 - 493931520*x + 348696576))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 5
19018052) - 3029638713748420756426308089806504*sqrt(31)*sqrt(2)*(28180*x^8
- 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^

```
4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*
sqrt(259509026/713)*(sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)
*(5980372*x^7 - 8582986*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 +
12517632*x^2 - sqrt(2)*(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*
x^4 + 9050328*x^3 + 8640000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134
689869150937352^(1/4)*sqrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 -
29282112*x^4 + 37610208*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*
x^6 + 15927060*x^5 - 20630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*
x) - 22726656*x))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) -
 8186887989068712800954*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888
*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550
*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1
209600*x^2 - 1036800*x) + 3276288*x) - 37213127...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}}{{\left(5\,x^2+3\,x+2\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3, x)
```

3.72 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 \, dx$

Optimal. Leaf size=254

$$-\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648}-\frac{401135647(1-4x)(3-x}{335544320}$$

```
[Out] -9226119881/2147483648*(1-4*x)*(2*x^2-x+3)^(3/2)-401135647/335544320*(1-4*x
)*(2*x^2-x+3)^(5/2)+25250178739/5725224960*(2*x^2-x+3)^(7/2)+112244125/1226
83392*x*(2*x^2-x+3)^(7/2)+122595067/19169280*x^2*(2*x^2-x+3)^(7/2)+23460839
/532480*x^3*(2*x^2-x+3)^(7/2)+3684995/39936*x^4*(2*x^2-x+3)^(7/2)+1046225/9
984*x^5*(2*x^2-x+3)^(7/2)+13875/208*x^6*(2*x^2-x+3)^(7/2)+625/28*x^7*(2*x^2
-x+3)^(7/2)-14641852251147/137438953472*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1
/2)-636602271789/34359738368*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{122595067(2x^2-x+3)^{7/2}x^2}{19169280}+\frac{112244125(2x^2-x+3)^{7/2}x}{122683392}+\frac{25250178739(2x^2-x+3)^{7/2}}{5725224960}-\frac{401135647(1-4x)(2x^2-x+3)^{5/2}}{335544320}-\frac{9226119881(1-4x)(2x^2-x+3)^{3/2}}{2147483648}-\frac{636602271789(1-4x)\sqrt{2x^2-x+3}}{34359738368}+\frac{625}{28}(2x^2-x+3)^{7/2}x^7+\frac{13875}{208}(2x^2-x+3)^{7/2}x^6+\frac{1046225(2x^2-x+3)^{7/2}x^5}{9984}+\frac{3684995(2x^2-x+3)^{7/2}x^4}{39936}+\frac{23460839(2x^2-x+3)^{7/2}x^3}{532480}-\frac{14641852251147\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{68719476736\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-636602271789*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/34359738368 - (9226119881*(1
- 4*x)*(3 - x + 2*x^2)^(3/2))/2147483648 - (401135647*(1 - 4*x)*(3 - x + 2*
x^2)^(5/2))/335544320 + (25250178739*(3 - x + 2*x^2)^(7/2))/5725224960 + (1
12244125*x*(3 - x + 2*x^2)^(7/2))/122683392 + (122595067*x^2*(3 - x + 2*x^2
)^(7/2))/19169280 + (23460839*x^3*(3 - x + 2*x^2)^(7/2))/532480 + (3684995*
x^4*(3 - x + 2*x^2)^(7/2))/39936 + (1046225*x^5*(3 - x + 2*x^2)^(7/2))/9984
 + (13875*x^6*(3 - x + 2*x^2)^(7/2))/208 + (625*x^7*(3 - x + 2*x^2)^(7/2))/
28 - (14641852251147*ArcSinh[(1 - 4*x)/Sqrt[23]])/(68719476736*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx = \frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2} + \frac{1}{28}\int \left(3-x+2x^2\right)^{5/2}\Big(448+2688x$$

$$= \frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2} + \frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2} + \frac{1}{728}\int (3$$

$$= \frac{1046225x^5(3-x+2x^2)^{7/2}}{9984} + \frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2} + \frac{625}{28}x^7$$

$$= \frac{3684995x^4(3-x+2x^2)^{7/2}}{39936} + \frac{1046225x^5(3-x+2x^2)^{7/2}}{9984} + \frac{1387}{208}$$

$$= \frac{23460839x^3(3-x+2x^2)^{7/2}}{532480} + \frac{3684995x^4(3-x+2x^2)^{7/2}}{39936} + \frac{104}{}$$

$$= \frac{122595067x^2(3-x+2x^2)^{7/2}}{19169280} + \frac{23460839x^3(3-x+2x^2)^{7/2}}{532480} +$$

$$= \frac{112244125x(3-x+2x^2)^{7/2}}{122683392} + \frac{122595067x^2(3-x+2x^2)^{7/2}}{19169280} +$$

$$= \frac{25250178739(3-x+2x^2)^{7/2}}{5725224960} + \frac{112244125x(3-x+2x^2)^{7/2}}{122683392} +$$

$$= -\frac{401135647(1-4x)\left(3-x+2x^2\right)^{5/2}}{335544320} + \frac{25250178739(3-x+2}{5725224960}$$

$$= -\frac{9226119881(1-4x)\left(3-x+2x^2\right)^{3/2}}{2147483648} - \frac{401135647(1-4x)\,(3}{33554432}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)\,(}{2147483}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)\,(}{2147483}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)\,(}{2147483}$$

Mathematica [A]
time = 1.28, size = 115, normalized size = 0.45

$$\frac{4\sqrt{3-x+2x^2}\left(10820567498568669+12071614275862524x+50064174038215008x^2+142490931553577856x^3+257786732552566784x^4+405468382284161024x^5+485091164642279424x^6+530502956133122048x^7+439064558846345216x^8+363646430503501824x^9+204932411660697600x^{10}+137233466130432000x^{11}+37398427729920000x^{12}+25125558681600000x^{13}\right)-59958384968446965\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{562812514467840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 500641740
38215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 4054683822
84161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 4390645588
46345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 137233466
130432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13) - 5995838
4968446965*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/562812514467840
```

Maple [A]
time = 0.14, size = 204, normalized size = 0.80

method	result
risch	$\frac{(25125558681600000x^{13}+37398427729920000x^{12}+137233466130432000x^{11}+204932411660697600x^{10}+363646430503501824x^{9}+439}{}$
trager	$\left(\frac{1250}{7}x^{13}+\frac{48375}{182}x^{12}+\frac{1217225}{1248}x^{11}+\frac{50895515}{34944}x^{10}+\frac{172023939}{66560}x^{9}+\frac{52340574127}{16773120}x^{8}+\frac{2023708176167}{536739840}x^{7}+\frac{24673}{715}\right.$
default	$\frac{25250178739(2x^2-x+3)^{\frac{7}{2}}}{5725224960}+\frac{401135647(4x-1)(2x^2-x+3)^{\frac{5}{2}}}{335544320}+\frac{636602271789(4x-1)\sqrt{2x^2-x+3}}{34359738368}+\frac{14641852251147\sqrt{}}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 25250178739/5725224960*(2*x^2-x+3)^(7/2)+401135647/335544320*(4*x-1)*(2*x^2
-x+3)^(5/2)+636602271789/34359738368*(4*x-1)*(2*x^2-x+3)^(1/2)+146418522511
47/137438953472*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+9226119881/214748364
8*(4*x-1)*(2*x^2-x+3)^(3/2)+112244125/122683392*x*(2*x^2-x+3)^(7/2)+1225950
67/19169280*x^2*(2*x^2-x+3)^(7/2)+23460839/532480*x^3*(2*x^2-x+3)^(7/2)+368
4995/39936*x^4*(2*x^2-x+3)^(7/2)+1046225/9984*x^5*(2*x^2-x+3)^(7/2)+13875/2
08*x^6*(2*x^2-x+3)^(7/2)+625/28*x^7*(2*x^2-x+3)^(7/2)
```

Maxima [A]
time = 0.51, size = 235, normalized size = 0.93

$$\frac{625}{28}\left(2x^2-x+3\right)^{\frac{7}{2}}x^7+\frac{13875}{208}\left(2x^2-x+3\right)^{\frac{7}{2}}x^6+\frac{1046225}{9984}\left(2x^2-x+3\right)^{\frac{7}{2}}x^5+\frac{3684995}{39936}\left(2x^2-x+3\right)^{\frac{7}{2}}x^4+\frac{23460839}{532480}\left(2x^2-x+3\right)^{\frac{7}{2}}x^3+\frac{122595067}{19169280}\left(2x^2-x+3\right)^{\frac{7}{2}}x^2+\frac{112244125}{122683392}\left(2x^2-x+3\right)^{\frac{7}{2}}x+\frac{25250178739}{5725224960}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{401135647}{83886080}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{401135647}{335544320}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{9226119881}{536870912}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{9226119881}{2147483648}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{636602271789}{8589934592}\sqrt{2x^2-x+3}\,x+\frac{14641852251147}{137438953472}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)-\frac{636602271789}{34359738368}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/28*(2*x^2 - x + 3)^(7/2)*x^7 + 13875/208*(2*x^2 - x + 3)^(7/2)*x^6 + 10
46225/9984*(2*x^2 - x + 3)^(7/2)*x^5 + 3684995/39936*(2*x^2 - x + 3)^(7/2)*
x^4 + 23460839/532480*(2*x^2 - x + 3)^(7/2)*x^3 + 122595067/19169280*(2*x^2
 - x + 3)^(7/2)*x^2 + 112244125/122683392*(2*x^2 - x + 3)^(7/2)*x + 2525017
8739/5725224960*(2*x^2 - x + 3)^(7/2) + 401135647/83886080*(2*x^2 - x + 3)^
```

```
(5/2)*x - 401135647/335544320*(2*x^2 - x + 3)^(5/2) + 9226119881/536870912*
(2*x^2 - x + 3)^(3/2)*x - 9226119881/2147483648*(2*x^2 - x + 3)^(3/2) + 636
602271789/8589934592*sqrt(2*x^2 - x + 3)*x + 14641852251147/137438953472*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 636602271789/34359738368*sqrt(2*x^
2 - x + 3)
```

Fricas [A]
time = 2.91, size = 118, normalized size = 0.46

$$\frac{1}{140703128616960}\left(25125558681600000\,x^{13}+37398427729920000\,x^{12}+137233466130432000\,x^{11}+204932411660697600\,x^{10}+363646430503501824\,x^{9}+439064558846345216\,x^{8}+530502956133122048\,x^{7}+485091164642279424\,x^{6}+405468382284161024\,x^{5}+257786732552566784\,x^{4}+142490931553577856\,x^{3}+50064174038215008\,x^{2}+12071614275862524\,x+10820567498568669\right)\sqrt{2\,x^{2}-x+3}+\frac{14641852251147}{274877906944}\,\sqrt{2}\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/140703128616960*(25125558681600000*x^13 + 37398427729920000*x^12 + 137233
466130432000*x^11 + 204932411660697600*x^10 + 363646430503501824*x^9 + 4390
64558846345216*x^8 + 530502956133122048*x^7 + 485091164642279424*x^6 + 4054
68382284161024*x^5 + 257786732552566784*x^4 + 142490931553577856*x^3 + 5006
4174038215008*x^2 + 12071614275862524*x + 10820567498568669)*sqrt(2*x^2 - x
 + 3) + 14641852251147/274877906944*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(2x^{2}-x+3\right)^{\frac{5}{2}}\left(5x^{2}+3x+2\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**4, x)
```

Giac [A]
time = 4.18, size = 113, normalized size = 0.44

$$\frac{1}{140703128616960}\left(4\left(8\left(4\left(16\left(4\left(8\left(4\left(32\left(12\left(200\left(20\left(240\left(260\,x+387\right)x+340823\right)x+10179103\right)x+3612502719\right)x+52340574127\right)x+2023708176167\right)x+7401903757359\right)x+49495652134297\right)x+125872428004183\right)x+1113210402762327\right)x+1564505438694219\right)x+3017903568965631\right)x+10820567498568669\right)\sqrt{2\,x^{2}-x+3}-\frac{14641852251147}{137438953472}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^{2}-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/140703128616960*(4*(8*(4*(16*(4*(8*(4*(32*(12*(200*(20*(240*(260*x + 387)
*x + 340823)*x + 10179103)*x + 3612502719)*x + 52340574127)*x + 20237081761
67)*x + 7401903757359)*x + 49495652134297)*x + 125872428004183)*x + 1113210
402762327)*x + 1564505438694219)*x + 3017903568965631)*x + 1082056749856866
9)*sqrt(2*x^2 - x + 3) - 14641852251147/137438953472*sqrt(2)*log(-2*sqrt(2)
*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4,x)
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4, x)
```

3.73 $\int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^3 \, dx$

Optimal. Leaf size=212

$$-\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824}-\frac{6660225(1-4x)\left(3-x+2x^2\right)^{3/2}}{67108864}-\frac{57915(1-4x)\left(3-x+2x^2\right)^{5/2}}{2097152}-$$

```
[Out] -6660225/67108864*(1-4*x)*(2*x^2-x+3)^(3/2)-57915/2097152*(1-4*x)*(2*x^2-x+
3)^(5/2)-1696165/2752512*(2*x^2-x+3)^(7/2)+509257/294912*x*(2*x^2-x+3)^(7/2
)+80483/9216*x^2*(2*x^2-x+3)^(7/2)+3823/256*x^3*(2*x^2-x+3)^(7/2)+1175/96*x
^4*(2*x^2-x+3)^(7/2)+125/24*x^5*(2*x^2-x+3)^(7/2)-10569777075/4294967296*ar
csinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-459555525/1073741824*(1-4*x)*(2*x^2-x+
3)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{80483\left(2x^2-x+3\right)^{7/2}x^2}{9216}+\frac{509257\left(2x^2-x+3\right)^{7/2}x}{294912}-\frac{1696165\left(2x^2-x+3\right)^{7/2}}{2752512}-\frac{57915(1-4x)\left(2x^2-x+3\right)^{5/2}}{2097152}-\frac{6660225(1-4x)\left(2x^2-x+3\right)^{3/2}}{67108864}-\frac{459555525(1-4x)\sqrt{2x^2-x+3}}{1073741824}+\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5+\frac{1175}{96}\left(2x^2-x+3\right)^{7/2}x^4+\frac{3823}{256}\left(2x^2-x+3\right)^{7/2}x^3-\frac{10569777075\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2147483648\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-459555525*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1073741824 - (6660225*(1 - 4*x)*
(3 - x + 2*x^2)^(3/2))/67108864 - (57915*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/2
097152 - (1696165*(3 - x + 2*x^2)^(7/2))/2752512 + (509257*x*(3 - x + 2*x^2
)^(7/2))/294912 + (80483*x^2*(3 - x + 2*x^2)^(7/2))/9216 + (3823*x^3*(3 - x
 + 2*x^2)^(7/2))/256 + (1175*x^4*(3 - x + 2*x^2)^(7/2))/96 + (125*x^5*(3 -
x + 2*x^2)^(7/2))/24 - (10569777075*ArcSinh[(1 - 4*x)/Sqrt[23]])/(214748364
8*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3\,dx = \frac{125}{24}x^5\left(3-x+2x^2\right)^{7/2}+\frac{1}{24}\int \left(3-x+2x^2\right)^{5/2}\Big(192+864x+$$

$$= \frac{1175}{96}x^4\left(3-x+2x^2\right)^{7/2}+\frac{125}{24}x^5\left(3-x+2x^2\right)^{7/2}+\frac{1}{528}\int (3-$$

$$= \frac{3823}{256}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1175}{96}x^4\left(3-x+2x^2\right)^{7/2}+\frac{125}{24}x^5(3-$$

$$= \frac{80483x^2\left(3-x+2x^2\right)^{7/2}}{9216}+\frac{3823}{256}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1175}{96}x^4(3$$

$$= \frac{509257x\left(3-x+2x^2\right)^{7/2}}{294912}+\frac{80483x^2\left(3-x+2x^2\right)^{7/2}}{9216}+\frac{3823}{256}x^3($$

$$= -\frac{1696165\left(3-x+2x^2\right)^{7/2}}{2752512}+\frac{509257x\left(3-x+2x^2\right)^{7/2}}{294912}+$$

$$= -\frac{57915(1-4x)\left(3-x+2x^2\right)^{5/2}}{2097152}-\frac{1696165\left(3-x+2x^2\right)^{7/2}}{2752512}+$$

$$= -\frac{6660225(1-4x)\left(3-x+2x^2\right)^{3/2}}{67108864}-\frac{57915(1-4x)\left(3-x+2x^2\right.}{2097152}$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824}-\frac{6660225(1-4x)\left(3-x+\right.}{67108864}$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824}-\frac{6660225(1-4x)\left(3-x+\right.}{67108864}$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824}-\frac{6660225(1-4x)\left(3-x+\right.}{67108864}$$

Mathematica [A]
time = 1.02, size = 105, normalized size = 0.50

$$\frac{4\sqrt{3-x+2x^2}\left(-1191399152715+4560943728924x+10060731582048x^2+20384824684416x^3+26186527209472x^4+34378613923840x^5+28347538538496x^6+27835561148416x^7+14341894045696x^8+12943588589568x^9+2395786444800x^{10}+2818572288000x^{11}\right)-665895955725\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{270582939648}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1191399152715 + 4560943728924*x + 10060731582048*x
^2 + 20384824684416*x^3 + 26186527209472*x^4 + 34378613923840*x^5 + 2834753
8538496*x^6 + 27835561148416*x^7 + 14341894045696*x^8 + 12943588589568*x^9
```

```
+ 2395786444800*x^10 + 2818572288000*x^11) - 665895955725*Sqrt[2]*Log[1 - 4
*x + 2*Sqrt[6 - 2*x + 4*x^2]])/270582939648
```

Maple [A]
time = 0.12, size = 170, normalized size = 0.80

method	result
risch	$\frac{(2818572288000x^{11}+2395786444800x^{10}+12943588589568x^{9}+14341894045696x^{8}+27835561148416x^{7}+28347538538496x^{6}+3437861}{6764573}$
trager	$\left(\frac{125}{3}x^{11}+\frac{425}{12}x^{10}+\frac{6123}{32}x^{9}+\frac{244241}{1152}x^{8}+\frac{15169177}{36864}x^{7}+\frac{144183037}{344064}x^{6}+\frac{4196608145}{8257536}x^{5}+\frac{1826627177}{4718592}x^{4}+\frac{5308}{176}\right.$
default	$-\frac{1696165(2x^2-x+3)^{\frac{7}{2}}}{2752512}+\frac{57915(4x-1)(2x^2-x+3)^{\frac{5}{2}}}{2097152}+\frac{459555525(4x-1)\sqrt{2x^2-x+3}}{1073741824}+\frac{10569777075\sqrt{2}\ \operatorname{arcsinh}\left(4\right.}{4294967296}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1696165/2752512*(2*x^2-x+3)^(7/2)+57915/2097152*(4*x-1)*(2*x^2-x+3)^(5/2)+
459555525/1073741824*(4*x-1)*(2*x^2-x+3)^(1/2)+10569777075/4294967296*2^(1/
2)*arcsinh(4/23*23^(1/2)*(x-1/4))+6660225/67108864*(4*x-1)*(2*x^2-x+3)^(3/2
)+509257/294912*x*(2*x^2-x+3)^(7/2)+80483/9216*x^2*(2*x^2-x+3)^(7/2)+3823/2
56*x^3*(2*x^2-x+3)^(7/2)+1175/96*x^4*(2*x^2-x+3)^(7/2)+125/24*x^5*(2*x^2-x+
3)^(7/2)
```

Maxima [A]
time = 0.50, size = 201, normalized size = 0.95

$$\frac{125}{24}\,(2\,x^2-x+3)^{\frac{7}{2}}x^5+\frac{1175}{96}\,(2\,x^2-x+3)^{\frac{7}{2}}x^4+\frac{3823}{256}\,(2\,x^2-x+3)^{\frac{7}{2}}x^3+\frac{80483}{9216}\,(2\,x^2-x+3)^{\frac{7}{2}}x^2+\frac{509257}{294912}\,(2\,x^2-x+3)^{\frac{7}{2}}x-\frac{1696165}{2752512}\,(2\,x^2-x+3)^{\frac{7}{2}}+\frac{57915}{524288}\,(2\,x^2-x+3)^{\frac{5}{2}}x-\frac{57915}{2097152}\,(2\,x^2-x+3)^{\frac{5}{2}}+\frac{6660225}{16777216}\,(2\,x^2-x+3)^{\frac{3}{2}}x-\frac{6660225}{67108864}\,(2\,x^2-x+3)^{\frac{3}{2}}+\frac{459555525}{268435456}\,\sqrt{2\,x^2-x+3}\,x+\frac{10569777075}{4294967296}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{459555525}{1073741824}\,\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 125/24*(2*x^2 - x + 3)^(7/2)*x^5 + 1175/96*(2*x^2 - x + 3)^(7/2)*x^4 + 3823
/256*(2*x^2 - x + 3)^(7/2)*x^3 + 80483/9216*(2*x^2 - x + 3)^(7/2)*x^2 + 509
257/294912*(2*x^2 - x + 3)^(7/2)*x - 1696165/2752512*(2*x^2 - x + 3)^(7/2)
+ 57915/524288*(2*x^2 - x + 3)^(5/2)*x - 57915/2097152*(2*x^2 - x + 3)^(5/2
) + 6660225/16777216*(2*x^2 - x + 3)^(3/2)*x - 6660225/67108864*(2*x^2 - x
+ 3)^(3/2) + 459555525/268435456*sqrt(2*x^2 - x + 3)*x + 10569777075/429496
7296*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 459555525/1073741824*sqrt(2
*x^2 - x + 3)
```

Fricas [A]
time = 3.74, size = 108, normalized size = 0.51

$$\frac{1}{67645734912}\,\left(2818572288000\,x^{11}+2395786444800\,x^{10}+12943588589568\,x^{9}+14341894045696\,x^{8}+27835561148416\,x^{7}+28347538538496\,x^{6}+34378613923840\,x^{5}+26186527209472\,x^{4}+20384824684416\,x^{3}+10060731582048\,x^{2}+4560943728924\,x-1191399152715\right)\sqrt{2\,x^2-x+3}+\frac{10569777075}{8589934592}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/67645734912*(2818572288000*x^11 + 2395786444800*x^10 + 12943588589568*x^9
 + 14341894045696*x^8 + 27835561148416*x^7 + 28347538538496*x^6 + 343786139
23840*x^5 + 26186527209472*x^4 + 20384824684416*x^3 + 10060731582048*x^2 +
4560943728924*x - 1191399152715)*sqrt(2*x^2 - x + 3) + 10569777075/85899345
92*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 2
5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2 - x + 3\right)^{\frac{5}{2}} \left(5x^2 + 3x + 2\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3, x)
```

Giac [A]
time = 5.25, size = 103, normalized size = 0.49

$$\frac{1}{67645734912}\left(4\left(8\left(4\left(16\left(4\left(8\left(28\left(32\left(12\left(200\left(20\,x+17\right)x+18369\right)x+244241\right)x+15169177\right)x+432549111\right)x+4196608145\right)x+12786390239\right)x+159256442847\right)x+314397861939\right)x+1140235932231\right)x-1191399152715\right)\sqrt{2\,x^2-x+3}-\frac{10569777075}{4294967296}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/67645734912*(4*(8*(4*(16*(4*(8*(28*(32*(12*(200*(20*x + 17)*x + 18369)*x
+ 244241)*x + 15169177)*x + 432549111)*x + 4196608145)*x + 12786390239)*x +
 159256442847)*x + 314397861939)*x + 1140235932231)*x - 1191399152715)*sqrt
(2*x^2 - x + 3) - 10569777075/4294967296*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x
- sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(2\,x^2-x+3\right)}^{5/2}\,{\left(5\,x^2+3\,x+2\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3, x)
```

3.74 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 \, dx$

Optimal. Leaf size=170

$$-\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{1547(1-4x)\left(3-x+2x^2\right)^{5/2}}{98304}+\frac{23225}{}$$

```
[Out] -177905/3145728*(1-4*x)*(2*x^2-x+3)^(3/2)-1547/98304*(1-4*x)*(2*x^2-x+3)^(5
/2)+23225/43008*(2*x^2-x+3)^(7/2)+8467/4608*x*(2*x^2-x+3)^(7/2)+305/144*x^2
*(2*x^2-x+3)^(7/2)+5/4*x^3*(2*x^2-x+3)^(7/2)-94111745/67108864*arcsinh(1/23
*(1-4*x)*23^(1/2))*2^(1/2)-4091815/16777216*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{305}{144}x^2\left(2x^2-x+3\right)^{7/2}+\frac{8467x\left(2x^2-x+3\right)^{7/2}}{4608}+\frac{23225\left(2x^2-x+3\right)^{7/2}}{43008}-\frac{1547(1-4x)\left(2x^2-x+3\right)^{5/2}}{98304}-\frac{177905(1-4x)\left(2x^2-x+3\right)^{3/2}}{3145728}-\frac{4091815(1-4x)\sqrt{2x^2-x+3}}{16777216}+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}-\frac{94111745\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{33554432\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (-4091815*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16777216 - (177905*(1 - 4*x)*(3 -
x + 2*x^2)^(3/2))/3145728 - (1547*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/98304 +
(23225*(3 - x + 2*x^2)^(7/2))/43008 + (8467*x*(3 - x + 2*x^2)^(7/2))/4608 +
 (305*x^2*(3 - x + 2*x^2)^(7/2))/144 + (5*x^3*(3 - x + 2*x^2)^(7/2))/4 - (9
4111745*ArcSinh[(1 - 4*x)/Sqrt[23]])/(33554432*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2\,dx = \frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1}{20}\int \left(3-x+2x^2\right)^{5/2}\left(80+240x+355\right.$$

$$=\frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+\frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1}{360}\int \left(3-x+\right.$$

$$=\frac{8467x\left(3-x+2x^2\right)^{7/2}}{4608}+\frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+\frac{5}{4}x^3\left(3-x+\right.$$

$$=\frac{23225\left(3-x+2x^2\right)^{7/2}}{43008}+\frac{8467x\left(3-x+2x^2\right)^{7/2}}{4608}+\frac{305}{144}x^2\left(3-x\right.$$

$$=-\frac{1547(1-4x)\left(3-x+2x^2\right)^{5/2}}{98304}+\frac{23225\left(3-x+2x^2\right)^{7/2}}{43008}+\frac{846}{}$$

$$=-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{1547(1-4x)\left(3-x+2x^2\right)}{98304}$$

$$=-\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x\right.}{3145728}$$

$$=-\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x\right.}{3145728}$$

$$=-\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x\right.}{3145728}$$

Mathematica [A]
time = 0.78, size = 95, normalized size = 0.56

$$\frac{4\sqrt{3-x+2x^2}\,(14824182519+39533249652x+42992644128x^2+77872272000x^3+57147467776x^4+75389820928x^5+26401898496x^6+44163137536x^7+2055208960x^8+10569646080x^9)-5929039935\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{4227858432}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(14824182519 + 39533249652*x + 42992644128*x^2 + 778
72272000*x^3 + 57147467776*x^4 + 75389820928*x^5 + 26401898496*x^6 + 441631
37536*x^7 + 2055208960*x^8 + 10569646080*x^9) - 5929039935*Sqrt[2]*Log[1 -
4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/4227858432
```

Maple [A]
time = 0.12, size = 136, normalized size = 0.80

method	result
risch	$\frac{(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+77872272000x^3+42992644128}{1056964608}$
trager	$\left(10x^9+\frac{35}{18}x^8+\frac{24067}{576}x^7+\frac{134287}{5376}x^6+\frac{9202859}{129024}x^5+\frac{3986291}{73728}x^4+\frac{202792375}{2752512}x^3+\frac{63977149}{1572864}x^2+\frac{3294437471}{88080384}x\right.$
default	$\frac{23225(2x^2-x+3)^{\frac{7}{2}}}{43008}+\frac{1547(4x-1)(2x^2-x+3)^{\frac{5}{2}}}{98304}+\frac{4091815(4x-1)\sqrt{2x^2-x+3}}{16777216}+\frac{94111745\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\ (x-}{23}\right.}{67108864}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 23225/43008*(2*x^2-x+3)^(7/2)+1547/98304*(4*x-1)*(2*x^2-x+3)^(5/2)+4091815/
16777216*(4*x-1)*(2*x^2-x+3)^(1/2)+94111745/67108864*2^(1/2)*arcsinh(4/23*2
3^(1/2)*(x-1/4))+177905/3145728*(4*x-1)*(2*x^2-x+3)^(3/2)+8467/4608*x*(2*x^
2-x+3)^(7/2)+305/144*x^2*(2*x^2-x+3)^(7/2)+5/4*x^3*(2*x^2-x+3)^(7/2)
```

Maxima [A]
time = 0.55, size = 167, normalized size = 0.98

$$\frac{5}{4}\left(2x^2-x+3\right)^{\frac{7}{2}}x^3+\frac{305}{144}\left(2x^2-x+3\right)^{\frac{7}{2}}x^2+\frac{8467}{4608}\left(2x^2-x+3\right)^{\frac{7}{2}}x+\frac{23225}{43008}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{1547}{24576}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{1547}{98304}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{177905}{786432}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{177905}{3145728}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{4091815}{4194304}\sqrt{2x^2-x+3}\,x+\frac{94111745}{67108864}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{4091815}{16777216}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*(2*x^2 - x + 3)^(7/2)*x^3 + 305/144*(2*x^2 - x + 3)^(7/2)*x^2 + 8467/46
08*(2*x^2 - x + 3)^(7/2)*x + 23225/43008*(2*x^2 - x + 3)^(7/2) + 1547/24576
*(2*x^2 - x + 3)^(5/2)*x - 1547/98304*(2*x^2 - x + 3)^(5/2) + 177905/786432
```

```
*(2*x^2 - x + 3)^(3/2)*x - 177905/3145728*(2*x^2 - x + 3)^(3/2) + 4091815/4
194304*sqrt(2*x^2 - x + 3)*x + 94111745/67108864*sqrt(2)*arcsinh(1/23*sqrt(
23)*(4*x - 1)) - 4091815/16777216*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 2.64, size = 98, normalized size = 0.58

$$\frac{1}{1056964608}\left(10569646080\,x^9+2055208960\,x^8+44163137536\,x^7+26401898496\,x^6+75389820928\,x^5+57147467776\,x^4+77872272000\,x^3+42992644128\,x^2+39533249652\,x+14824182519\right)\sqrt{2\,x^2-x+3}+\frac{94111745}{134217728}\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/1056964608*(10569646080*x^9 + 2055208960*x^8 + 44163137536*x^7 + 26401898
496*x^6 + 75389820928*x^5 + 57147467776*x^4 + 77872272000*x^3 + 42992644128
*x^2 + 39533249652*x + 14824182519)*sqrt(2*x^2 - x + 3) + 94111745/13421772
8*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2, x)
```

Giac [A]
time = 4.18, size = 93, normalized size = 0.55

$$\frac{1}{1056964608}\left(4\left(8\left(4\left(16\left(4\left(8\left(28\left(160\left(36\,x+7\right)x+24067\right)x+402861\right)x+9202859\right)x+27904037\right)x+608377125\right)x+1343520129\right)x+9883312413\right)x+14824182519\right)\sqrt{2\,x^2-x+3}-\frac{94111745}{67108864}\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/1056964608*(4*(8*(4*(16*(4*(8*(28*(160*(36*x + 7)*x + 24067)*x + 402861)*
x + 9202859)*x + 27904037)*x + 608377125)*x + 1343520129)*x + 9883312413)*x
 + 14824182519)*sqrt(2*x^2 - x + 3) - 94111745/67108864*sqrt(2)*log(-2*sqrt
(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{5/2}\,{\left(5\,x^2+3\,x+2\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2, x)
```

3.75 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right) \, dx$

Optimal. Leaf size=128

$$-\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}+\frac{141}{448}(3-$$

```
[Out] -31855/98304*(1-4*x)*(2*x^2-x+3)^(3/2)-277/3072*(1-4*x)*(2*x^2-x+3)^(5/2)+1
41/448*(2*x^2-x+3)^(7/2)+5/16*x*(2*x^2-x+3)^(7/2)-16851295/2097152*arcsinh(
1/23*(1-4*x)*23^(1/2))*2^(1/2)-732665/524288*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}+\frac{141}{448}\left(2x^2-x+3\right)^{7/2}-\frac{277(1-4x)\left(2x^2-x+3\right)^{5/2}}{3072}-\frac{31855(1-4x)\left(2x^2-x+3\right)^{3/2}}{98304}-\frac{732665(1-4x)\sqrt{2x^2-x+3}}{524288}-\frac{16851295\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1048576\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-732665*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/524288 - (31855*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/98304 - (277*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/3072 + (141*(3
- x + 2*x^2)^(7/2))/448 + (5*x*(3 - x + 2*x^2)^(7/2))/16 - (16851295*ArcSin
h[(1 - 4*x)/Sqrt[23]])/(1048576*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)\,dx = \frac{5}{16}x\left(3-x+2x^2\right)^{7/2}+\frac{1}{16}\int\left(17+\frac{141x}{2}\right)\left(3-x+2x^2\right)^{5/2}\,dx$$

$$= \frac{141}{448}\left(3-x+2x^2\right)^{7/2}+\frac{5}{16}x\left(3-x+2x^2\right)^{7/2}+\frac{277}{128}\int\left(3-x+2$$

$$= -\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}+\frac{141}{448}\left(3-x+2x^2\right)^{7/2}+\frac{5}{16}x\left(3-\right.$$

$$= -\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/}}{98304}$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/}}{98304}$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/}}{98304}$$

Mathematica [A]
time = 0.56, size = 85, normalized size = 0.66

$$\frac{4\sqrt{3-x+2x^2}\left(58536675+148957444x+67272352x^2+172684416x^3-1619968x^4+118808576x^5-13565952x^6+27525120x^7\right)-353877195\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{44040192}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(58536675 + 148957444*x + 67272352*x^2 + 172684416*x
^3 - 1619968*x^4 + 118808576*x^5 - 13565952*x^6 + 27525120*x^7) - 353877195
*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/44040192
```

Maple [A]
time = 0.10, size = 102, normalized size = 0.80

method	result
risch	$\frac{(27525120x^7-13565952x^6+118808576x^5-1619968x^4+172684416x^3+67272352x^2+148957444x+58536675)\sqrt{2x^2-x+3}}{11010048} +$
trager	$\left(\frac{5}{2}x^7 - \frac{69}{56}x^6 + \frac{14503}{1344}x^5 - \frac{113}{768}x^4 + \frac{449699}{28672}x^3 + \frac{300323}{49152}x^2 + \frac{37239361}{2752512}x + \frac{19512225}{3670016}\right)\sqrt{2x^2-x+3} + \frac{16851}{}$
default	$\frac{5x(2x^2-x+3)^{\frac{7}{2}}}{16} + \frac{141(2x^2-x+3)^{\frac{7}{2}}}{448} + \frac{277(4x-1)(2x^2-x+3)^{\frac{5}{2}}}{3072} + \frac{31855(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{98304} + \frac{732665(4x-1)\sqrt{2x^2-x}}{524288}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*x*(2*x^2-x+3)^(7/2)+141/448*(2*x^2-x+3)^(7/2)+277/3072*(4*x-1)*(2*x^2-
x+3)^(5/2)+31855/98304*(4*x-1)*(2*x^2-x+3)^(3/2)+732665/524288*(4*x-1)*(2*x
^2-x+3)^(1/2)+16851295/2097152*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.50, size = 133, normalized size = 1.04

$$\frac{5}{16}\left(2x^2-x+3\right)^{\frac{7}{2}}x + \frac{141}{448}\left(2x^2-x+3\right)^{\frac{7}{2}} + \frac{277}{768}\left(2x^2-x+3\right)^{\frac{5}{2}}x - \frac{277}{3072}\left(2x^2-x+3\right)^{\frac{5}{2}} + \frac{31855}{24576}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{31855}{98304}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{732665}{131072}\sqrt{2x^2-x+3}\,x + \frac{16851295}{2097152}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{732665}{524288}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(7/2)*x + 141/448*(2*x^2 - x + 3)^(7/2) + 277/768*(2*x
^2 - x + 3)^(5/2)*x - 277/3072*(2*x^2 - x + 3)^(5/2) + 31855/24576*(2*x^2 -
 x + 3)^(3/2)*x - 31855/98304*(2*x^2 - x + 3)^(3/2) + 732665/131072*sqrt(2*
x^2 - x + 3)*x + 16851295/2097152*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
- 732665/524288*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 3.21, size = 88, normalized size = 0.69

$$\frac{1}{11010048}\left(27525120x^7 - 13565952x^6 + 118808576x^5 - 1619968x^4 + 172684416x^3 + 67272352x^2 + 148957444x + 58536675\right)\sqrt{2x^2-x+3} + \frac{16851295}{4194304}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/11010048*(27525120*x^7 - 13565952*x^6 + 118808576*x^5 - 1619968*x^4 + 172
684416*x^3 + 67272352*x^2 + 148957444*x + 58536675)*sqrt(2*x^2 - x + 3) + 1
6851295/4194304*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2 - x + 3\right)^{\frac{5}{2}} \cdot \left(5x^2 + 3x + 2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2), x)
```

Giac [**A**]
time = 3.20, size = 83, normalized size = 0.65

$$\frac{1}{11010048}\left(4\left(8\left(4\left(16\left(4\left(24\left(140x - 69\right)x + 14503\right)x - 791\right)x + 1349097\right)x + 2102261\right)x + 37239361\right)x + 58536675\right)\sqrt{2x^2 - x + 3} - \frac{16851295}{2097152}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 1/11010048*(4*(8*(4*(16*(4*(24*(140*x - 69)*x + 14503)*x - 791)*x + 1349097
)*x + 2102261)*x + 37239361)*x + 58536675)*sqrt(2*x^2 - x + 3) - 16851295/2
097152*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2 - x + 3\right)}^{5/2}\,\left(5\,x^2 + 3\,x + 2\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2), x)
```

3.76 $\int \frac{\left(3-x+2x^2\right)^{5/2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=222

$$-\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000}-\frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2}-\frac{7216203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}}-\frac{121\sqrt{\frac{11}{31}}}{}$$

```
[Out] -1/600*(103-60*x)*(2*x^2-x+3)^(3/2)-7216203/1600000*arcsinh(1/23*(1-4*x)*23
^(1/2))*2^(1/2)-1/80000*(226249-99620*x)*(2*x^2-x+3)^(1/2)-121/96875*arctan
(1/62*(196-443*2^(1/2)-x*(690+247*2^(1/2)))*682^(1/2)/(-15457+25000*2^(1/2)
)^(1/2)/(2*x^2-x+3)^(1/2))*(-5270837+8525000*2^(1/2))^(1/2)+121/96875*arcta
nh(1/62*(196-x*(690-247*2^(1/2))+443*2^(1/2))*682^(1/2)/(15457+25000*2^(1/2
))^(1/2)/(2*x^2-x+3)^(1/2))*(5270837+8525000*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {991, 1080, 1090, 633, 221, 1049, 1043, 212, 210}

$$-\frac{121\sqrt{\frac{11}{31}\left(25000\sqrt{2}-15457\right)}\,\mathrm{ArcTan}\left(\frac{\sqrt{\frac{11}{62\left(25000\sqrt{2}-15457\right)}}\left(-\left(\left(690+247\sqrt{2}\right)x\right)-443\sqrt{2}+196\right)}{\sqrt{2x^2-x+3}}\right)}{3125}-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}-\frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000}+\frac{121\sqrt{\frac{11}{31}\left(15457+25000\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(15457+25000\sqrt{2}\right)}}\left(-\left(\left(690-247\sqrt{2}\right)x\right)+443\sqrt{2}+196\right)}{\sqrt{2x^2-x+3}}\right)}{3125}-\frac{7216203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] -1/80000*((226249 - 99620*x)*Sqrt[3 - x + 2*x^2]) - ((103 - 60*x)*(3 - x +
2*x^2)^(3/2))/600 - (7216203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(800000*Sqrt[2])
- (121*Sqrt[(11*(-15457 + 25000*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(-15457 +
 25000*Sqrt[2]))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x))/Sqrt[3 - x +
 2*x^2]])/3125 + (121*Sqrt[(11*(15457 + 25000*Sqrt[2]))/31]*ArcTanh[(Sqrt[1
1/(62*(15457 + 25000*Sqrt[2]))]*(196 + 443*Sqrt[2] - (690 - 247*Sqrt[2])*x)
)/Sqrt[3 - x + 2*x^2]])/3125
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 991

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
```

```
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = -\frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{1}{300}\int \frac{\left(-\frac{4731}{2}+\frac{6135x}{4}-\frac{14943x^2}{4}\right)\sqrt{3-x+}}{2+3x+5x^2}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{\frac{32052}{8}}{\sqrt{3}}}{}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} + \frac{\int \sqrt{3}}{}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{\int \frac{-702}{}}{}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{721620}{}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{721620}{}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.55, size = 238, normalized size = 1.07

$$\frac{20\sqrt{3-x+2x^2}\left(-802347+412060x-106400x^2+48000x^3\right)-21648609\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)-2044416\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{368\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+22\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1-119\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]}{4800000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (20*Sqrt[3 - x + 2*x^2]*(-802347 + 412060*x - 106400*x^2 + 48000*x^3) - 216
48609*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]] - 2044416*RootSum[-56
- 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (368*Log[-(Sqrt[2]*
x) + Sqrt[3 - x + 2*x^2] - #1] + 22*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
 2*x^2] - #1]*#1 - 119*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(
-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/4800000
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4859 vs. $2(165) = 330$.

time = 0.81, size = 4860, normalized size = 21.89

method	result
trager	Expression too large to display
risch	$\frac{(48000x^3-106400x^2+412060x-802347)\sqrt{2x^2-x+3}}{240000}+\frac{7216203\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{1600000}+\frac{121\sqrt{\frac{8(\sqrt{2}-1+\ldots}{(\sqrt{2}+1-x\ldots}}}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^3*(2*x^2-x+3)^(1/2)-133/300*x^2*(2*x^2-x+3)^(1/2)+20603/12000*x*(2*x^
2-x+3)^(1/2)-267449/80000*(2*x^2-x+3)^(1/2)+7216203/1600000*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+4/33034375*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(
1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(75195*2^(1
/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(
1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41)
)^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+8
2*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1
/2)))*(-775687+549362*2^(1/2))^(1/2)+106294*(-8866+6820*2^(1/2))^(1/2)*arct
an(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/
2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)
^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+3201
6)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+2
3)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2
)+108099046*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2
^(1/2)-158290154*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*
(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/
2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1
```

+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+6/6606875*(8*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3
*2^(1/2))^(1/2)*2^(1/2)*(10915*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/
11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(2
3*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2
^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+149
18*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1
/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))
^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82
(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/
2)))*(-775687+549362*2^(1/2))^(1/2)-5052938*arctanh(31/2*(8*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/
2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-51565338*arctanh(31/2*(8*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))
^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2
^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/
2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820
2^(1/2))^(1/2)-21/1321375(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^
(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(4245*2^(1/2)*(-886
6+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23
*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(
6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-7
75687+549362*2^(1/2))^(1/2)+6154*(-8866+6820*2^(1/2))^(1/2)*arctan(1/116924
87*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(
1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)
-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+12325786*
arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-35941
4*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/
2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(
1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-
x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-37/528550*(8*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)
2^(1/2)(2365*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-77568
7+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-
x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10
368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^

```
4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. 2(163) = 326.
time = 1.84, size = 2010, normalized size = 9.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 121/96875000*6050^(1/4)*sqrt(31)*sqrt(2)*sqrt(-772850000*sqrt(2) + 25000000
00)*arctan(1/254496437500*(722441500000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) + 2300*(4*6
050^(3/4)*sqrt(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 127
2680*x^3 - 1081800*x^2 - sqrt(2)*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1
083344*x^4 + 393672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 104
3712) + 5*6050^(1/4)*sqrt(31)*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 11
1531424*x^4 + 156761280*x^3 - 168192000*x^2 - sqrt(2)*(712757*x^7 - 1023330
3*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417
152*x + 37407744) - 74815488*x + 106417152))*sqrt(2*x^2 - x + 3)*sqrt(-7728
50000*sqrt(2) + 2500000000) - sqrt(10/5711)*(314105000*sqrt(31)*sqrt(2)*(12
3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5
+ 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - (4*60
50^(3/4)*sqrt(31)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 369
3960*x^3 + 4195584*x^2 + 22*sqrt(2)*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4
464*x^4 - 49464*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sq
rt(31)*(160956*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*
x^3 - 296027136*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 -
 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x
))*sqrt(2*x^2 - x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqr
t(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x
^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6
```

```
 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt
((6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt
(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2)*(2*x^
2 - x + 3) - 431180500*x + 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x
^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096
*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*
x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 9488
7936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^
4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 121/96875000*60
50^(1/4)*sqrt(31)*sqrt(2)*sqrt(-772850000*sqrt(2) + 2500000000)*arctan(-1/2
54496437500*(722441500000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270
*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x
^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3961
44*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2300*(4*6050^(3/4)*sqrt
(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 108
1800*x^2 - sqrt(2)*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1083344*x^4 + 3
93672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 1043712) + 5*6050
^(1/4)*sqrt(31)*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 111531424*x^4 +
156761280*x^3 - 168192000*x^2 - sqrt(2)*(712757*x^7 - 10233303*x^6 + 485297
68*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417152*x + 374077
44) - 74815488*x + 106417152))*sqrt(2*x^2 - x + 3)*sqrt(-772850000*sqrt(2)
+ 2500000000) - sqrt(10/5711)*(314105000*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2
 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4
- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + (4*6050^(3/4)*sqrt(
31)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419
5584*x^2 + 22*sqrt(2)*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4464*x^4 - 4946
4*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sqrt(31)*(160956
*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*x^3 - 29602713
6*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4
 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x))*sqrt(2*x^2
- x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqrt(31)*(254591*
x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*
x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6050^(1/4)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt(-772850000*s
qrt(2) + 2500000000) - 139919500*x^2 - 125642000*sqrt(2)*(2*x^2 - x + 3) +
431180500*x - 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x^8 - 9696916*
x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 379814
40*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x
^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(25851
91*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{5x^2 + 3x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(2\,x^2-x+3\right)}^{5/2}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2), x)
```

3.77 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=255

$$-\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750}+\frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2}+\frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)}-\frac{4799\sinh^{-1}\left(\right.}{2500\sqrt{2}}$$

```
[Out] 4/155*(4-5*x)*(2*x^2-x+3)^(3/2)+1/31*(3+10*x)*(2*x^2-x+3)^(5/2)/(5*x^2+3*x+
2)-4799/5000*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/7750*(1277+2240*x)*(2
*x^2-x+3)^(1/2)-11/1201250*arctanh(1/62*(21136+x*(87710-54423*2^(1/2))-3328
7*2^(1/2))*682^(1/2)/(-224510383+194487500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2)
)*(-76558040603+66320237500*2^(1/2))^(1/2)+11/1201250*arctan(1/62*(21136+33
287*2^(1/2)+x*(87710+54423*2^(1/2)))*682^(1/2)/(224510383+194487500*2^(1/2)
)^(1/2)/(2*x^2-x+3)^(1/2))*(76558040603+66320237500*2^(1/2))^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {985, 1080, 1090, 633, 221, 1049, 1043, 212, 210}

$$\frac{11\sqrt{\frac{11}{31}\left(224510383+194487500\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{62\left(224510383+194487500\sqrt{2}\right)}}\left(\left(87710+54423\sqrt{2}\right)x+33287\sqrt{2}+21136\right)}{\sqrt{2x^2-x+3}}\right)}{38750}+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\left(2x^2-x+3\right)^{3/2}-\frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750}-\frac{11\sqrt{\frac{11}{31}\left(194487500\sqrt{2}-224510383\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(194487500\sqrt{2}-224510383\right)}}\left(\left(87710-54423\sqrt{2}\right)x-33287\sqrt{2}+21136\right)}{\sqrt{2x^2-x+3}}\right)}{38750}-\frac{4799\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2500\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] -1/7750*((1277 + 2240*x)*Sqrt[3 - x + 2*x^2]) + (4*(4 - 5*x)*(3 - x + 2*x^2
)^(3/2))/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) -
(4799*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2500*Sqrt[2]) + (11*Sqrt[(11*(224510383
 + 194487500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*Sqrt[
2]))]*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x
^2]])/38750 - (11*Sqrt[(11*(-224510383 + 194487500*Sqrt[2]))/31]*ArcTanh[(S
qrt[11/(62*(-224510383 + 194487500*Sqrt[2]))]*(21136 - 33287*Sqrt[2] + (877
10 - 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/38750
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 985

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
```

```
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(-\frac{75}{2}+15x+80x^2\right)}{2+3x+5x^2}\,dx$$

$$= \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{(87660-54300x-53760x}{2+3x+}}{186}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)(3-}{31(2+3x}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)(3-}{31(2+3x}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)(3-}{31(2+3x}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)(3-}{31(2+3x}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)(3-}{31(2+3x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.72, size = 433, normalized size = 1.70

$$\frac{\frac{500\sqrt{3-x+2x^2}\left(8996+9289x-12555x^2+3100x^3\right)}{2+3x+5x^2} - 3719225\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right) + 30008\,\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{5237\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+2880\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+2225\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right] - 242\sqrt{2}\,\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{639994\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)-22980\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+1175\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]}{3875000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((500*Sqrt[3 - x + 2*x^2]*(8996 + 9289*x - 12555*x^2 + 3100*x^3))/(2 + 3*x
+ 5*x^2) - 3719225*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]] + 30008*R
ootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (5237*Lo
g[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2880*Sqrt[2]*Log[-(Sqrt[2]*x)
```

```
+ Sqrt[3 - x + 2*x^2] - #1]*#1 + 2225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 242*Sq
rt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (
639994*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 22980*Log[-(S
qrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1175*Sqrt[2]*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*
#1^3) & ])/3875000
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40027 vs. $2(194) = 388$.
time = 0.96, size = 40028, normalized size = 156.97

method	result
trager	Expression too large to display
risch	$\frac{(3100x^3-12555x^2+9289x+8996)\sqrt{2x^2-x+3}}{38750x^2+23250x+15500}+\frac{4799\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{5000}+\frac{11\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3}{}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2161 vs. $2(194) = 388$.

time = 3.76, size = 2161, normalized size = 8.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/13227599224357079000000*(38925001324*1464599010050^(1/4)*sqrt(155590)*sqrt
(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) + 388975000)*arctan(1
/2969754470638663638199958 75*(110935670*sqrt(155590)*(4*1464599010050^(3/4)
*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5 - 298338397*x^4 + 15
6222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62947113*x^6 + 1354639
29*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 - 122404608*x + 714528
00) - 142905600*x + 122404608) + 2411645*1464599010050^(1/4)*sqrt(62)*(3035
566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4 + 807721920*x^3 - 57
9888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204950152*x^5 - 4510041
40*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x + 268655616) - 5373112
32*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000)
+ 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 70
4270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(87
46*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 +
396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(77795/920561)*
(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374*x^7 - 85793239*x^6
+ 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 601344*x^2 - 4*sqrt(2)*(1
7889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^4 + 15593832*x^3 + 5
8564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599010050^(1/4)*sqrt(62
)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984960*x^4 + 949582080*
x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036*x^6 + 505509740*x^5
 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 725677056*x) - 183223296
*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000) + 759924265600
1778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x
^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^
7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 103
6800*x) + 3276288*x) + 345420120727353550*sqrt(31)*(254591*x^8 - 4815126*x^
7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*
x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*
x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(1464599010050^(1/4)*sqrt(15
5590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 387
58*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 6744561519183110*x^2 -
6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 2752882252
7278000)/x^2) + 957985313109246334903212 5*sqrt(31)*(2828123*x^8 - 9696916*x
^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3798144
0*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^
4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(258519
1*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x
```

```
^3 - 34615296*x^2 - 24772608*x + 18579456)) + 38925001324*1464599010050^(1/
4)*sqrt(155590)*sqrt(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) +
 388975000)*arctan(1/296975447063866363819995875*(110935670*sqrt(155590)*(4
*1464599010050^(3/4)*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5
- 298338397*x^4 + 156222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62
947113*x^6 + 135463929*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 -
122404608*x + 71452800) - 142905600*x + 122404608) + 2411645*1464599010050^
(1/4)*sqrt(62)*(3035566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4
+ 807721920*x^3 - 579888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204
950152*x^5 - 451004140*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x +
268655616) - 537311232*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*s
qrt(2) + 388975000) - 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x
^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 984
96*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710
*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
 sqrt(77795/920561)*(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374
*x^7 - 85793239*x^6 + 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 60134
4*x^2 - 4*sqrt(2)*(17889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^
4 + 15593832*x^3 + 58564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599
010050^(1/4)*sqrt(62)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984
960*x^4 + 949582080*x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036
*x^6 + 505509740*x^5 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 7256
77056*x) - 183223296*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 38897
5000) - 759924265600177810O*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 157
8888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(1
5550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
 + 1209600*x^2 - 1036800*x) + 3276288*x) - 345420120727353550*sqrt(31)*(254
591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219
328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{15625,[8]%%%}+%%%{%%{[-37500,0]:[1,0,-2]%%},[7]%%%}+%%%{-6125
0,[6]%
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{5/2}}{\left(5\,x^2+3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2, x)
```

3.78 $\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=281

$$\frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050}+\frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2}+\frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)}-\frac{4}{125}\sqrt{2}\ \mathrm{si}$$

```
[Out] 1/62*(3+10*x)*(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2+1/3844*(769+2336*x)*(2*x^2-
x+3)^(3/2)/(5*x^2+3*x+2)-4/125*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/480
50*(11359-12920*x)*(2*x^2-x+3)^(1/2)-1/29791000*arctanh(1/62*(3957722+x*(98
32420-6895071*2^(1/2))-2937349*2^(1/2))*682^(1/2)/(-3531015707557+249885207
1250*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(2937349-1978861*2^(1/2))*(-11+44*2^
(1/2))^(1/2)+1/29791000*arctan(1/62*(3957722+2937349*2^(1/2)+x*(9832420+689
5071*2^(1/2)))*682^(1/2)/(3531015707557+2498852071250*2^(1/2))^(1/2)/(2*x^2
-x+3)^(1/2))*(2937349+1978861*2^(1/2))*(11+44*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$,
Rules used = {985, 1068, 1080, 1090, 633, 221, 1049, 1043, 212, 210}

$$\frac{\sqrt{11\left(1+4\sqrt{2}\right)}\left(2937349+1978861\sqrt{2}\right)\mathrm{ArcTan}\left(\frac{\sqrt{\frac{11}{62\left(3531015707557+2498852071250\sqrt{2}\right)}}\left(\left(9832420+6895071\sqrt{2}\right)x+2937349\sqrt{2}+3957722\right)}{\sqrt{2x^2-x+3}}\right)}{29791000}+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}+\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{3844\left(5x^2+3x+2\right)}+\frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050}-\frac{\left(2937349-1978861\sqrt{2}\right)\sqrt{11\left(4\sqrt{2}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(2498852071250\sqrt{2}-3531015707557\right)}}\left(\left(9832420-6895071\sqrt{2}\right)x-2937349\sqrt{2}+3957722\right)}{\sqrt{2x^2-x+3}}\right)}{29791000}-\frac{4}{125}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((11359 - 12920*x)*Sqrt[3 - x + 2*x^2])/48050 + ((3 + 10*x)*(3 - x + 2*x^2)
^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + ((769 + 2336*x)*(3 - x + 2*x^2)^(3/2))/(
3844*(2 + 3*x + 5*x^2)) - (4*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/125 + (Sq
rt[11*(1 + 4*Sqrt[2])]*(2937349 + 1978861*Sqrt[2])*ArcTan[(Sqrt[11/(62*(353
1015707557 + 2498852071250*Sqrt[2]))]*(3957722 + 2937349*Sqrt[2] + (9832420
 + 6895071*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000 - ((2937349 - 197886
1*Sqrt[2])*Sqrt[11*(-1 + 4*Sqrt[2])]*ArcTanh[(Sqrt[11/(62*(-3531015707557 +
 2498852071250*Sqrt[2]))]*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*S
qrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 985

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
```

```
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
 - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \frac{(3+10x)\,(3-x+2x^2)^{5/2}}{62\,(2+3x+5x^2)^2} - \frac{1}{62}\int \frac{(3-x+2x^2)^{3/2}\left(-\frac{195}{2}+35x+40x^2\right)}{(2+3x+5x^2)^2}\,dx$$

$$= \frac{(3+10x)\,(3-x+2x^2)^{5/2}}{62\,(2+3x+5x^2)^2} + \frac{(769+2336x)\,(3-x+2x^2)^{3/2}}{3844\,(2+3x+5x^2)} + \frac{\int \frac{\left(\frac{66735}{4}-7375x-25840\ldots\right.}{2+3x\ldots}}{9\ldots}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\,(3-x+2x^2)^{5/2}}{62\,(2+3x+5x^2)^2} + \frac{(769+2336x)\,(3\ldots}{3844\,(2+3\ldots}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\,(3-x+2x^2)^{5/2}}{62\,(2+3x+5x^2)^2} + \frac{(769+2336x)\,(3\ldots}{3844\,(2+3\ldots}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\,(3-x+2x^2)^{5/2}}{62\,(2+3x+5x^2)^2} + \frac{(769+2336x)\,(3\ldots}{3844\,(2+3\ldots}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\,(3-x+2x^2)^{5/2}}{62\,(2+3x+5x^2)^2} + \frac{(769+2336x)\,(3\ldots}{3844\,(2+3\ldots}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\,(3-x+2x^2)^{5/2}}{62\,(2+3x+5x^2)^2} + \frac{(769+2336x)\,(3\ldots}{3844\,(2+3\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.86, size = 616, normalized size = 2.19

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((15812500*Sqrt[3 - x + 2*x^2]*(22552 + 69621*x + 93872*x^2 + 97155*x^3))/(
2 + 3*x + 5*x^2)^2 - 4420600000*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^
```

```
2]] + 972532000*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*
#1^4 & , (3781*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 630*Sqrt[2]*L
og[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 150*Log[-(Sqrt[2]*x) + Sqr
t[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^
3) & ] + 682*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3
 - 5*#1^4 & , (4978708507*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
#1] - 165870920*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 111095502
5*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] +
 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 11*Sqrt[2]*RootSum[-56 - 26*Sqrt[2
]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (492740319684*Sqrt[2]*Log[-(Sq
rt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 128644699540*Log[-(Sqrt[2]*x) + Sqrt
[3 - x + 2*x^2] - #1]*#1 + 55365920925*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 -
x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]
)/138143750000
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119457 vs. $2(220) = 440$.
time = 1.09, size = 119458, normalized size = 425.12

method	result
trager	Expression too large to display
risch	$\frac{11(97155x^3+93872x^2+69621x+22552)\sqrt{2x^2-x+3}}{96100(5x^2+3x+2)^2}+\frac{4\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{125}+\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+3\ldots}}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2240 vs. $2(220) = 440$.
time = 3.51, size = 2240, normalized size = 7.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/758714159921174808909075728000*(3184949732636*3868444992270541948232^(1/4
)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*s
qrt(3531015707557*sqrt(2) + 4997704142500)*arctan(1/45354848806294031039917
896246958932041502 31*(2850690442882*sqrt(1999081657)*(2*3868444992270541948
232^(3/4)*sqrt(62)*(2627559914*x^7 - 10187615527*x^6 + 21362956024*x^5 - 34
451465819*x^4 + 17321103240*x^3 - 8320757400*x^2 - sqrt(2)*(1893366636*x^7
- 7237484076*x^6 + 15226003533*x^5 - 24262105817*x^4 + 12127036096*x^3 - 56
64787848*x^2 - 13367586816*x + 9338025600) - 18676051200*x + 13367586816) +
 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(400116332*x^7 - 6149336
082*x^6 + 32552996440*x^5 - 74427496472*x^4 + 96235107840*x^3 - 61219656000
*x^2 - sqrt(2)*(286685371*x^7 - 4395067059*x^6 + 23180544704*x^5 - 52748573
780*x^4 + 68065744032*x^3 - 42544702944*x^2 - 48625837056*x + 34092306432)
- 68184612864*x + 48625837056))*sqrt(2*x^2 - x + 3)*sqrt(3531015707557*sqrt
(2) + 4997704142500) + 128749248224318539726218544184915678053296 88*sqrt(31
)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4
- 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 78
3113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 115
4304*x - 456192) - sqrt(1999081657/828550919)*(sqrt(1999081657)*(2*38684449
92270541948232^(3/4)*sqrt(62)*(9351066298*x^7 - 13433496653*x^6 + 433103458
23*x^5 - 17374572240*x^4 + 20927636280*x^3 + 18483199488*x^2 - sqrt(2)*(683
9273266*x^7 - 9809465289*x^6 + 31524099699*x^5 - 12024617744*x^4 + 13914887
256*x^3 + 14839341696*x^2 - 14839341696*x) - 18483199488*x) + 61971531367*3
868444992270541948232^(1/4)*sqrt(62)*(1427210918*x^7 - 18462714328*x^6 + 71
210222920*x^5 - 92387041920*x^4 + 119489780160*x^3 + 68726817792*x^2 - sqrt
(2)*(1033310523*x^7 - 13365477772*x^6 + 51521534980*x^5 - 66583614528*x^4 +
 85122955872*x^3 + 53108877312*x^2 - 53108877312*x) - 68726817792*x))*sqrt(
2*x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 45164233298567
21284677540671884*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 -
 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 -
 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600
*x^2 - 1036800*x) + 3276288*x) + 20529196953894187657625184872 2*sqrt(31)*(2
```

```
54591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742
19328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*
x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(38684
44992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(2141441*x + 1076175) - 3217616*x - 1065266)*sqrt(353101570755
7*sqrt(2) + 4997704142500) - 155990877430002205517374*x^2 - 140073440957553
000872744*sqrt(2)*(2*x^2 - x + 3) + 480706581467965980267826*x - 6366974588
97968185785200)/x^2) + 146305963891271067870702891119222361424201*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 31849
49732636*3868444992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25
*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(3531015707557*sqrt(2) + 49977041425
00)*arctan(1/453548488062940310399178962469589320415023l*(2850690442882*sqr
t(1999081657)*(2*3868444992270541948232^(3/4)*sqrt(62)*(2627559914*x^7 - 10
187615527*x^6 + 21362956024*x^5 - 34451465819*x^4 + 17321103240*x^3 - 83207
57400*x^2 - sqrt(2)*(1893366636*x^7 - 7237484076*x^6 + 15226003533*x^5 - 24
262105817*x^4 + 12127036096*x^3 - 5664787848*x^2 - 13367586816*x + 93380256
00) - 18676051200*x + 13367586816) + 61971531367*3868444992270541948232^(1/
4)*sqrt(62)*(400116332*x^7 - 6149336082*x^6 + 32552996440*x^5 - 74427496472
*x^4 + 96235107840*x^3 - 61219656000*x^2 - sqrt(2)*(286685371*x^7 - 4395067
059*x^6 + 23180544704*x^5 - 52748573780*x^4 + 68065744032*x^3 - 42544702944
*x^2 - 48625837056*x + 34092306432) - 68184612864*x + 48625837056))*sqrt(2*
x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) - 1287492482243185
397262185441849156780532968 8*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3
96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(1999081657/8285
50919)*(sqrt(1999081657)*(2*3868444992270541948232^(3/4)*sqrt(62)*(93510662
98*x^7 - 13433496653*x^6 + 43310345823*x^5 - 17374572240*x^4 + 20927636280*
x^3 + 18483199488*x^2 - sqrt(2)*(6839273266*x^7 - 9809465289*x^6 + 31524099
699*x^5 - 12024617744*x^4 + 13914887256*x^3 + 14839341696*x^2 - 14839341696
*x) - 18483199488*x) + 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(1
427210918*x^7 - 18462714328*x^6 + 71210222920*x...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**3, x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,inf

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{5/2}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3,x)

[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3, x)

3.79 $\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=185

$$\frac{16493087661\sqrt{3-x+2x^2}}{29360128}+\frac{1572007407x\sqrt{3-x+2x^2}}{7340032}-\frac{15428243x^2\sqrt{3-x+2x^2}}{131072}-\frac{19750457x^3\sqrt{3-}}{229376}$$

```
[Out] 2899366573/16777216*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+16493087661/2936
0128*(2*x^2-x+3)^(1/2)+1572007407/7340032*x*(2*x^2-x+3)^(1/2)-15428243/1310
72*x^2*(2*x^2-x+3)^(1/2)-19750457/229376*x^3*(2*x^2-x+3)^(1/2)+686531/6144*
x^4*(2*x^2-x+3)^(1/2)+2116475/10752*x^5*(2*x^2-x+3)^(1/2)+57375/448*x^6*(2*
x^2-x+3)^(1/2)+625/16*x^7*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1675, 654, 633, 221}

$$-\frac{15428243\sqrt{2x^2-x+3}\,x^2}{131072}+\frac{1572007407\sqrt{2x^2-x+3}\,x}{7340032}+\frac{16493087661\sqrt{2x^2-x+3}}{29360128}+\frac{625}{16}\sqrt{2x^2-x+3}\,x^7+\frac{57375}{448}\sqrt{2x^2-x+3}\,x^6+\frac{2116475\sqrt{2x^2-x+3}\,x^5}{10752}+\frac{686531\sqrt{2x^2-x+3}\,x^4}{6144}-\frac{19750457\sqrt{2x^2-x+3}\,x^3}{229376}+\frac{2899366573\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8388608\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (16493087661*Sqrt[3 - x + 2*x^2])/29360128 + (1572007407*x*Sqrt[3 - x + 2*x
^2])/7340032 - (15428243*x^2*Sqrt[3 - x + 2*x^2])/131072 - (19750457*x^3*Sq
rt[3 - x + 2*x^2])/229376 + (686531*x^4*Sqrt[3 - x + 2*x^2])/6144 + (211647
5*x^5*Sqrt[3 - x + 2*x^2])/10752 + (57375*x^6*Sqrt[3 - x + 2*x^2])/448 + (6
25*x^7*Sqrt[3 - x + 2*x^2])/16 + (2899366573*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
8388608*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{625}{16}x^7\sqrt{3-x+2x^2} + \frac{1}{16}\int \frac{256+1536x+6016x^2+14976x^3+28176x^4+3744}{\sqrt{3-x+2x^2}}$$

$$= \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7\sqrt{3-x+2x^2} + \frac{1}{224}\int \frac{3584+21504x+84224}{}$$

$$= \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7\sqrt{3-x+2x^2} +$$

$$= \frac{686531x^4\sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{57375}{448}x^6\sqrt{3-x+2x^2} +$$

$$= -\frac{19750457x^3\sqrt{3-x+2x^2}}{229376} + \frac{686531x^4\sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5\sqrt{3-x+2}}{10752}$$

$$= -\frac{15428243x^2\sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3\sqrt{3-x+2x^2}}{229376} + \frac{686531x^4\sqrt{3-x+}}{6144}$$

$$= \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3\sqrt{3-x}}{229376}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x}}{131072}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x}}{131072}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x}}{131072}$$

Mathematica [A]
time = 0.64, size = 85, normalized size = 0.46

$$\frac{4\sqrt{3-x+2x^2}\,(49479262983+18864088884x-10367779296x^2-7584175488x^3+9842108416x^4+17338163200x^5+11280384000x^6+3440640000x^7)+60886698033\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{352321536}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(49479262983 + 18864088884*x - 10367779296*x^2 - 758
4175488*x^3 + 9842108416*x^4 + 17338163200*x^5 + 11280384000*x^6 + 34406400
```

```
00*x^7) + 60886698033*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/35232
1536
```

Maple [A]
time = 0.13, size = 147, normalized size = 0.79

method	result
risch	$\frac{(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x^2+18864088884x+49479262983)\sqrt{}}{88080384}$
trager	$\left(\frac{625}{16}x^7+\frac{57375}{448}x^6+\frac{2116475}{10752}x^5+\frac{686531}{6144}x^4-\frac{19750457}{229376}x^3-\frac{15428243}{131072}x^2+\frac{1572007407}{7340032}x+\frac{16493087661}{29360128}\right)\sqrt{2x^2}$
default	$\frac{16493087661\sqrt{2x^2-x+3}}{29360128}-\frac{2899366573\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{16777216}+\frac{2116475x^5\sqrt{2x^2-x+3}}{10752}+\frac{57375x^6\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16493087661/29360128*(2*x^2-x+3)^(1/2)-2899366573/16777216*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))+2116475/10752*x^5*(2*x^2-x+3)^(1/2)+57375/448*x^6*(2
*x^2-x+3)^(1/2)+625/16*x^7*(2*x^2-x+3)^(1/2)+1572007407/7340032*x*(2*x^2-x+
3)^(1/2)-15428243/131072*x^2*(2*x^2-x+3)^(1/2)-19750457/229376*x^3*(2*x^2-x
+3)^(1/2)+686531/6144*x^4*(2*x^2-x+3)^(1/2)
```

Maxima [A]
time = 0.52, size = 148, normalized size = 0.80

$$\frac{625}{16}\sqrt{2x^2-x+3}\,x^7+\frac{57375}{448}\sqrt{2x^2-x+3}\,x^6+\frac{2116475}{10752}\sqrt{2x^2-x+3}\,x^5+\frac{686531}{6144}\sqrt{2x^2-x+3}\,x^4-\frac{19750457}{229376}\sqrt{2x^2-x+3}\,x^3-\frac{15428243}{131072}\sqrt{2x^2-x+3}\,x^2+\frac{1572007407}{7340032}\sqrt{2x^2-x+3}\,x-\frac{2899366573}{16777216}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{16493087661}{29360128}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 625/16*sqrt(2*x^2 - x + 3)*x^7 + 57375/448*sqrt(2*x^2 - x + 3)*x^6 + 211647
5/10752*sqrt(2*x^2 - x + 3)*x^5 + 686531/6144*sqrt(2*x^2 - x + 3)*x^4 - 197
50457/229376*sqrt(2*x^2 - x + 3)*x^3 - 15428243/131072*sqrt(2*x^2 - x + 3)*
x^2 + 1572007407/7340032*sqrt(2*x^2 - x + 3)*x - 2899366573/16777216*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16493087661/29360128*sqrt(2*x^2 - x +
3)
```

Fricas [A]
time = 3.33, size = 88, normalized size = 0.48

$$\frac{1}{88080384}\left(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x^2+18864088884x+49479262983\right)\sqrt{2x^2-x+3}+\frac{2899366573}{33554432}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/88080384*(3440640000*x^7 + 11280384000*x^6 + 17338163200*x^5 + 9842108416
*x^4 - 7584175488*x^3 - 10367779296*x^2 + 18864088884*x + 49479262983)*sqrt
(2*x^2 - x + 3) + 2899366573/33554432*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^4}{\sqrt{2x^2 - x + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/sqrt(2*x**2 - x + 3), x)
```

Giac [A]
time = 4.18, size = 83, normalized size = 0.45

$$\frac{1}{88080384}\left(4\left(8\left(4\left(16\left(100\left(120\left(140\,x+459\right)x+84659\right)x+4805717\right)x-59251371\right)x-323993103\right)x+4716022221\right)x+49479262983\right)\sqrt{2\,x^2-x+3}+\frac{2899366573}{16777216}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/88080384*(4*(8*(4*(16*(100*(120*(140*x + 459)*x + 84659)*x + 4805717)*x -
 59251371)*x - 323993103)*x + 4716022221)*x + 49479262983)*sqrt(2*x^2 - x +
 3) + 2899366573/16777216*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^4}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2), x)
```

3.80 $\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{203373\sqrt{3-x+2x^2}}{32768}-\frac{372783x\sqrt{3-x+2x^2}}{8192}-\frac{3387x^2\sqrt{3-x+2x^2}}{1024}+\frac{8185}{256}x^3\sqrt{3-x+2x^2}+\frac{1355}{48}x^4\sqrt{}$$

```
[Out] -9267707/131072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-203373/32768*(2*x^2-
x+3)^(1/2)-372783/8192*x*(2*x^2-x+3)^(1/2)-3387/1024*x^2*(2*x^2-x+3)^(1/2)+
8185/256*x^3*(2*x^2-x+3)^(1/2)+1355/48*x^4*(2*x^2-x+3)^(1/2)+125/12*x^5*(2*
x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1675, 654, 633, 221}

$$-\frac{3387\sqrt{2x^2-x+3}\,x^2}{1024}-\frac{372783\sqrt{2x^2-x+3}\,x}{8192}-\frac{203373\sqrt{2x^2-x+3}}{32768}+\frac{125}{12}\sqrt{2x^2-x+3}\,x^5+\frac{1355}{48}\sqrt{2x^2-x+3}\,x^4+\frac{8185}{256}\sqrt{2x^2-x+3}\,x^3-\frac{9267707\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-203373*Sqrt[3 - x + 2*x^2])/32768 - (372783*x*Sqrt[3 - x + 2*x^2])/8192 -
 (3387*x^2*Sqrt[3 - x + 2*x^2])/1024 + (8185*x^3*Sqrt[3 - x + 2*x^2])/256 +
 (1355*x^4*Sqrt[3 - x + 2*x^2])/48 + (125*x^5*Sqrt[3 - x + 2*x^2])/12 - (92
67707*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{12}\int \frac{96+432x+1368x^2+2484x^3+1545x^4+\frac{6775x^5}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{120}\int \frac{960+4320x+13680x^2 -}{\sqrt{3-x+}}$$

$$= \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{960}\int$$

$$= -\frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}$$

$$= -\frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{135}{48}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{818}{25}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{818}{25}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{818}{25}$$

Mathematica [A]
time = 0.45, size = 75, normalized size = 0.52

$$\frac{4\sqrt{3-x+2x^2}\left(-610119-4473396x-325152x^2+3143040x^3+2775040x^4+1024000x^5\right)-27803121\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{393216}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-610119 - 4473396*x - 325152*x^2 + 3143040*x^3 + 27
75040*x^4 + 1024000*x^5) - 27803121*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x +
4*x^2]])/393216
```

Maple [A]
time = 0.14, size = 113, normalized size = 0.79

method	result
risch	$\frac{(1024000x^5+2775040x^4+3143040x^3-325152x^2-4473396x-610119)\sqrt{2x^2-x+3}}{98304}+\frac{9267707\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right.}{131072}$
trager	$\left(\frac{125}{12}x^5+\frac{1355}{48}x^4+\frac{8185}{256}x^3-\frac{3387}{1024}x^2-\frac{372783}{8192}x-\frac{203373}{32768}\right)\sqrt{2x^2-x+3}+\frac{9267707\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{R}\right.}{}$
default	$-\frac{203373\sqrt{2x^2-x+3}}{32768}+\frac{9267707\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{131072}+\frac{125x^5\sqrt{2x^2-x+3}}{12}-\frac{372783x\sqrt{2x^2-x}}{8192}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -203373/32768*(2*x^2-x+3)^(1/2)+9267707/131072*2^(1/2)*arcsinh(4/23*23^(1/2
)*(x-1/4))+125/12*x^5*(2*x^2-x+3)^(1/2)-372783/8192*x*(2*x^2-x+3)^(1/2)-338
7/1024*x^2*(2*x^2-x+3)^(1/2)+8185/256*x^3*(2*x^2-x+3)^(1/2)+1355/48*x^4*(2*
x^2-x+3)^(1/2)
```

Maxima [A]
time = 0.51, size = 114, normalized size = 0.80

$$\frac{125}{12}\sqrt{2x^2-x+3}\,x^5+\frac{1355}{48}\sqrt{2x^2-x+3}\,x^4+\frac{8185}{256}\sqrt{2x^2-x+3}\,x^3-\frac{3387}{1024}\sqrt{2x^2-x+3}\,x^2-\frac{372783}{8192}\sqrt{2x^2-x+3}\,x+\frac{9267707}{131072}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{203373}{32768}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/12*sqrt(2*x^2 - x + 3)*x^5 + 1355/48*sqrt(2*x^2 - x + 3)*x^4 + 8185/256
*sqrt(2*x^2 - x + 3)*x^3 - 3387/1024*sqrt(2*x^2 - x + 3)*x^2 - 372783/8192*
sqrt(2*x^2 - x + 3)*x + 9267707/131072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x -
 1)) - 203373/32768*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 1.50, size = 78, normalized size = 0.55

$$\frac{1}{98304}\left(1024000\,x^5+2775040\,x^4+3143040\,x^3-325152\,x^2-4473396\,x-610119\right)\sqrt{2\,x^2-x+3}+\frac{9267707}{262144}\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(1024000*x^5 + 2775040*x^4 + 3143040*x^3 - 325152*x^2 - 4473396*x -
 610119)*sqrt(2*x^2 - x + 3) + 9267707/262144*sqrt(2)*log(-4*sqrt(2)*sqrt(2
*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+3x+2)^3}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/sqrt(2*x**2 - x + 3), x)
```

Giac [A]
time = 2.61, size = 73, normalized size = 0.51

$$\frac{1}{98304}\left(4\left(8\left(20\left(16\left(100\,x+271\right)x+4911\right)x-10161\right)x-1118349\right)x-610119\right)\sqrt{2\,x^2-x+3}-\frac{9267707}{131072}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\,\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/98304*(4*(8*(20*(16*(100*x + 271)*x + 4911)*x - 10161)*x - 1118349)*x - 6
10119)*sqrt(2*x^2 - x + 3) - 9267707/131072*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^3}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2), x)
```

3.81 $\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{11373\sqrt{3-x+2x^2}}{1024}+\frac{3443}{768}x\sqrt{3-x+2x^2}+\frac{655}{96}x^2\sqrt{3-x+2x^2}+\frac{25}{8}x^3\sqrt{3-x+2x^2}+\frac{30725\sinh^{-1}\left(\cdots\right)}{2048\sqrt{2}}$$

```
[Out] 30725/4096*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-11373/1024*(2*x^2-x+3)^(1
/2)+3443/768*x*(2*x^2-x+3)^(1/2)+655/96*x^2*(2*x^2-x+3)^(1/2)+25/8*x^3*(2*x
^2-x+3)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1675, 654, 633, 221}

$$\frac{655}{96}\sqrt{2x^2-x+3}\,x^2+\frac{3443}{768}\sqrt{2x^2-x+3}\,x-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{25}{8}\sqrt{2x^2-x+3}\,x^3+\frac{30725\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-11373*Sqrt[3 - x + 2*x^2])/1024 + (3443*x*Sqrt[3 - x + 2*x^2])/768 + (655
*x^2*Sqrt[3 - x + 2*x^2])/96 + (25*x^3*Sqrt[3 - x + 2*x^2])/8 + (30725*ArcS
inh[(1 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{8}\int \frac{32+96x+7x^2+\frac{655x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{48}\int \frac{192-1389x+\frac{3443x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{192}\int \frac{-7\ldots}{\sqrt{3\ldots}}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{\ldots}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{\ldots}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{\ldots}$$

Mathematica [A]
time = 0.30, size = 65, normalized size = 0.64

$$\frac{4\sqrt{3-x+2x^2}\,(-34119+13772x+20960x^2+9600x^3)+92175\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{12288}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
[Out] (4*Sqrt[3 - x + 2*x^2]*(-34119 + 13772*x + 20960*x^2 + 9600*x^3) + 92175*Sq
rt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/12288
```

Maple [A]
time = 0.12, size = 79, normalized size = 0.78

method	result
risch	$\frac{(9600x^3+20960x^2+13772x-34119)\sqrt{2x^2-x+3}}{3072}-\frac{30725\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{4096}$
trager	$\left(\frac{25}{8}x^3+\frac{655}{96}x^2+\frac{3443}{768}x-\frac{11373}{1024}\right)\sqrt{2x^2-x+3}+\frac{30725\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-\right.}{4096}$
default	$\frac{25x^3\sqrt{2x^2-x+3}}{8}+\frac{655x^2\sqrt{2x^2-x+3}}{96}+\frac{3443x\sqrt{2x^2-x+3}}{768}-\frac{11373\sqrt{2x^2-x+3}}{1024}-\frac{30725\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/8*x^3*(2*x^2-x+3)^(1/2)+655/96*x^2*(2*x^2-x+3)^(1/2)+3443/768*x*(2*x^2-x
+3)^(1/2)-11373/1024*(2*x^2-x+3)^(1/2)-30725/4096*2^(1/2)*arcsinh(4/23*23^(
1/2)*(x-1/4))
```

Maxima [A]
time = 0.50, size = 80, normalized size = 0.79

$$\frac{25}{8}\sqrt{2\,x^2-x+3}\,x^3+\frac{655}{96}\sqrt{2\,x^2-x+3}\,x^2+\frac{3443}{768}\sqrt{2\,x^2-x+3}\,x-\frac{30725}{4096}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{11373}{1024}\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 25/8*sqrt(2*x^2 - x + 3)*x^3 + 655/96*sqrt(2*x^2 - x + 3)*x^2 + 3443/768*sq
rt(2*x^2 - x + 3)*x - 30725/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) -
 11373/1024*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 2.10, size = 68, normalized size = 0.67

$$\frac{1}{3072}\left(9600\,x^3+20960\,x^2+13772\,x-34119\right)\sqrt{2\,x^2-x+3}+\frac{30725}{8192}\sqrt{2}\,\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(9600*x^3 + 20960*x^2 + 13772*x - 34119)*sqrt(2*x^2 - x + 3) + 30725
/8192*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x -
 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(5x^2+3x+2\right)^2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/sqrt(2*x**2 - x + 3), x)
```

Giac [A]
time = 4.84, size = 63, normalized size = 0.62

$$\frac{1}{3072}\left(4\left(40\left(60\,x+131\right)x+3443\right)x-34119\right)\sqrt{2\,x^2-x+3}+\frac{30725}{4096}\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(40*(60*x + 131)*x + 3443)*x - 34119)*sqrt(2*x^2 - x + 3) + 30725
/4096*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^2}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2), x)
```

3.82 $\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=59

$$\frac{39}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] 17/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+39/16*(2*x^2-x+3)^(1/2)+5/4*x*
(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1675, 654, 633, 221}

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x+\frac{39}{16}\sqrt{2x^2-x+3}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (39*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*x^2])/4 + (17*ArcSinh[(1
- 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx &= \frac{5}{4}x\sqrt{3-x+2x^2}+\frac{1}{4}\int \frac{-7+\frac{39x}{2}}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{39}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}-\frac{17}{32}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{39}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}-\frac{17\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{32\sqrt{46}}\\
&= \frac{39}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.16, size = 55, normalized size = 0.93

$$\frac{1}{64}\left(4(39+20x)\sqrt{3-x+2x^2}+17\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*(39 + 20*x)*Sqrt[3 - x + 2*x^2] + 17*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*
x + 4*x^2]])/64
```

Maple [**A**]
time = 0.10, size = 45, normalized size = 0.76

method	result
risch	$\frac{(20x+39)\sqrt{2x^2-x+3}}{16}-\frac{17\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{64}$

default	$\frac{5x\sqrt{2x^2-x+3}}{4}+\frac{39\sqrt{2x^2-x+3}}{16}-\frac{17\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{64}$
trager	$\left(\frac{5x}{4}+\frac{39}{16}\right)\sqrt{2x^2-x+3}-\frac{17\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x-\operatorname{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-x+3}\right)}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/4*x*(2*x^2-x+3)^(1/2)+39/16*(2*x^2-x+3)^(1/2)-17/64*2^(1/2)*arcsinh(4/23*
23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.51, size = 46, normalized size = 0.78

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x-\frac{17}{64}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{39}{16}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2*x^2 - x + 3)*x - 17/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
+ 39/16*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 1.77, size = 58, normalized size = 0.98

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(20x+39)+\frac{17}{128}\sqrt{2}\,\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/128*sqrt(2)*log(4*sqrt(2)*sqrt(2*
x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{5x^2+3x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/sqrt(2*x**2 - x + 3), x)
```

Giac [**A**]
time = 4.93, size = 53, normalized size = 0.90

$$\frac{1}{16}\sqrt{2\,x^2-x+3}\,(20\,x+39)+\frac{17}{64}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{5\,x^2+3\,x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

3.83 $\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx$

Optimal. Leaf size=148

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(7+3\sqrt{2}+\left(13+10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)-\sqrt{\frac{1}{682}\left(-13+10\sqrt{\right.}}$$

```
[Out] -1/682*arctanh(1/31*(7+x*(13-10*2^(1/2))-3*2^(1/2))*341^(1/2)/(-13+10*2^(1/
2))^(1/2)/(2*x^2-x+3)^(1/2))*(-8866+6820*2^(1/2))^(1/2)+1/682*arctan(1/31*(
7+3*2^(1/2)+x*(13+10*2^(1/2)))*341^(1/2)/(13+10*2^(1/2))^(1/2)/(2*x^2-x+3)^
(1/2))*(8866+6820*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1000, 1043, 210, 212}

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)-\sqrt{\frac{1}{682}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(10\sqrt{2}-13\right)}}\left(\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))]*(7 + 3*
Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] - Sqrt[(-13 + 10*Sqrt[
2])/682]*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))]*(7 - 3*Sqrt[2] + (13 - 1
0*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = -\frac{\int \frac{11-11\sqrt{2}-11x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx}{22\sqrt{2}} + \frac{\int \frac{11+11\sqrt{2}-11x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx}{22\sqrt{2}}$$

$$= -\left(\frac{1}{2}\left(11\left(20-13\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-3751\left(13-10\sqrt{2}\right)-11x^2}\,d\right.\right.$$

$$= \sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(7+3\sqrt{2}+\left(\right.\right.}{\sqrt{3-x+2x^2}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.19, size = 135, normalized size = 0.91

$$\text{RootSum}\left[-56-26\sqrt{2}\,\#1+17\#1^2+6\sqrt{2}\,\#1^3-5\#1^4\&,\frac{\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)+2\sqrt{2}\,\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1}{-13\sqrt{2}+17\#1+9\sqrt{2}\,\#1^2-10\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) &
 ]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(109) = 218$.
time = 0.57, size = 684, normalized size = 4.62

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+465124\,\mathrm{RootOf}\left(232562\_Z^4+4433\_Z^2+25\right)^2+8866\right)\ln\left(\frac{-18721241\,\mathrm{RootOf}\left(\_Z^2+465124\,\mathrm{RootOf}\left(232562\_Z^4+443\right.\right.}{}\right.}{}$
default	$\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\;\sqrt{2}\left(369\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\arctan\left(\frac{\sqrt{-7}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21142*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(1/2)*(-8866+6820*2^(1/2))^(1/2
)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*
(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2
)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-
x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2)
)^(1/2)+520*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^
(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(
1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+
1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(
```

```
8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+465124*arctanh(31/2*(8*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1
/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arctanh(31/2*(8*(2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^
(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)
^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/
(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-886
6+6820*2^(1/2))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2002 vs. $2(109) = 218$.
time = 2.37, size = 2002, normalized size = 13.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
[Out] -1/845680*sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(13*sqrt(2) + 20)*(13*sq
rt(2) - 20)*log(1240*(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) + 7595*x^2 + 6820*s
qrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) + 1/845680*sqrt(341)*200^(1/
4)*sqrt(31)*sqrt(5)*sqrt(13*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-1240*(sqrt
(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3
*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(2)*(2*x^2 - x + 3) + 2
3405*x - 31000)/x^2) - 1/6820*sqrt(341)*200^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*s
qrt(2) + 20)*arctan(1/2762875*(14260*sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*
(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838*x^5 - 93342*x^4 + 45376*x^3 - 1
8288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^
3 - 19080*x^2 - 34560*x + 27648) - 55296*x + 34560) + 5*200^(1/4)*(18463*x^
7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - sq
rt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5 - 2237548*x^4 + 2920320*x^3 - 20
05920*x^2 - 1990656*x + 1534464) - 3068928*x + 1990656))*sqrt(13*sqrt(2) +
20) + 7843000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 13852
56*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*
x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546
```

048*x - 539136) + 1154304*x - 456192) - 2*sqrt(310)*(sqrt(341)*sqrt(5)*sqrt
(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 - 44014*x^6 + 139674*x^5 - 42464*x
^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 3
7360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x) - 89856*x) + 5*200^(1/4)*(69479
*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 -
sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*x^5 - 2560320*x^4 + 3506400*x^3
+ 1202688*x^2 - 1202688*x) - 4478976*x))*sqrt(13*sqrt(2) + 20) + 550*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) + 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^
5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 7
6*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144
820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*
(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(
2)*(2*x^2 - x + 3) + 23405*x - 31000)/x^2) + 89125*sqrt(31)*(2828123*x^8 -
9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3
+ 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 +
15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936)
)/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4
4249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 1/6820*sqrt(341)*200
^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/2762875*(14260*sqrt(3
41)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838
*x^5 - 93342*x^4 + 45376*x^3 - 18288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 +
40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - 34560*x + 27648) - 55296*x
+ 34560) + 5*200^(1/4)*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4
+ 4140576*x^3 - 2378592*x^2 - sqrt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5
- 2237548*x^4 + 2920320*x^3 - 2005920*x^2 - 1990656*x + 1534464) - 3068928*
x + 1990656))*sqrt(13*sqrt(2) + 20) - 7843000*sqrt(31)*sqrt(2)*(28180*x^8 -
254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x
^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4
- 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*s
qrt(310)*(sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 -
44014*x^6 + 139674*x^5 - 42464*x^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454
*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x)
- 89856*x) + 5*200^(1/4)*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x
^4 + 5347296*x^3 + 4478976*x^2 - sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*
x^5 - 2560320*x^4 + 3506400*x^3 + 1202688*x^2 - 1202688*x) - 4478976*x))*sq
rt(13*sqrt(2) + 20) - 550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 15788
88*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(155
50*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 +
1209600*x^2 - 1036800*x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*
x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 16895692
8*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 361
8*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((sqrt(341)*200^(1/4)*sqrt(3

```
1)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2
) + 20) + 7595*x^2 + 6820*sqrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) -
 89125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
 254146592*x^4 - 249300096*x^3 + 37981440*x^2 -...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3} \cdot (5x^2 + 3x + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^2 - x + 3}\,\left(5\,x^2 + 3\,x + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)), x)
```

3.84 $\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=188

$$\frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\,(2119+}{\sqrt{3-x+2x^2}}\right.}{1364}$$

```
[Out] 1/682*(4+65*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/930248*arctanh(1/31*(2119+
x*(5751-3935*2^(1/2))-1816*2^(1/2))*341^(1/2)/(-2343727+1678700*2^(1/2))^(1
/2)/(2*x^2-x+3)^(1/2))*(-1598421814+1144873400*2^(1/2))^(1/2)+1/930248*arct
an(1/31*(2119+1816*2^(1/2)+x*(5751+3935*2^(1/2)))*341^(1/2)/(2343727+167870
0*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(1598421814+1144873400*2^(1/2))^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {988, 1049, 1043, 212, 210}

$$\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)}{1364}+\frac{\sqrt{2x^2-x+3}\,(65x+4)}{682\,(5x^2+3x+2)}-\frac{\sqrt{\frac{1}{682}\left(1678700\sqrt{2}-2343727\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(1678700\sqrt{2}-2343727\right)}}\left(\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)}{1364}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727 +
 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))]*(2
119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1364 -
 (Sqrt[(-2343727 + 1678700*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-2343727 + 1
678700*Sqrt[2]))]*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/1364
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)} - \frac{\int \frac{-1826+\frac{2255x}{2}}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx}{7502}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)} - \frac{\int \frac{\frac{121}{2}\left(537-332\sqrt{2}\right)-\frac{121}{2}\left(127-205\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx}{165044\sqrt{2}}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)} - \frac{1}{496}\left(11\left(3357400-2343727\sqrt{2}\right)\right)\text{Subs}$$

$$\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\ \tan^{-1}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)} + \frac{\ }{\ }$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.36, size = 230, normalized size = 1.22

$$\frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)} + \frac{\text{RootSum}\left[-10580-2024\sqrt{2}\#1+68\#1^2+44\sqrt{2}\#1^3-5\#1^4\&,\frac{-9430\sqrt{2}\log\left(\sqrt{2}(-1+4x)-4\sqrt{3-x+2x^2}+\#1\right)+4492\log\left(\sqrt{2}(-1+4x)-4\sqrt{3-x+2x^2}+\#1\right)\#1+205\sqrt{2}\log\left(\sqrt{2}(-1+4x)-4\sqrt{3-x+2x^2}+\#1\right)\#1^2}{-506\sqrt{2}+34\#1+33\sqrt{2}\#1^2-5\#1^3}\&\right]}{682\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + RootSum[-10580 -
 2024*Sqrt[2]*#1 + 68*#1^2 + 44*Sqrt[2]*#1^3 - 5*#1^4 & , (-9430*Sqrt[2]*Lo
g[Sqrt[2]*(-1 + 4*x) - 4*Sqrt[3 - x + 2*x^2] + #1] + 4492*Log[Sqrt[2]*(-1 +
 4*x) - 4*Sqrt[3 - x + 2*x^2] + #1]*#1 + 205*Sqrt[2]*Log[Sqrt[2]*(-1 + 4*x)
 - 4*Sqrt[3 - x + 2*x^2] + #1]*#1^2)/(-506*Sqrt[2] + 34*#1 + 33*Sqrt[2]*#1^
2 - 5*#1^3) & ]/(682*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5224 vs. $2(140) = 280$.
time = 0.63, size = 5225, normalized size = 27.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x^2 - x + 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2102 vs. 2(140) = 280.
time = 3.57, size = 2102, normalized size = 11.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/263043507934399808*(8422204*563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)
*(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2) + 3357400)*arctan(1/710190022151725
4683789*(47876524*sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(2950932*x^7 -
11691762*x^6 + 24397746*x^5 - 40053004*x^4 + 20309552*x^3 - 10145376*x^2 -
sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17801494*x^5 - 27869789*x^4 + 13808040
*x^3 - 6172200*x^2 - 15724800*x + 10596096) - 21192192*x + 15724800) + 5203
97*563606738^(1/4)*sqrt(341)*(226651*x^7 - 3496629*x^6 + 18614024*x^5 - 428
60780*x^4 + 55586592*x^3 - 36274464*x^2 - sqrt(2)*(168871*x^7 - 2579646*x^6
 + 13533020*x^5 - 30582616*x^4 + 39345120*x^3 - 23947200*x^2 - 28449792*x +
 19450368) - 38900736*x + 28449792))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(
2) + 3357400) + 20160232886887690715272*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(33
574/2191)*(sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(10257392*x^7 - 147733
68*x^6 + 47877288*x^5 - 20710528*x^4 + 26321472*x^3 + 17079552*x^2 - sqrt(2
)*(8292238*x^7 - 11867543*x^6 + 37968813*x^5 - 13449840*x^4 + 14570280*x^3
+ 20176128*x^2 - 20176128*x) - 17079552*x) + 520397*563606738^(1/4)*sqrt(34
1)*(795513*x^7 - 10292932*x^6 + 39734380*x^5 - 51864768*x^4 + 68281632*x^3
+ 34255872*x^2 - 8*sqrt(2)*(77213*x^7 - 998548*x^6 + 3846220*x^5 - 4943520*
x^4 + 6215760*x^3 + 4318272*x^2 - 4318272*x) - 34255872*x))*sqrt(2*x^2 - x
+ 3)*sqrt(2343727*sqrt(2) + 3357400) + 421088065768678*sqrt(31)*sqrt(2)*(12
3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5
+ 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 19140
366625849*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5
```

```
+ 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*
x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 14482
0224*x))*sqrt(-(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
 x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357
400) - 1731948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 53372285
80187*x - 7069176927400)/x^2) + 22909355553281466719*sqrt(31)*(2828123*x^8
 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x
^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^
5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 948879
36))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4
+ 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 8422204*563606738
^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2)
 + 3357400)*arctan(1/710190022151725468378 9*(47876524*sqrt(33574)*(22*56360
6738^(3/4)*sqrt(341)*(2950932*x^7 - 11691762*x^6 + 24397746*x^5 - 40053004*
x^4 + 20309552*x^3 - 10145376*x^2 - sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17
801494*x^5 - 27869789*x^4 + 13808040*x^3 - 6172200*x^2 - 15724800*x + 10596
096) - 21192192*x + 15724800) + 520397*563606738^(1/4)*sqrt(341)*(226651*x^
7 - 3496629*x^6 + 18614024*x^5 - 42860780*x^4 + 55586592*x^3 - 36274464*x^2
 - sqrt(2)*(168871*x^7 - 2579646*x^6 + 13533020*x^5 - 30582616*x^4 + 393451
20*x^3 - 23947200*x^2 - 28449792*x + 19450368) - 38900736*x + 28449792))*sq
rt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) - 20160232886887690715272
*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549
144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*
x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53913
6) + 1154304*x - 456192) - 2*sqrt(33574/2191)*(sqrt(33574)*(22*563606738^(3
/4)*sqrt(341)*(10257392*x^7 - 14773368*x^6 + 47877288*x^5 - 20710528*x^4 +
26321472*x^3 + 17079552*x^2 - sqrt(2)*(8292238*x^7 - 11867543*x^6 + 3796881
3*x^5 - 13449840*x^4 + 14570280*x^3 + 20176128*x^2 - 20176128*x) - 17079552
*x) + 520397*563606738^(1/4)*sqrt(341)*(795513*x^7 - 10292932*x^6 + 3973438
0*x^5 - 51864768*x^4 + 68281632*x^3 + 34255872*x^2 - 8*sqrt(2)*(77213*x^7 -
 998548*x^6 + 3846220*x^5 - 4943520*x^4 + 6215760*x^3 + 4318272*x^2 - 43182
72*x) - 34255872*x))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) -
421088065768678*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3
293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 1
18051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x
^2 - 1036800*x) + 3276288*x) - 19140366625849*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((563606738^(1/4)*sqrt(335
74)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x
 - 225)*sqrt(2343727*sqrt(2) + 3357400) + 17319...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3}\,(5x^2 + 3x + 2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^2 - x + 3}\,\left(5\,x^2 + 3\,x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2), x)
```

3.85 $\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=223

$$\frac{(4+65x)\sqrt{3-x+2x^2}}{1364\,(2+3x+5x^2)^2}+\frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\,(2+3x+5x^2)}+\frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\ \tan}{}$$

```
[Out] 1/1364*(4+65*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2+1/1860496*(26794+86265*x)
*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-25/2537716544*arctanh(1/31*(123161+x*(2946
69-208915*2^(1/2))-85754*2^(1/2))*341^(1/2)/(-6414867847+4536374600*2^(1/2)
)^(1/2)/(2*x^2-x+3)^(1/2))*(-4374939871654+3093807477200*2^(1/2))^(1/2)+25/
2537716544*arctan(1/31*(123161+85754*2^(1/2)+x*(294669+208915*2^(1/2)))*341
^(1/2)/(6414867847+4536374600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(4374939871
654+3093807477200*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {988, 1074, 1049, 1043, 212, 210}

$$\frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(6414867847+4536374600\sqrt{2}\right)}}\left(\left(294669+208915\sqrt{2}\right)x+85754\sqrt{2}+123161\right)}{\sqrt{2x^2-x+3}}\right)}{3720992}+\frac{\sqrt{2x^2-x+3}\,(65x+4)}{1364\,(5x^2+3x+2)^2}+\frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496\,(5x^2+3x+2)}-\frac{25\sqrt{\frac{1}{682}\left(4536374600\sqrt{2}-6414867847\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(4536374600\sqrt{2}-6414867847\right)}}\left(\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+123161\right)}{\sqrt{2x^2-x+3}}\right)}{3720992}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + ((26794 + 862
65*x)*Sqrt[3 - x + 2*x^2])/(1860496*(2 + 3*x + 5*x^2)) + (25*Sqrt[(64148678
47 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600*
Sqrt[2]))]*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/3720992 - (25*Sqrt[(-6414867847 + 4536374600*Sqrt[2])/682]*Ar
cTanh[(Sqrt[11/(31*(-6414867847 + 4536374600*Sqrt[2]))]*(123161 - 85754*Sqr
t[2] + (294669 - 208915*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/3720992
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^3}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5775+\frac{6479x}{2}-2860x^2}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx}{15004}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\int \cdots}{\cdots}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\int \cdots}{\cdots}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{(6\cdots}{\cdots}$$

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$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} + \cdots$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.63, size = 396, normalized size = 1.78

$$\frac{\sqrt{3-x+2x^2}\left(59044+341572x+392765x^2+431325x^3\right)}{1860496\left(2+3x+5x^2\right)^2} + \frac{3\,\text{RootSum}\left[-56-26\sqrt{2}\,\#1+17\#1^2+6\sqrt{2}\,\#1^3-5\#1^4\&,\frac{-42330420383\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+11629301740\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1-2992879225\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\,\#1^2-10\#1^3}\&\right]}{49210119200} - \frac{16\,\text{RootSum}\left[-56-26\sqrt{2}\,\#1+17\#1^2+6\sqrt{2}\,\#1^3-5\#1^4\&,\frac{-720397\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+129160\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1-65525\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\,\#1^2-10\#1^3}\&\right]}{4509725}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(59044 + 341572*x + 392765*x^2 + 431325*x^3))/(1860496
*(2 + 3*x + 5*x^2)^2) + (3*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2
]*#1^3 - 5*#1^4 & , (-42330420383*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
#1] + 11629301740*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 -
 2992879225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2]
 + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/49210119200 - (16*RootSum[-56 - 2
6*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-720397*Log[-(Sqrt[2]
*x) + Sqrt[3 - x + 2*x^2] - #1] + 129160*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3
- x + 2*x^2] - #1]*#1 - 65525*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*
#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/4509725
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 13039 vs. $2(171) = 342$.
time = 0.65, size = 13040, normalized size = 58.48

method	result
trager	Expression too large to display
risch	$\frac{(431325x^3+392765x^2+341572x+59044)\sqrt{2x^2-x+3}}{1860496(5x^2+3x+2)^2} + \frac{25\sqrt{\frac{8(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2} + \frac{3\sqrt{2}(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2} + 8 - 3\sqrt{2}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(171) = 342$.
time = 2.86, size = 2183, normalized size = 9.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/212344000027477426346822144*(46113488900*4115738902305032^(1/4)*sqrt(226
81873)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(6414867
847*sqrt(2) + 9072749200)*arctan(1/3836668309294009530058322373948769*(6468
8701796*sqrt(22681873)*(11*4115738902305032^(3/4)*sqrt(341)*(160344708*x^7
 - 615873378*x^6 + 1294230774*x^5 - 2070733376*x^4 + 1037098288*x^3 - 489164
544*x^2 - sqrt(2)*(112700446*x^7 - 434839553*x^6 + 912850886*x^5 - 14661276
91*x^4 + 735661560*x^3 - 350098200*x^2 - 799200000*x + 567316224) - 1134632
448*x + 799200000) + 703138063*4115738902305032^(1/4)*sqrt(341)*(12162569*x
^7 - 186616851*x^6 + 985490056*x^5 - 2246141620*x^4 + 2900382048*x^3 - 1823
848416*x^2 - sqrt(2)*(8564099*x^7 - 131508024*x^6 + 695288980*x^5 - 1587105
104*x^4 + 2050714080*x^3 - 1296806400*x^2 - 1457077248*x + 1033108992) - 20
66217984*x + 1457077248))*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2) + 907
2749200) + 10891187458641059311133302222822312*sqrt(31)*sqrt(2)*(28180*x^8
 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^
4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*
sqrt(45363746/479849)*(sqrt(22681873)*(11*4115738902305032^(3/4)*sqrt(341)*
(576322648*x^7 - 827050092*x^6 + 2660713572*x^5 - 1032439232*x^4 + 12116047
68*x^3 + 1213394688*x^2 - sqrt(2)*(403157522*x^7 - 578844217*x^6 + 18641293
47*x^5 - 735062160*x^4 + 873708120*x^3 + 823986432*x^2 - 823986432*x) - 121
3394688*x) + 703138063*4115738902305032^(1/4)*sqrt(341)*(43684647*x^7 - 565
067708*x^6 + 2178643220*x^5 - 2819241792*x^4 + 3618371808*x^3 + 2197767168*
x^2 - 2*sqrt(2)*(15328963*x^7 - 198290348*x^6 + 764653220*x^5 - 990717120*x
^4 + 1276256160*x^3 + 755350272*x^2 - 755350272*x) - 2197767168*x))*sqrt(2*
x^2 - x + 3)*sqrt(6414867847*sqrt(2) + 9072749200) + 1683630550043672623393
22*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) + 7652866136562148288151*sqrt(31)*(254591*x^8 - 4815126*x^7
 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x
^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x
^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(4115738902305032^(1/4)*sqrt(
22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012)
 - 93199*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187
753*x^2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*
x - 2091880597633419400)/x^2) + 123763493848193855808332979804799*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 46113
488900*4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(2)*(25*x^4 + 30
*x^3 + 29*x^2 + 12*x + 4)*sqrt(6414867847*sqrt(2) + 9072749200)*arctan(1/38
36668309294009530058322373948769*(64688701796*sqrt(22681873)*(11*4115738902
305032^(3/4)*sqrt(341)*(160344708*x^7 - 615873378*x^6 + 1294230774*x^5 - 20
```

```
70733376*x^4 + 1037098288*x^3 - 489164544*x^2 - sqrt(2)*(112700446*x^7 - 43
4839553*x^6 + 912850886*x^5 - 1466127691*x^4 + 735661560*x^3 - 350098200*x^
2 - 799200000*x + 567316224) - 1134632448*x + 799200000) + 703138063*411573
8902305032^(1/4)*sqrt(341)*(12162569*x^7 - 186616851*x^6 + 985490056*x^5 -
2246141620*x^4 + 2900382048*x^3 - 1823848416*x^2 - sqrt(2)*(8564099*x^7 - 1
31508024*x^6 + 695288980*x^5 - 1587105104*x^4 + 2050714080*x^3 - 1296806400
*x^2 - 1457077248*x + 1033108992) - 2066217984*x + 1457077248))*sqrt(2*x^2
- x + 3)*sqrt(6414867847*sqrt(2) + 9072749200) - 108911874586410593111333302
222822312*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x
^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7
+ 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*
x - 539136) + 1154304*x - 456192) - 2*sqrt(45363746/479849)*(sqrt(22681873)
*(11*4115738902305032^(3/4)*sqrt(341)*(576322648*x^7 - 827050092*x^6 + 2660
713572*x^5 - 1032439232*x^4 + 1211604768*x^3 + 1213394688*x^2 - sqrt(2)*(40
3157522*x^7 - 578844217*x^6 + 1864129347*x^5 - 735062160*x^4 + 873708120*x^
3 + 823986432*x^2 - 823986432*x) - 1213394688*x) + 703138063*41157389023050
32^(1/4)*sqrt(341)*(43684647*x^7 - 565067708*x^6 + 2178643220*x^5 - 2819241
792*x^4 + 3618371808*x^3 + 2197767168*x^2 - 2*sqrt(2)*(15328963*x^7 - 19829
0348*x^6 + 764653220*x^5 - 990717120*x^4 + 1276256160*x^3 + 755350272*x^2 -
 755350272*x) - 2197767168*x))*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2)
+ 9072749200) - 168363055004367262339322*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3}\,(5x^2 + 3x + 2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2\,x^2-x+3}\,\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3),x)
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3), x)
```

3.86 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}}-\frac{31009685\sqrt{3-x+2x^2}}{65536}-\frac{8992487x\sqrt{3-x+2x^2}}{16384}-\frac{111315x^2\sqrt{3-x+2x^2}}{2048}+\frac{7942}{512}$$

```
[Out] -310445587/262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-14641/1472*(101+79
*x)/(2*x^2-x+3)^(1/2)-31009685/65536*(2*x^2-x+3)^(1/2)-8992487/16384*x*(2*x
^2-x+3)^(1/2)-111315/2048*x^2*(2*x^2-x+3)^(1/2)+79425/512*x^3*(2*x^2-x+3)^(
1/2)+10075/96*x^4*(2*x^2-x+3)^(1/2)+625/24*x^5*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1674, 1675, 654, 633, 221}

$$-\frac{111315\sqrt{2x^2-x+3}\,x^2}{2048}-\frac{8992487\sqrt{2x^2-x+3}\,x}{16384}-\frac{31009685\sqrt{2x^2-x+3}}{65536}-\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}+\frac{625}{24}\sqrt{2x^2-x+3}\,x^5+\frac{10075}{96}\sqrt{2x^2-x+3}\,x^4+\frac{79425}{512}\sqrt{2x^2-x+3}\,x^3-\frac{310445587\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) - (31009685*Sqrt[3 - x + 2
*x^2])/65536 - (8992487*x*Sqrt[3 - x + 2*x^2])/16384 - (111315*x^2*Sqrt[3 -
 x + 2*x^2])/2048 + (79425*x^3*Sqrt[3 - x + 2*x^2])/512 + (10075*x^4*Sqrt[3
 - x + 2*x^2])/96 + (625*x^5*Sqrt[3 - x + 2*x^2])/24 - (310445587*ArcSinh[(
1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{2821893}{256} - \frac{661181x}{128} - \frac{488267x^2}{64} + \frac{143635x^3}{32} + \frac{213325x^4}{16} + 83}{\sqrt{3-x+2x^2}}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{625}{24}x^5\sqrt{3-x+2x^2} + \frac{1}{138}\int \frac{\frac{8465679}{64} - \frac{1983543x}{32} - \frac{1464801x}{16}}{\sqrt{3}}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x^5\sqrt{3-x+2x^2} + \frac{\int \frac{4232839}{32}}{}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{1007}{96}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79}{5}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{11}{}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{11}{}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{11}{}$$

Mathematica [A]
time = 0.90, size = 85, normalized size = 0.51

$$\frac{\frac{4(-10961697147-8859305979x-2534760678x^2-2613624504x^3+230669760x^4+1281670400x^5+831385600x^6+235520000x^7)}{\sqrt{3-x+2x^2}} - 21420745503\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{18087936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] ((4*(-10961697147 - 8859305979*x - 2534760678*x^2 - 2613624504*x^3 + 230669
760*x^4 + 1281670400*x^5 + 831385600*x^6 + 235520000*x^7))/Sqrt[3 - x + 2*x
^2] - 21420745503*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/18087936
```

Maple [A]

time = 0.13, size = 166, normalized size = 1.00

method	result
risch	$\frac{235520000x^7+831385600x^6+1281670400x^5+230669760x^4-2613624504x^3-2534760678x^2-8859305979x-10961697147}{4521984\sqrt{2x^2-x+3}}+\frac{310445}{}$
trager	$\frac{235520000x^7+831385600x^6+1281670400x^5+230669760x^4-2613624504x^3-2534760678x^2-8859305979x-10961697147}{4521984\sqrt{2x^2-x+3}}+\frac{310445}{}$
default	$\frac{\frac{1234044515x}{3014656}-\frac{1234044515}{12058624}}{\sqrt{2x^2-x+3}}+\frac{8825x^6}{48\sqrt{2x^2-x+3}}+\frac{217675x^5}{768\sqrt{2x^2-x+3}}+\frac{625x^7}{12\sqrt{2x^2-x+3}}+\frac{52235x^4}{1024\sqrt{2x^2-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1234044515/12058624*(4*x-1)/(2*x^2-x+3)^(1/2)+8825/48*x^6/(2*x^2-x+3)^(1/2)
+217675/768*x^5/(2*x^2-x+3)^(1/2)+625/12*x^7/(2*x^2-x+3)^(1/2)+52235/1024*x
^4/(2*x^2-x+3)^(1/2)-4734827/8192*x^3/(2*x^2-x+3)^(1/2)-18367831/32768*x^2/
(2*x^2-x+3)^(1/2)-310445587/131072*x/(2*x^2-x+3)^(1/2)-1217267299/524288/(2
*x^2-x+3)^(1/2)+310445587/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]

time = 0.53, size = 148, normalized size = 0.89

$$\frac{625\,x^7}{12\,\sqrt{2\,x^2-x+3}}+\frac{8825\,x^6}{48\,\sqrt{2\,x^2-x+3}}+\frac{217675\,x^5}{768\,\sqrt{2\,x^2-x+3}}+\frac{52235\,x^4}{1024\,\sqrt{2\,x^2-x+3}}-\frac{4734827\,x^3}{8192\,\sqrt{2\,x^2-x+3}}-\frac{18367831\,x^2}{32768\,\sqrt{2\,x^2-x+3}}+\frac{310445587}{262144}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{2953101993\,x}{1507328\,\sqrt{2\,x^2-x+3}}-\frac{3653899049}{1507328\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 625/12*x^7/sqrt(2*x^2 - x + 3) + 8825/48*x^6/sqrt(2*x^2 - x + 3) + 217675/7
68*x^5/sqrt(2*x^2 - x + 3) + 52235/1024*x^4/sqrt(2*x^2 - x + 3) - 4734827/8
192*x^3/sqrt(2*x^2 - x + 3) - 18367831/32768*x^2/sqrt(2*x^2 - x + 3) + 3104
45587/262144*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 2953101993/1507328*
x/sqrt(2*x^2 - x + 3) - 3653899049/1507328/sqrt(2*x^2 - x + 3)
```

Fricas [A]

time = 2.10, size = 112, normalized size = 0.67

$$\frac{21420745503\,\sqrt{2}\,(2\,x^2-x+3)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\,(235520000\,x^7+831385600\,x^6+1281670400\,x^5+230669760\,x^4-2613624504\,x^3-2534760678\,x^2-8859305979\,x-10961697147)\sqrt{2\,x^2-x+3}}{36175872\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/36175872*(21420745503*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(235520000*x^7 + 831385600*x^6
+ 1281670400*x^5 + 230669760*x^4 - 2613624504*x^3 - 2534760678*x^2 - 885930
5979*x - 10961697147)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(3/2), x)
```

Giac [A]
time = 2.70, size = 82, normalized size = 0.49

$$-\frac{310445587}{262144}\sqrt{2}\,\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{(46\,(4\,(40\,(20\,(16\,(100\,x+353)x+8707)x+31341)x-14204481)x-55103493)x-8859305979)x-10961697147}{4521984\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -310445587/262144*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
+ 1) + 1/4521984*((46*(4*(40*(20*(16*(100*x + 353)*x + 8707)*x + 31341)*x -
 14204481)*x - 55103493)*x - 8859305979)*x - 10961697147)/sqrt(2*x^2 - x +
3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^4}{{\left(2\,x^2-x+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2), x)
```

3.87 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}} \, dx$

Optimal. Leaf size=124

$$-\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}}-\frac{181561\sqrt{3-x+2x^2}}{2048}+\frac{15565}{512}x\sqrt{3-x+2x^2}+\frac{1825}{64}x^2\sqrt{3-x+2x^2}+\frac{125}{16}x^3\sqrt{3-$$

```
[Out] 1168881/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1331/368*(17-45*x)/(2*x
^2-x+3)^(1/2)-181561/2048*(2*x^2-x+3)^(1/2)+15565/512*x*(2*x^2-x+3)^(1/2)+1
825/64*x^2*(2*x^2-x+3)^(1/2)+125/16*x^3*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{1825}{64}\sqrt{2x^2-x+3}\,x^2+\frac{15565}{512}\sqrt{2x^2-x+3}\,x-\frac{181561\sqrt{2x^2-x+3}}{2048}-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}+\frac{125}{16}\sqrt{2x^2-x+3}\,x^3+\frac{1168881\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) - (181561*Sqrt[3 - x + 2*x^2]
)/2048 + (15565*x*Sqrt[3 - x + 2*x^2])/512 + (1825*x^2*Sqrt[3 - x + 2*x^2])
/64 + (125*x^3*Sqrt[3 - x + 2*x^2])/16 + (1168881*ArcSinh[(1 - 4*x)/Sqrt[23
]])/(4096*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{110285}{64}-\frac{19067x}{32}+\frac{22195x^2}{16}+\frac{13225x^3}{8}+\frac{2875x^4}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{125}{16}x^3\sqrt{3-x+2x^2} + \frac{1}{92}\int \frac{-\frac{110285}{8}-\frac{19067x}{4}+\frac{18515x^2}{4}+\cdots}{\sqrt{3-x+2x^2}}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \frac{125}{16}x^3\sqrt{3-x+2x^2} + \frac{1}{552}\int \cdots$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \frac{125}{16}x^3\sqrt{\cdots}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{\cdots}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{\cdots}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{\cdots}$$

Mathematica [A]
time = 0.64, size = 75, normalized size = 0.60

$$\frac{\frac{4(-15423965+16138403x-5754186x^2+2624760x^3+2318400x^4+736000x^5)}{\sqrt{3-x+2x^2}}+26884263\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{188416}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
[Out] ((4*(-15423965 + 16138403*x - 5754186*x^2 + 2624760*x^3 + 2318400*x^4 + 736
000*x^5))/Sqrt[3 - x + 2*x^2] + 26884263*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2
*x + 4*x^2]])/188416
```

Maple [A]
time = 0.13, size = 132, normalized size = 1.06

method	result
risch	$\frac{736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965}{47104\sqrt{2x^2-x+3}}-\frac{1168881\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$
trager	$\frac{736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965}{47104\sqrt{2x^2-x+3}}-\frac{1168881\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x-\right.}{8192}$
default	$\frac{\frac{5392543x}{94208}-\frac{5392543}{376832}}{\sqrt{2x^2-x+3}}+\frac{125x^5}{8\sqrt{2x^2-x+3}}+\frac{1575x^4}{32\sqrt{2x^2-x+3}}+\frac{14265x^3}{256\sqrt{2x^2-x+3}}-\frac{125091x^2}{1024\sqrt{2x^2-x+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
[Out] 5392543/376832*(4*x-1)/(2*x^2-x+3)^(1/2)+125/8*x^5/(2*x^2-x+3)^(1/2)+1575/3
2*x^4/(2*x^2-x+3)^(1/2)+14265/256*x^3/(2*x^2-x+3)^(1/2)-125091/1024*x^2/(2*
x^2-x+3)^(1/2)+1168881/4096*x/(2*x^2-x+3)^(1/2)-5130399/16384/(2*x^2-x+3)^(
1/2)-1168881/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.51, size = 114, normalized size = 0.92

$$\frac{125\,x^5}{8\,\sqrt{2\,x^2-x+3}}+\frac{1575\,x^4}{32\,\sqrt{2\,x^2-x+3}}+\frac{14265\,x^3}{256\,\sqrt{2\,x^2-x+3}}-\frac{125091\,x^2}{1024\,\sqrt{2\,x^2-x+3}}-\frac{1168881}{8192}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{16138403\,x}{47104\,\sqrt{2\,x^2-x+3}}-\frac{15423965}{47104\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
[Out] 125/8*x^5/sqrt(2*x^2 - x + 3) + 1575/32*x^4/sqrt(2*x^2 - x + 3) + 14265/256
*x^3/sqrt(2*x^2 - x + 3) - 125091/1024*x^2/sqrt(2*x^2 - x + 3) - 1168881/81
```

```
92*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16138403/47104*x/sqrt(2*x^2 -
 x + 3) - 15423965/47104/sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 1.78, size = 102, normalized size = 0.82

$$\frac{26884263 \sqrt{2} (2 x^2 - x + 3) \log \left(4 \sqrt{2} \sqrt{2 x^2 - x + 3} (4 x - 1) - 32 x^2 + 16 x - 25\right) + 8 (736000 x^5 + 2318400 x^4 + 2624760 x^3 - 5754186 x^2 + 16138403 x - 15423965) \sqrt{2 x^2 - x + 3}}{376832 (2 x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/376832*(26884263*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3
)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(736000*x^5 + 2318400*x^4 + 2624760*x
^3 - 5754186*x^2 + 16138403*x - 15423965)*sqrt(2*x^2 - x + 3))/(2*x^2 - x +
 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)
```

Giac [A]
time = 4.18, size = 72, normalized size = 0.58

$$\frac{1168881}{8192} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2} x - \sqrt{2 x^2 - x + 3}\right) + 1\right) + \frac{(46 (20 (40 (20 x + 63) x + 2853) x - 125091) x + 16138403) x - 15423965}{47104 \sqrt{2 x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1168881/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
+ 1/47104*((46*(20*(40*(20*x + 63)*x + 2853)*x - 125091)*x + 16138403)*x -
15423965)/sqrt(2*x^2 - x + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5 x^2 + 3 x + 2)^3}{(2 x^2 - x + 3)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2), x)
```

3.88 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}} \, dx$

Optimal. Leaf size=82

$$\frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{415}{32}\sqrt{3-x+2x^2}+\frac{25}{8}x\sqrt{3-x+2x^2}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

```
[Out] -223/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+121/92*(19-7*x)/(2*x^2-x+3)
^(1/2)+415/32*(2*x^2-x+3)^(1/2)+25/8*x*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}+\frac{25}{8}x\sqrt{2x^2-x+3}+\frac{415}{32}\sqrt{2x^2-x+3}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (415*Sqrt[3 - x + 2*x^2])/32 +
(25*x*Sqrt[3 - x + 2*x^2])/8 - (223*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2
])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{1173}{16}+\frac{1955x}{8}+\frac{575x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{1}{46}\int \frac{-138+\frac{9545x}{8}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{223}{64}\int \frac{1}{\sqrt{3-x+2}}$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{223\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right)}{64}$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} - \frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.}{64\sqrt{2}}$$

Mathematica [A]
time = 0.49, size = 65, normalized size = 0.79

$$\frac{47027-9421x+16790x^2+4600x^3}{736\sqrt{3-x+2x^2}} - \frac{223\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (47027 - 9421*x + 16790*x^2 + 4600*x^3)/(736*Sqrt[3 - x + 2*x^2]) - (223*Lo
g[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(64*Sqrt[2])
```

Maple [A]
time = 0.13, size = 98, normalized size = 1.20

method	result
risch	$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}+\frac{223\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{128}$
trager	$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}+\frac{223\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x-\mathrm{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-x+3}\right.}{128}$
default	$\frac{25x^3}{4\sqrt{2x^2-x+3}}+\frac{365x^2}{16\sqrt{2x^2-x+3}}-\frac{223x}{64\sqrt{2x^2-x+3}}+\frac{15761}{256\sqrt{2x^2-x+3}}-\frac{13713(4x-1)}{5888\sqrt{2x^2-x+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/4*x^3/(2*x^2-x+3)^(1/2)+365/16*x^2/(2*x^2-x+3)^(1/2)-223/64*x/(2*x^2-x+3
)^(1/2)+15761/256/(2*x^2-x+3)^(1/2)-13713/5888*(4*x-1)/(2*x^2-x+3)^(1/2)+22
3/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.50, size = 80, normalized size = 0.98

$$\frac{25\,x^3}{4\,\sqrt{2\,x^2-x+3}}+\frac{365\,x^2}{16\,\sqrt{2\,x^2-x+3}}+\frac{223}{128}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{9421\,x}{736\,\sqrt{2\,x^2-x+3}}+\frac{47027}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 25/4*x^3/sqrt(2*x^2 - x + 3) + 365/16*x^2/sqrt(2*x^2 - x + 3) + 223/128*sqr
t(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 9421/736*x/sqrt(2*x^2 - x + 3) + 47
027/736/sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 1.75, size = 92, normalized size = 1.12

$$\frac{5129\,\sqrt{2}\,(2\,x^2-x+3)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\,(4600\,x^3+16790\,x^2-9421\,x+47027)\sqrt{2\,x^2-x+3}}{5888\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(5129*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*
x - 1) - 32*x^2 + 16*x - 25) + 8*(4600*x^3 + 16790*x^2 - 9421*x + 47027)*sq
rt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(3/2), x)
```

Giac [A]
time = 2.51, size = 62, normalized size = 0.76

$$-\frac{223}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x - \sqrt{2\,x^2 - x + 3}\right) + 1\right) + \frac{(230\,(20\,x + 73)x - 9421)x + 47027}{736\,\sqrt{2\,x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -223/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
736*((230*(20*x + 73)*x - 9421)*x + 47027)/sqrt(2*x^2 - x + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2 + 3\,x + 2\right)}^2}{{\left(2\,x^2 - x + 3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2), x)
```

3.89 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

```
[Out] -5/4*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-11/23*(5+3*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1674, 12, 633, 221}

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
```

```
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{115}{4\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{5}{2}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{2\sqrt{46}} \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 55, normalized size = 1.22

$$-\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}-\frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*Log[1 - 4*x + 2*Sqrt[6 - 2*x
+ 4*x^2]])/(2*Sqrt[2])
```

Maple [A]
time = 0.10, size = 64, normalized size = 1.42

method	result	size
risch	$-\frac{11(5+3x)}{23\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{4}$	35

trager	$-\frac{11(5+3x)}{23\sqrt{2x^2-x+3}}-\frac{5\,\mathrm{RootOf}(\_Z^2-2)\ln\left(-4\,\mathrm{RootOf}(\_Z^2-2)x+4\sqrt{2x^2-x+3}+\mathrm{RootOf}(\_Z^2-2)\right)}{4}$	60
default	$-\frac{5x}{2\sqrt{2x^2-x+3}}-\frac{17}{8\sqrt{2x^2-x+3}}+\frac{\frac{49x}{46}-\frac{49}{184}}{\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{4}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/2*x/(2*x^2-x+3)^(1/2)-17/8/(2*x^2-x+3)^(1/2)+49/184*(4*x-1)/(2*x^2-x+3)^
(1/2)+5/4*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.50, size = 46, normalized size = 1.02

$$\frac{5}{4}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{33\,x}{23\,\sqrt{2\,x^2-x+3}}-\frac{55}{23\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 33/23*x/sqrt(2*x^2 - x + 3)
- 55/23/sqrt(2*x^2 - x + 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.
time = 1.68, size = 82, normalized size = 1.82

$$\frac{115\,\sqrt{2}\,(2\,x^2-x+3)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)-88\,\sqrt{2\,x^2-x+3}\,(3\,x+5)}{184\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/184*(115*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) - 88*sqrt(2*x^2 - x + 3)*(3*x + 5))/(2*x^2 - x +
 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{5x^2+3x+2}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

Giac [**A**]
time = 3.51, size = 53, normalized size = 1.18

$$-\frac{5}{4}\sqrt{2}\,\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{11\,(3\,x+5)}{23\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -5/4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/23*
(3*x + 5)/sqrt(2*x^2 - x + 3)
```

Mupad [**B**]
time = 0.23, size = 87, normalized size = 1.93

$$\frac{5\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{4}+\frac{3\,(2\,x-12)}{23\,\sqrt{2\,x^2-x+3}}-\frac{10\,\left(\frac{11\,x}{2}+\frac{3}{2}\right)}{23\,\sqrt{2\,x^2-x+3}}+\frac{16\,x-4}{23\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] (5*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/4 + (3*(2*
x - 12))/(23*(2*x^2 - x + 3)^(1/2)) - (10*((11*x)/2 + 3/2))/(23*(2*x^2 - x
+ 3)^(1/2)) + (16*x - 4)/(23*(2*x^2 - x + 3)^(1/2))
```

3.90 $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx$

Optimal. Leaf size=176

$$\frac{13-6x}{253\sqrt{3-x+2x^2}}+\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(61+4\sqrt{2}+\left(69+65\sqrt{\right.\right.}}{\sqrt{3-x+2x^2}}\right.$$

```
[Out] 1/253*(13-6*x)/(2*x^2-x+3)^(1/2)-1/15004*arctanh(1/31*(61+x*(69-65*2^(1/2))
-4*2^(1/2))*341^(1/2)/(-247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-168454+
341000*2^(1/2))^(1/2)+1/15004*arctan(1/31*(61+4*2^(1/2)+x*(69+65*2^(1/2)))*
341^(1/2)/(247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(168454+341000*2^(1/2)
)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {988, 1049, 1043, 212, 210}

$$\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{253\sqrt{2x^2-x+3}}-\frac{1}{22}\sqrt{\frac{1}{682}\left(500\sqrt{2}-247\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(500\sqrt{2}-247\right)}}\left(\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcTa
n[(Sqrt[11/(31*(247 + 500*Sqrt[2]))]*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*x)
)/Sqrt[3 - x + 2*x^2]])/22 - (Sqrt[(-247 + 500*Sqrt[2])/682]*ArcTanh[(Sqrt[
11/(31*(-247 + 500*Sqrt[2]))]*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x))/Sqrt[
3 - x + 2*x^2]])/22
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx &= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{\int \frac{-1012-\frac{1265x}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx}{2783} \\ &= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{\int \frac{\frac{2783}{2}\left(3+8\sqrt{2}\right)-\frac{2783}{2}\left(13-5\sqrt{2}\right)x}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx}{61226\sqrt{2}} - \frac{\int \frac{\frac{2783}{2}\left(\cdots\right.}{\sqrt{\cdots}}}{\cdots} \\ &= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{1}{8}\left(253\left(1000-247\sqrt{2}\right)\right)\operatorname{Subst}\left(\int \frac{\cdots}{-\frac{240097\cdots}{4}}\cdots\right. \\ &= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{\cdots}{31\left(2\cdots\right.}}}{\cdots}\right.\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.35, size = 199, normalized size = 1.13

$$\frac{13-6x}{253\sqrt{3-x+2x^2}}+\frac{1}{22}\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{23\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+16\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1-5\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^
2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (23*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
- #1] + 16*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 5*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqr
t[2]*#1^2 - 10*#1^3) & ]/22
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(128) = 256$.
time = 0.64, size = 718, normalized size = 4.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/22/(2*x^2-x+3)^(1/2)-3/506*(4*x-1)/(2*x^2-x+3)^(1/2)+1/465124*(8*(2^(1/2)
-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/
2))^(1/2)*2^(1/2)*(2197*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/1169248
```

```
7*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+
1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1
/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-
1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+3070*(-886
6+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23
*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(
6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-7
75687+549362*2^(1/2))^(1/2)+1712502*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2
)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-886
6+6820*2^(1/2))^(1/2))*2^(1/2)-6617446*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(
1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-
8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^
2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))
^(1/2)
```

<u>Maxima</u> **[F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(3/2)), x)
```

<u>Fricas</u> **[B]** Leaf count of result is larger than twice the leaf count of optimal. 2083 vs. $2(128) = 256$.
time = 2.80, size = 2083, normalized size = 11.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
[Out] -1/50921775520*(339388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*
sqrt(247*sqrt(2) + 1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(
2*x^2 - x + 3)*(22*50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x
^4 + 235088*x^3 - 46944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5
- 427391*x^4 + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 17
2800) + 5*50^(1/4)*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 +
 24624288*x^3 - 10862496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^
5 - 12895624*x^4 + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19
```

989504*x + 10533888))*sqrt(247*sqrt(2) + 1000) + 933317000*sqrt(31)*sqrt(2)
*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*
x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5
+ 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x -
456192) - 2*sqrt(310/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3
/4)*(246848*x^7 - 348192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 10298
88*x^2 - sqrt(2)*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*
x^3 - 269568*x^2 + 269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 667694
8*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(
2)*(38689*x^7 - 502244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689
984*x^2 + 1689984*x) - 46199808*x))*sqrt(247*sqrt(2) + 1000) + 65450*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) + 2975*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*
x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 -
 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1
44820224*x))*sqrt(-(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 81
1580*sqrt(2)*(2*x^2 - x + 3) + 2785195*x - 3689000)/x^2) + 10605875*sqrt(31
)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4
 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*
x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064
064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 -
 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 339
388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*sqrt(247*sqrt(2) +
1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*
50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 4
6944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 2343
60*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^(1/4)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 +
17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888
))*sqrt(247*sqrt(2) + 1000) - 933317000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(31
0/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(246848*x^7 - 3
48192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - sqrt(2)*(4
6522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 +
269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 6676948*x^6 + 25569820*x^
5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(2)*(38689*x^7 - 502
244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689984*x^2 + 1689984*x
) - 46199808*x))*sqrt(247*sqrt(2) + 1000) - 65450*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38

```
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 2975*sqrt(
31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4
 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 -
 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((
sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 3
8) + x - 75)*sqrt(247*sqrt(2) + 1000) + 903805*x^2 + 811580*sqrt(2)*(2*x^2
- x + 3) - 2785195*x + 3689000)/x^2) - 10605875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 23*sqrt(341)*50^(1/4)*s
qrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} \cdot (5x^2 + 3x + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^2+3\,x+2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)),x)
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)), x)
```

3.91 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=211

$$-\frac{6315-2306x}{345092\sqrt{3-x+2x^2}}+\frac{4+65x}{682\sqrt{3-x+2x^2}\,(2+3x+5x^2)}+\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\,\tan^{-1}}{}$$

```
[Out] 1/345092*(-6315+2306*x)/(2*x^2-x+3)^(1/2)+1/682*(4+65*x)/(5*x^2+3*x+2)/(2*x
^2-x+3)^(1/2)-1/20465456*arctanh(1/31*(12611+x*(45519-29065*2^(1/2))-16454*
2^(1/2))*341^(1/2)/(-129694447+103775000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*
(-88451612854+70774550000*2^(1/2))^(1/2)+1/20465456*arctan(1/31*(12611+1645
4*2^(1/2)+x*(45519+29065*2^(1/2)))*341^(1/2)/(129694447+103775000*2^(1/2))^
(1/2)/(2*x^2-x+3)^(1/2))*(88451612854+70774550000*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {988, 1074, 1049, 1043, 212, 210}

$$\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(129694447+103775000\sqrt{2}\right)}}\left(\left(45519+29065\sqrt{2}\right)x+16454\sqrt{2}+12611\right)}{\sqrt{2x^2-x+3}}\right)}{30008}-\frac{6315-2306x}{345092\sqrt{2x^2-x+3}}+\frac{65x+4}{682\sqrt{2x^2-x+3}\,(5x^2+3x+2)}-\frac{\sqrt{\frac{1}{682}\left(103775000\sqrt{2}-129694447\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(103775000\sqrt{2}-129694447\right)}}\left(\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611\right)}{\sqrt{2x^2-x+3}}\right)}{30008}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -1/345092*(6315 - 2306*x)/Sqrt[3 - x + 2*x^2] + (4 + 65*x)/(682*Sqrt[3 - x
+ 2*x^2]*(2 + 3*x + 5*x^2)) + (Sqrt[(129694447 + 103775000*Sqrt[2])/682]*Ar
cTan[(Sqrt[11/(31*(129694447 + 103775000*Sqrt[2]))]*(12611 + 16454*Sqrt[2]
+ (45519 + 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/30008 - (Sqrt[(-1296944
47 + 103775000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-129694447 + 103775000*S
qrt[2]))]*(12611 - 16454*Sqrt[2] + (45519 - 29065*Sqrt[2])*x))/Sqrt[3 - x +
 2*x^2]])/30008
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx = \frac{4+65x}{682\sqrt{3-x+2x^2}\,(2+3x+5x^2)} - \frac{\int \frac{-1782+\frac{3333x}{2}-2860x^2}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx}{7502}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\,(2+3x+5x^2)} - \frac{\int \cdots}{\cdots}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\,(2+3x+5x^2)} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\,(2+3x+5x^2)} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\,(2+3x+5x^2)} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.67, size = 414, normalized size = 1.96

$$\frac{\sqrt{3-x+2x^2}\,(-10606+18557x-24657x^2+11530x^3)}{345092\,(6+7x+16x^2+x^3+10x^4)} - \frac{1}{484}\text{RootSum}\left[-56-26\sqrt{2}\,\#1+17\#1^2+6\sqrt{2}\,\#1^3-5\#1^4\&, \frac{225\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)+8\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1-15\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\,\#1^2-10\#1^3}\&\right] + \frac{\text{RootSum}\left[-56-26\sqrt{2}\,\#1+17\#1^2+6\sqrt{2}\,\#1^3-5\#1^4\&, \frac{8623\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)+9624\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1+1565\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\,\#1^2-10\#1^3}\&\right]}{30008\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-10606 + 18557*x - 24657*x^2 + 11530*x^3))/(345092*(6
 + 7*x + 16*x^2 + x^3 + 10*x^4)) - RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 +
6*Sqrt[2]*#1^3 - 5*#1^4 & , (225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #
1] + 8*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 15*Log[-(S
qrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2
]*#1^2 - 10*#1^3) & ]/484 + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[
2]*#1^3 - 5*#1^4 & , (8623*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1] + 9624*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1565*Sqrt[2]*
Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9
*Sqrt[2]*#1^2 - 10*#1^3) & ]/(30008*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5941 vs. $2(159) = 318$.

time = 0.70, size = 5942, normalized size = 28.16

method	result
trager	Expression too large to display
risch	$\frac{11530x^3-24657x^2+18557x-10606}{345092(5x^2+3x+2)\sqrt{2x^2-x+3}} + \frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(1173047\sqrt{\right.}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2173 vs. $2(159) = 318$.

time = 2.81, size = 2173, normalized size = 10.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

[Out] 1/29889247038841109870720*(35183643812*3446160200^(1/4)*sqrt(20755)*sqrt(34
1)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sqrt(2) + 20755
0000)*arctan(1/2437871055247532640924125*(59193260*sqrt(20755)*(11*34461602
00^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^5 - 311740976
*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 - 65709803*x^6
+ 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2 - 126316800*
x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(1/4)*sqrt(341
)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4 + 425065248*x
^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106575580*x^5 -
236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x + 143534592) -
287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 2075
50000) + 6920408156831705561333000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7
 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2
)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x
^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(41510/3
97951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x^7 - 96938292
*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 38700288*x^2 - sqrt(
2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^4 + 79054920*
x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446160200^(1/4)*
sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358592*x^4 + 506
287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948*x^6 + 128512
220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 173705472*x) - 14
4903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 207550000) + 11691
2097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329
3072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118
051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2
 - 1036800*x) + 3276288*x) + 5314186228782025*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(3446160200^(1/4)*sqrt(2
0755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 18
949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 - 3
49243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 1587470711561
000)/x^2) + 78641001782178472287875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 5
3385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2
 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 55
68*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8
 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3
4615296*x^2 - 24772608*x + 18579456)) + 35183643812*3446160200^(1/4)*sqrt(2
0755)*sqrt(341)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sq
rt(2) + 207550000)*arctan(1/2437871055247532640924125*(59193260*sqrt(20755)
*(11*3446160200^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^
5 - 311740976*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 -
65709803*x^6 + 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2
 - 126316800*x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(

```
1/4)*sqrt(341)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4
+ 425065248*x^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106
575580*x^5 - 236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x +
143534592) - 287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*s
qrt(2) + 207550000) - 6920408156831705561333000*sqrt(31)*sqrt(2)*(28180*x^8
 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496
*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x
^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2
*sqrt(41510/397951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x
^7 - 96938292*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 3870028
8*x^2 - sqrt(2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^
4 + 79054920*x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446
160200^(1/4)*sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358
592*x^4 + 506287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948
*x^6 + 128512220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 1737
05472*x) - 144903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 20755
0000) - 116912097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578
888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15
550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
+ 1209600*x^2 - 1036800*x) + 3276288*x) - 5314186228782025*sqrt(31)*(254591
*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328
*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 7...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2),x)
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2), x)
```

3.92 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}}+\frac{4+65x}{1364\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}+\frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2}\,(2+3x+5x^2)}-$$

```
[Out] 1/941410976*(-4353943+6508666*x)/(2*x^2-x+3)^(1/2)+1/1364*(4+65*x)/(5*x^2+3
*x+2)^2/(2*x^2-x+3)^(1/2)+5/1860496*(7318+17315*x)/(5*x^2+3*x+2)/(2*x^2-x+3
)^(1/2)-3/55829763968*arctanh(1/31*(5538393+x*(13785797-9662095*2^(1/2))-41
23702*2^(1/2))*341^(1/2)/(-13874275807943+9819738650000*2^(1/2))^(1/2)/(2*x
^2-x+3)^(1/2))*(-9462256101017126+6697061759300000*2^(1/2))^(1/2)+3/5582976
3968*arctan(1/31*(5538393+4123702*2^(1/2)+x*(13785797+9662095*2^(1/2)))*341
^(1/2)/(13874275807943+9819738650000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(946
2256101017126+6697061759300000*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {988, 1074, 1049, 1043, 212, 210}

$$\frac{3\sqrt{\frac{1}{682}\left(13874275807943+9819738650000\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(13874275807943+9819738650000\sqrt{2}\right)}}\left(\left(13785797+9662095\sqrt{2}\right)x+4123702\sqrt{2}+5538393\right)}{\sqrt{2x^2-x+3}}\right)}{81861824}-\frac{4353943-6508666x}{941410976\sqrt{2x^2-x+3}}+\frac{5(17315x+7318)}{1860496\sqrt{2x^2-x+3}\,(5x^2+3x+2)}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\,(5x^2+3x+2)^2}-\frac{3\sqrt{\frac{1}{682}\left(9819738650000\sqrt{2}-13874275807943\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(9819738650000\sqrt{2}-13874275807943\right)}}\left(\left(13785797-9662095\sqrt{2}\right)x-4123702\sqrt{2}+5538393\right)}{\sqrt{2x^2-x+3}}\right)}{81861824}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -1/941410976*(4353943 - 6508666*x)/Sqrt[3 - x + 2*x^2] + (4 + 65*x)/(1364*S
qrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + (5*(7318 + 17315*x))/(1860496*Sqr
t[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (3*Sqrt[(13874275807943 + 98197386500
00*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2
]))]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/81861824 - (3*Sqrt[(-13874275807943 + 9819738650000*Sqrt[2])/
682]*ArcTanh[(Sqrt[11/(31*(-13874275807943 + 9819738650000*Sqrt[2]))]*(5538
393 - 4123702*Sqrt[2] + (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/81861824
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
```

```
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5731+\frac{7557x}{2}-5720x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364\sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^2} + \frac{5(7318+17315x}{1860496\sqrt{3-x+2x^2}\,(2+}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^2}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^2}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^2}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.12, size = 607, normalized size = 2.47

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((4*(22374044 + 161806828*x + 175833195*x^2 + 277167774*x^3 + 86411405*x^4
+ 162716650*x^5))/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) - 176824*Sqrt[2
]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-491
*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 208*Log[-(Sqrt[2]*x
) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 5*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
+ 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] +
 124*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^
```

```
4 & , (7194481*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 37984
56*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 575915*Sqrt[2]*Log[-(S
qrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2
]*#1^2 - 10*#1^3) & ] - 7*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6
*Sqrt[2]*#1^3 - 5*#1^4 & , (143178771*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1] - 105962920*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1
 + 6180225*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*
Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/3765643904
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 18980 vs. $2(190) = 380$.
time = 0.86, size = 18981, normalized size = 77.16

method	result
trager	Expression too large to display
risch	$\frac{162716650x^5+86411405x^4+277167774x^3+175833195x^2+161806828x+22374044}{941410976(5x^2+3x+2)^2\sqrt{2x^2-x+3}} + \frac{3\sqrt{\frac{8(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}\,(\sqrt{2}-1-}{(\sqrt{2}+1-x)}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2263 vs. $2(190) = 380$.
time = 2.74, size = 2263, normalized size = 9.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/33652296632397026886019646994897920*(920746859815884*1928545343086076450^
(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^
3 + 83*x^2 + 32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arcta
n(1/225227015528909794375187692534722839169237 5*(2800589462980*sqrt(1963947
730)*(22*1928545343086076450^(3/4)*sqrt(341)*(7361410004*x^7 - 28555361914*
x^6 + 59872788262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2
 - sqrt(2)*(5311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 6800681868
3*x^4 + 33985514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) -
52334913024*x + 37489478400) + 30441189815*1928545343086076450^(1/4)*sqrt(3
41)*(560592897*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 +
134890825824*x^3 - 85859939808*x^2 - sqrt(2)*(402019087*x^7 - 6162703212*x^
6 + 32499503540*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 -
 68177562624*x + 47773380096) - 95546760192*x + 68177562624))*sqrt(2*x^2 -
x + 3)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 6393541085981955453 2
31134497759874144159000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x
^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8
 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144
*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(1963947730/3471424
919)*(sqrt(1963947730)*(22*1928545343086076450^(3/4)*sqrt(341)*(26184810824
*x^7 - 37618468196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x
^3 + 51583129344*x^2 - sqrt(2)*(19194187986*x^7 - 27528525721*x^6 + 8845761
3411*x^5 - 33685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 4176467481
6*x) - 51583129344*x) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(19
98926311*x^7 - 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 16744
6420704*x^3 + 96037622784*x^2 - 22*sqrt(2)*(65886479*x^7 - 852213084*x^6 +
3285070260*x^5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259
776*x) - 96037622784*x))*sqrt(2*x^2 - x + 3)*sqrt(13874275807943*sqrt(2) +
19639477300000) + 2282926923240949861309948624550*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 1037694056
01861357332270392025*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt(-(1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(
341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*
```

```
x - 1471975)*sqrt(13874275807943*sqrt(2) + 19639477300000) - 16051926912456
8199977215*x^2 - 14413975186695919997954O*sqrt(2)*(2*x^2 - x + 3) + 4946614
21179791799929785*x - 65518069030435999990700O)/x^2) + 72653875977067675604
8992556563622061836 25*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 -
142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)
*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*
x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7
+ 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 -
24772608*x + 18579456)) + 920746859815884*1928545343086076450^(1/4)*sqrt(19
63947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 +
32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arctan(1/225227015
52890979437518769253472283916923 75*(2800589462980*sqrt(1963947730)*(22*1928
545343086076450^(3/4)*sqrt(341)*(7361410004*x^7 - 28555361914*x^6 + 5987278
8262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2 - sqrt(2)*(5
311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 68006818683*x^4 + 33985
514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) - 52334913024*x
 + 37489478400) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(56059289
7*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 + 134890825824*
x^3 - 85859939808*x^2 - sqrt(2)*(402019087*x^7 - 6162703212*x^6 + 324995035
40*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 - 68177562624*
x + 47773380096) - 95546760192*x + 68177562624))*sqrt(2*x^2 - x + 3)*sqrt(1
3874275807943*sqrt(2) + 19639477300000) - 63935410859819554532311344977598 7
4144159000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*
x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7
 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048
*x - 539136) + 1154304*x - 456192) - 2*sqrt(1963947730/3471424919)*(sqrt(19
63947730)*(22*1928545343086076450^(3/4)*sqrt(341)*(26184810824*x^7 - 376184
68196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x^3 + 51583129
344*x^2 - sqrt(2)*(19194187986*x^7 - 27528525721*x^6 + 88457613411*x^5 - 33
685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 41764674816*x) - 515831
29344*x) + 30441189815*1928545343086076450^(1/4...
```

<u>**Sympy**</u> **[F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

<u>**Giac**</u> **[F(-2)]**

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3), x)
```

3.93 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}}+\frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}}-\frac{1308645\sqrt{3-x+2x^2}}{4096}+\frac{526075x\sqrt{3-x+2x^2}}{3072}+\frac{38375}{384}$$

```
[Out] -14641/4416*(101+79*x)/(2*x^2-x+3)^(3/2)+16955197/16384*arcsinh(1/23*(1-4*x
)*23^(1/2))*2^(1/2)+1331/101568*(7409+116368*x)/(2*x^2-x+3)^(1/2)-1308645/4
096*(2*x^2-x+3)^(1/2)+526075/3072*x*(2*x^2-x+3)^(1/2)+38375/384*x^2*(2*x^2-
x+3)^(1/2)+625/32*x^3*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{38375}{384}\sqrt{2x^2-x+3}\,x^2+\frac{526075\sqrt{2x^2-x+3}\,x}{3072}-\frac{1308645\sqrt{2x^2-x+3}}{4096}+\frac{1331(116368x+7409)}{101568\sqrt{2x^2-x+3}}-\frac{14641(79x+101)}{4416\,(2x^2-x+3)^{3/2}}+\frac{625}{32}\sqrt{2x^2-x+3}\,x^3+\frac{16955197\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + (1331*(7409 + 116368*x
))/(101568*Sqrt[3 - x + 2*x^2]) - (1308645*Sqrt[3 - x + 2*x^2])/4096 + (526
075*x*Sqrt[3 - x + 2*x^2])/3072 + (38375*x^2*Sqrt[3 - x + 2*x^2])/384 + (62
5*x^3*Sqrt[3 - x + 2*x^2])/32 + (16955197*ArcSinh[(1 - 4*x)/Sqrt[23]])/(819
2*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{2}{69}\int \frac{\frac{3839123}{256} - \frac{1983543x}{128} - \frac{1464801x^2}{64} + \frac{430905x^3}{32} + \frac{639975x^4}{16} \cdots}{(3-x+2x^2)^{3/2}}$$

$$= -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{4\int \frac{-\frac{141812733}{256} - \frac{1880595x}{16} + \frac{15512925x^2}{64} \cdots}{\sqrt{3-x+2\cdots}}}{1587}$$

$$= -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{625}{32}x^3\sqrt{3-x+2x^2} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{38375}{384}x^2\sqrt{3-x+2x^2} + \frac{62\cdots}{32\cdots}$$

$$= -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{526075x\sqrt{3-x+2x^2}}{3072} + \frac{38\cdots}{3\cdots}$$

$$= -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{52\cdots}{\cdots}$$

$$= -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{52\cdots}{\cdots}$$

$$= -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{52\cdots}{\cdots}$$

Mathematica [A]
time = 1.06, size = 85, normalized size = 0.58

$$\frac{-18974698519 + 49883864262x - 36481630395x^2 + 39848900984x^3 - 5076781260x^4 + 3504730800x^5 + 2090608000x^6 + 507840000x^7}{6500352\,(3-x+2x^2)^{3/2}} + \frac{16955197\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 5076781
260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 - x
+ 2*x^2)^(3/2)) + (16955197*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8192*S
qrt[2])
```

Maple [A]
time = 0.14, size = 214, normalized size = 1.46

method	result
risch	$\frac{507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+49883864262x-18974698519}{6500352(2x^2-x+3)^{\frac{3}{2}}} - \frac{16}{}$
trager	$\frac{507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+49883864262x-18974698519}{6500352(2x^2-x+3)^{\frac{3}{2}}} - \frac{16}{}$
default	$\frac{\frac{992926033x}{3250176}-\frac{992926033}{13000704}}{\sqrt{2x^2-x+3}} + \frac{138025x^5}{256(2x^2-x+3)^{\frac{3}{2}}} - \frac{799745x^4}{1024(2x^2-x+3)^{\frac{3}{2}}} + \frac{16955197x^3}{12288(2x^2-x+3)^{\frac{3}{2}}} - \frac{67488035x^2}{16384(2x^2-x+3)^{\frac{3}{2}}} + \frac{55167267}{131072(2x^2-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 992926033/13000704*(4*x-1)/(2*x^2-x+3)^(1/2)+138025/256*x^5/(2*x^2-x+3)^(3/
2)-799745/1024*x^4/(2*x^2-x+3)^(3/2)+16955197/12288*x^3/(2*x^2-x+3)^(3/2)-6
7488035/16384*x^2/(2*x^2-x+3)^(3/2)+55167267/131072*x/(2*x^2-x+3)^(3/2)+625
/8*x^7/(2*x^2-x+3)^(3/2)+30875/96*x^6/(2*x^2-x+3)^(3/2)+16955197/8192*x/(2*
x^2-x+3)^(1/2)+16955197/32768/(2*x^2-x+3)^(1/2)-2149616639/524288/(2*x^2-x+
3)^(3/2)+5141612725/36175872*(4*x-1)/(2*x^2-x+3)^(3/2)-16955197/16384*2^(1/
2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(118) = 236.
time = 0.51, size = 253, normalized size = 1.72

$$\frac{625\,x^7}{8\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{30875\,x^6}{96\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{138025\,x^5}{256\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{799745\,x^4}{1024\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{16955197}{13000704}x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right)-\frac{16955197}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{1203818987}{6500352}\sqrt{2\,x^2-x+3}+\frac{3536205583\,x}{3250176\,\sqrt{2\,x^2-x+3}}-\frac{2638851\,x^2}{512\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{257773037}{1083392\,\sqrt{2\,x^2-x+3}}+\frac{29484067\,x}{23552\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{374445479}{70656\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 625/8*x^7/(2*x^2 - x + 3)^(3/2) + 30875/96*x^6/(2*x^2 - x + 3)^(3/2) + 1380
25/256*x^5/(2*x^2 - x + 3)^(3/2) - 799745/1024*x^4/(2*x^2 - x + 3)^(3/2) -
16955197/13000704*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(
3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 -
 x + 3)^(3/2)) - 16955197/16384*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) +
1203818987/6500352*sqrt(2*x^2 - x + 3) + 3536205583/3250176*x/sqrt(2*x^2 -
x + 3) - 2638851/512*x^2/(2*x^2 - x + 3)^(3/2) + 257773037/1083392/sqrt(2*x
^2 - x + 3) + 29484067/23552*x/(2*x^2 - x + 3)^(3/2) - 374445479/70656/(2*x
^2 - x + 3)^(3/2)
```

Fricas [A]
time = 1.82, size = 132, normalized size = 0.90

$$\frac{26907897639 \sqrt{2} \left(4 x^{4}-4 x^{3}+13 x^{2}-6 x+9\right) \log \left(4 \sqrt{2} \sqrt{2 x^{2}-x+3} (4 x-1)-32 x^{2}+16 x-25\right)+8 \left(507840000 x^{7}+2090608000 x^{6}+3504730800 x^{5}-5076781260 x^{4}+39848900984 x^{3}-36481630395 x^{2}+49883864262 x-18974698519\right) \sqrt{2 x^{2}-x+3}}{52002816 \left(4 x^{4}-4 x^{3}+13 x^{2}-6 x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/52002816*(26907897639*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(4*sq
rt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(507840000*x^
7 + 2090608000*x^6 + 3504730800*x^5 - 5076781260*x^4 + 39848900984*x^3 - 36
481630395*x^2 + 49883864262*x - 18974698519)*sqrt(2*x^2 - x + 3))/(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^{2}+3x+2\right)^{4}}{\left(2x^{2}-x+3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(5/2), x)
```

Giac [A]
time = 5.79, size = 81, normalized size = 0.55

$$\frac{16955197}{16384} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^{2}-x+3}\right)+1\right)+\frac{((4 (2645 (20 (40 (60 x+247) x+16563) x-479847) x+9962225246) x-36481630395) x+49883864262) x-18974698519}{6500352 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] 16955197/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1
) + 1/6500352*(((4*(2645*(20*(40*(60*x + 247)*x + 16563)*x - 479847)*x + 99
62225246)*x - 36481630395)*x + 49883864262)*x - 18974698519)/(2*x^2 - x + 3
)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^4}{{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2), x)
```

3.94 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{1331(17-45x)}{1104\,(3-x+2x^2)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3-x+2x^2}-\frac{7495\sinh^{-1}\left(\frac{1-}{\sqrt{}}\right.}{128\sqrt{2}}$$

```
[Out] -1331/1104*(17-45*x)/(2*x^2-x+3)^(3/2)-7495/256*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)+121/8464*(10679-6744*x)/(2*x^2-x+3)^(1/2)+3175/64*(2*x^2-x+3)^(
1/2)+125/16*x*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{121(10679-6744x)}{8464\sqrt{2x^2-x+3}}+\frac{125}{16}x\sqrt{2x^2-x+3}+\frac{3175}{64}\sqrt{2x^2-x+3}-\frac{1331(17-45x)}{1104\,(2x^2-x+3)^{3/2}}-\frac{7495\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + (121*(10679 - 6744*x))/(
8464*Sqrt[3 - x + 2*x^2]) + (3175*Sqrt[3 - x + 2*x^2])/64 + (125*x*Sqrt[3 -
 x + 2*x^2])/16 - (7495*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{91275}{64}-\frac{57201x}{32}+\frac{66585x^2}{16}+\frac{39675x^3}{8}+\frac{8625x^4}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{1452105}{64}+\frac{277725x}{8}+\frac{198375x^2}{16}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{125}{16}x\sqrt{3-x+2x^2}+\frac{\int \frac{\frac{214245}{4}}{\sqrt{3-}}}{}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3}$$

Mathematica [A]
time = 0.79, size = 75, normalized size = 0.71

$$\frac{89784565-62463282x+101546529x^2-29423976x^3+16980900x^4+3174000x^5}{101568\,(3-x+2x^2)^{3/2}}-\frac{7495\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89784565 - 62463282*x + 101546529*x^2 - 29423976*x^3 + 16980900*x^4 + 3174
000*x^5)/(101568*(3 - x + 2*x^2)^(3/2)) - (7495*Log[1 - 4*x + 2*Sqrt[6 - 2*
x + 4*x^2]])/(128*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(84) = 168$.
time = 0.13, size = 180, normalized size = 1.71

method	result
risch	$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568(2x^2-x+3)^{\frac{3}{2}}}+\frac{7495\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\ (x-\frac{1}{4})}{23}\right)}{256}$
trager	$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568(2x^2-x+3)^{\frac{3}{2}}}+\frac{7495\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x-\right.}{256}$
default	$-\frac{3391139(4x-1)}{203136\sqrt{2x^2-x+3}}+\frac{125x^5}{4(2x^2-x+3)^{\frac{3}{2}}}+\frac{2675x^4}{16(2x^2-x+3)^{\frac{3}{2}}}-\frac{7495x^3}{192(2x^2-x+3)^{\frac{3}{2}}}+\frac{222809x^2}{256(2x^2-x+3)^{\frac{3}{2}}}-\frac{281177x}{2048(2x^2-x+3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3391139/203136*(4*x-1)/(2*x^2-x+3)^(1/2)+125/4*x^5/(2*x^2-x+3)^(3/2)+2675/
16*x^4/(2*x^2-x+3)^(3/2)-7495/192*x^3/(2*x^2-x+3)^(3/2)+222809/256*x^2/(2*x
^2-x+3)^(3/2)-281177/2048*x/(2*x^2-x+3)^(3/2)-7495/128*x/(2*x^2-x+3)^(1/2)-
7495/512/(2*x^2-x+3)^(1/2)+20961031/24576/(2*x^2-x+3)^(3/2)-14081711/565248
*(4*x-1)/(2*x^2-x+3)^(3/2)+7495/256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(84) = 168$.
time = 0.52, size = 219, normalized size = 2.09

$$\frac{125\,x^5}{4\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{2675\,x^4}{16\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{7495}{203136}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right)+\frac{7495}{256}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{532145}{101568}\sqrt{2\,x^2-x+3}-\frac{4515389\,x}{50784\,\sqrt{2\,x^2-x+3}}+\frac{7197\,x^2}{8\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{396211}{50784\,\sqrt{2\,x^2-x+3}}-\frac{269783\,x}{1104\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{1002137}{1104\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 125/4*x^5/(2*x^2 - x + 3)^(3/2) + 2675/16*x^4/(2*x^2 - x + 3)^(3/2) + 7495/
203136*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 7495/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 532145/101568*sqr
t(2*x^2 - x + 3) - 4515389/50784*x/sqrt(2*x^2 - x + 3) + 7197/8*x^2/(2*x^2
- x + 3)^(3/2) + 396211/50784/sqrt(2*x^2 - x + 3) - 269783/1104*x/(2*x^2 -
x + 3)^(3/2) + 1002137/1104/(2*x^2 - x + 3)^(3/2)
```

Fricas [A]
time = 1.76, size = 122, normalized size = 1.16

$$\frac{11894565\,\sqrt{2}\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\,(3174000\,x^5+16980900\,x^4-29423976\,x^3+101546529\,x^2-62463282\,x+89784565)\sqrt{2\,x^2-x+3}}{812544\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/812544*(11894565*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(3174000*x^5 + 16
980900*x^4 - 29423976*x^3 + 101546529*x^2 - 62463282*x + 89784565)*sqrt(2*x
^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^3}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(5/2), x)
```

Giac [A]
time = 4.29, size = 72, normalized size = 0.69

$$-\frac{7495}{256}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\,\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{3\,((4\,(13225\,(20\,x+107)x-2451998)x+33848843)x-20821094)x+89784565}{101568\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -7495/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/101568*(3*((4*(13225*(20*x + 107)*x - 2451998)*x + 33848843)*x - 20821094)
*x + 89784565)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^3}{{\left(2\,x^2-x+3\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2),x)
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2), x)
```

3.95 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{121(19-7x)}{276\,(3-x+2x^2)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

```
[Out] 121/276*(19-7*x)/(2*x^2-x+3)^(3/2)-25/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1
/2)-11/6348*(7351+2336*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1674, 12, 633, 221}

$$\frac{121(19-7x)}{276\,(2x^2-x+3)^{3/2}}-\frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) - (11*(7351 + 2336*x))/(6348*S
qrt[3 - x + 2*x^2]) - (25*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx &= \frac{121(19-7x)}{276\,(3-x+2x^2)^{3/2}}+\frac{2}{69}\int \frac{\frac{131}{16}+\frac{5865x}{8}+\frac{1725x^2}{4}}{(3-x+2x^2)^{3/2}}\,dx\\
&= \frac{121(19-7x)}{276\,(3-x+2x^2)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}+\frac{4\int \frac{39675}{16\sqrt{3-x+2x^2}}\,dx}{1587}\\
&= \frac{121(19-7x)}{276\,(3-x+2x^2)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}+\frac{25}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{121(19-7x)}{276\,(3-x+2x^2)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}+\frac{25\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4\right.}{4\sqrt{46}}\\
&= \frac{121(19-7x)}{276\,(3-x+2x^2)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.58, size = 65, normalized size = 0.96

$$-\frac{11(8623+714x+6183x^2+2336x^3)}{3174\,(3-x+2x^2)^{3/2}}-\frac{25\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(8623 + 714*x + 6183*x^2 + 2336*x^3))/(3174*(3 - x + 2*x^2)^(3/2)) - (
25*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(4*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(55) = 110.
time = 0.12, size = 146, normalized size = 2.15

method	result
risch	$-\frac{11(2336x^3+6183x^2+714x+8623)}{3174(2x^2-x+3)^{\frac{3}{2}}}+\frac{25\sqrt{2}\ \text{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{8}$
trager	$-\frac{11(2336x^3+6183x^2+714x+8623)}{3174(2x^2-x+3)^{\frac{3}{2}}}-\frac{25\,\text{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\text{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}+\text{RootOf}\left(\_Z^2-\right.\right.}{8}$
default	$-\frac{25x^3}{6(2x^2-x+3)^{\frac{3}{2}}}-\frac{145x^2}{8(2x^2-x+3)^{\frac{3}{2}}}-\frac{319x}{64(2x^2-x+3)^{\frac{3}{2}}}-\frac{15775}{768(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{8493x}{1472}-\frac{8493}{5888}}{(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{2267x}{529}-\frac{2267}{2116}}{\sqrt{2x^2-x+3}}-\frac{}{4\sqrt{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/6*x^3/(2*x^2-x+3)^(3/2)-145/8*x^2/(2*x^2-x+3)^(3/2)-319/64*x/(2*x^2-x+3
)^(3/2)-15775/768/(2*x^2-x+3)^(3/2)+8493/5888*(4*x-1)/(2*x^2-x+3)^(3/2)+226
7/2116*(4*x-1)/(2*x^2-x+3)^(1/2)-25/4*x/(2*x^2-x+3)^(1/2)-25/16/(2*x^2-x+3)
^(1/2)+25/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(55) = 110$.
time = 0.52, size = 185, normalized size = 2.72

$$\frac{25}{6348}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right)+\frac{25}{8}\,\sqrt{2}\,\text{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{1775}{3174}\,\sqrt{2\,x^2-x+3}+\frac{1017\,x}{529\,\sqrt{2\,x^2-x+3}}-\frac{15\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{6413}{3174\,\sqrt{2\,x^2-x+3}}-\frac{x}{138\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{2593}{138\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 25/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/
sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3
/2)) + 25/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 1775/3174*sqrt(2*x^2
 - x + 3) + 1017/529*x/sqrt(2*x^2 - x + 3) - 15*x^2/(2*x^2 - x + 3)^(3/2) -
 6413/3174/sqrt(2*x^2 - x + 3) - 1/138*x/(2*x^2 - x + 3)^(3/2) - 2593/138/(
2*x^2 - x + 3)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(55) = 110$.
time = 1.56, size = 112, normalized size = 1.65

$$\frac{39675\,\sqrt{2}\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)-88\,(2336\,x^3+6183\,x^2+714\,x+8623)\sqrt{2\,x^2-x+3}}{25392\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(39675*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sq
rt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 88*(2336*x^3 + 6183*x^2
 + 714*x + 8623)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(5/2), x)
```

Giac [A]
time = 3.97, size = 61, normalized size = 0.90

$$-\frac{25}{8}\sqrt{2}\,\log\left(-2\sqrt{2}\left(\sqrt{2}\,x - \sqrt{2\,x^2 - x + 3}\right) + 1\right) - \frac{11\,(((2336\,x + 6183)x + 714)x + 8623)}{3174\,(2\,x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -25/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/31
74*(((2336*x + 6183)*x + 714)*x + 8623)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5\,x^2 + 3\,x + 2)^2}{(2\,x^2 - x + 3)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2), x)
```

3.96 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} dx$

Optimal. Leaf size=47

$$-\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}-\frac{71(1-4x)}{529\sqrt{3-x+2x^2}}$$

```
[Out] -11/69*(5+3*x)/(2*x^2-x+3)^(3/2)-71/529*(1-4*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1674, 12, 627}

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 - x
 + 2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{5/2}}\,dx &= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{213}{4\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{71}{46}\int \frac{1}{\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}-\frac{71(1-4x)}{529\sqrt{3-x+2x^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 33, normalized size = 0.70

$$\frac{2(-952+1005x-639x^2+852x^3)}{1587\left(3-x+2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
[Out] (2*(-952 + 1005*x - 639*x^2 + 852*x^3))/(1587*(3 - x + 2*x^2)^(3/2))
```

Maple [A]
time = 0.12, size = 69, normalized size = 1.47

method	result	size
gosper	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
trager	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
risch	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
default	$-\frac{5x}{4(2x^2-x+3)^{\frac{3}{2}}}-\frac{29}{48(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{71x}{92}-\frac{71}{368}}{(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{284x}{529}-\frac{71}{529}}{\sqrt{2x^2-x+3}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
[Out] -5/4*x/(2*x^2-x+3)^(3/2)-29/48/(2*x^2-x+3)^(3/2)+71/368*(4*x-1)/(2*x^2-x+3)
^(3/2)+71/529*(4*x-1)/(2*x^2-x+3)^(1/2)
```

Maxima [A]
time = 0.29, size = 59, normalized size = 1.26

$$\frac{284\,x}{529\,\sqrt{2\,x^2-x+3}}-\frac{71}{529\,\sqrt{2\,x^2-x+3}}-\frac{11\,x}{23\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{55}{69\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 284/529*x/sqrt(2*x^2 - x + 3) - 71/529/sqrt(2*x^2 - x + 3) - 11/23*x/(2*x^2
 - x + 3)^(3/2) - 55/69/(2*x^2 - x + 3)^(3/2)
```

Fricas [A]
time = 2.07, size = 51, normalized size = 1.09

$$\frac{2\,(852\,x^3-639\,x^2+1005\,x-952)\sqrt{2\,x^2-x+3}}{1587\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1587*(852*x^3 - 639*x^2 + 1005*x - 952)*sqrt(2*x^2 - x + 3)/(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

Giac [A]
time = 4.19, size = 29, normalized size = 0.62

$$\frac{2\,(3\,(71\,(4\,x-3)x+335)x-952)}{1587\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] 2/1587*(3*(71*(4*x - 3)*x + 335)*x - 952)/(2*x^2 - x + 3)^(3/2)
```

Mupad [B]
time = 0.09, size = 29, normalized size = 0.62

$$\frac{2\,(852\,x^3-639\,x^2+1005\,x-952)}{1587\,(2\,x^2-x+3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] (2*(1005*x - 639*x^2 + 852*x^3 - 952))/(1587*(2*x^2 - x + 3)^(3/2))
```

3.97 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx$

Optimal. Leaf size=199

$$\frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}}+\frac{3603-658x}{128018\sqrt{3-x+2x^2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(-15457+25000\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(-15457+\right.}}}{}\right.$$

```
[Out] 1/759*(13-6*x)/(2*x^2-x+3)^(3/2)+1/128018*(3603-658*x)/(2*x^2-x+3)^(1/2)+1/
330088*arctan(1/31*(443-98*2^(1/2)+x*(247+345*2^(1/2)))*341^(1/2)/(-15457+2
5000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-10541674+17050000*2^(1/2))^(1/2)-1
/330088*arctanh(1/31*(443+x*(247-345*2^(1/2))+98*2^(1/2))*341^(1/2)/(15457+
25000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(10541674+17050000*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {988, 1074, 1049, 1043, 212, 210}

$$\frac{1}{484}\sqrt{\frac{1}{682}\left(25000\sqrt{2}-15457\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left(\left(247+345\sqrt{2}\right)x-98\sqrt{2}+443\right)}{\sqrt{2x^2-x+3}}\right)+\frac{3603-658x}{128018\sqrt{2x^2-x+3}}+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}-\frac{1}{484}\sqrt{\frac{1}{682}\left(15457+25000\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(15457+25000\sqrt{2}\right)}}\left(\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + (3603 - 658*x)/(128018*Sqrt[3 - x
+ 2*x^2]) + (Sqrt[(-15457 + 25000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(-15457
 + 25000*Sqrt[2]))]*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/484 - (Sqrt[(15457 + 25000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(1
5457 + 25000*Sqrt[2]))]*(443 + 98*Sqrt[2] + (247 - 345*Sqrt[2])*x))/Sqrt[3
- x + 2*x^2]])/484
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
```

```
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \frac{13-6x}{759(3-x+2x^2)^{3/2}} - \frac{\int \frac{-2772-\frac{3003x}{2}+660x^2}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx}{8349}$$

$$= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{\int \frac{-\frac{5184729}{2}-\frac{12481}{4}\cdots}{\sqrt{3-x+2x^2}\,(2\cdots}}{2323526\cdots}$$

$$= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{\int \frac{-\frac{2112297}{4}(11-54\sqrt{2}\cdots}{\sqrt{3-x+}\cdots}}{5111\cdots}$$

$$= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{1}{32}(17457(50000-\cdots$$

$$= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{1}{484}\sqrt{\frac{1}{682}(-1545\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.64, size = 209, normalized size = 1.05

$$\frac{39005-19767x+23592x^2-3948x^3}{384054\left(3-x+2x^2\right)^{3/2}}+\frac{1}{484}\text{RootSum}\left[-56-26\sqrt{2}\,\#1+17\#1^2+6\sqrt{2}\,\#1^3-5\#1^4\&,\frac{249\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)+108\sqrt{2}\,\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1-65\log\left(-\sqrt{2}\,x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\,\#1^2-10\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (39005 - 19767*x + 23592*x^2 - 3948*x^3)/(384054*(3 - x + 2*x^2)^(3/2)) + R
ootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (249*Log
[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 108*Sqrt[2]*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1]*#1 - 65*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
#1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/484
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(147) = 294$.
time = 0.63, size = 751, normalized size = 3.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 13/484/(2*x^2-x+3)^(1/2)-329/256036*(4*x-1)/(2*x^2-x+3)^(1/2)+1/10232728*(8
*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
8-3*2^(1/2))^(1/2)*2^(1/2)*(10111*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan
(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)
-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)
/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)
*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+
13910*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))
^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-4
1))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)
^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4
+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^
(1/2)))*(-775687+549362*2^(1/2))^(1/2)-993674*arctanh(31/2*(8*(2^(1/2)-1+x)
^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(
1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-42685698*arctanh(31/2*(8*(2^(1/2)-
1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2
))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3
*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(
1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+68
20*2^(1/2))^(1/2)+1/66/(2*x^2-x+3)^(3/2)-1/506*(4*x-1)/(2*x^2-x+3)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. $2(147) = 294$.
time = 2.39, size = 2133, normalized size = 10.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/370971467791584000*(1123856268*sqrt(341)*200^(1/4)*sqrt(2)*(4*x^4 - 4*x^3
 + 13*x^2 - 6*x + 9)*sqrt(-772850000*sqrt(2) + 2500000000)*arctan(-1/788938
9562500*(71300*sqrt(341)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(347404*x^7 - 90
7814*x^6 + 2112962*x^5 - 2166688*x^4 + 787344*x^3 + 304128*x^2 - sqrt(2)*(3
5898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 1081800*x^
2 - 518400*x + 1043712) - 2087424*x + 518400) + 5*200^(1/4)*(712757*x^7 - 1
0233303*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 -
sqrt(2)*(158647*x^7 - 2935272*x^6 + 19428740*x^5 - 55765712*x^4 + 78380640*
x^3 - 84096000*x^2 - 37407744*x + 53208576) - 106417152*x + 37407744))*sqrt
(-772850000*sqrt(2) + 2500000000) + 22395686500000*sqrt(31)*sqrt(2)*(28180*
x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98
496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 132071
0*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192)
- sqrt(310/5711)*(sqrt(341)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(1665224*x^7
- 2325796*x^6 + 7065036*x^5 - 196416*x^4 - 2176416*x^3 + 8895744*x^2 + sqrt
(2)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419
5584*x^2 - 4195584*x) - 8895744*x) + 5*200^(1/4)*(3246491*x^7 - 41888524*x^
6 + 159670660*x^5 - 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 2*sqrt(
2)*(40239*x^7 - 558044*x^6 + 2804660*x^5 - 9524160*x^4 + 34843680*x^3 - 740
06784*x^2 + 74006784*x) - 376648704*x))*sqrt(-772850000*sqrt(2) + 250000000
0) + 314105000*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 32
93072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 11
8051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^
2 - 1036800*x) + 3276288*x) + 14277500*sqrt(31)*(254591*x^8 - 4815126*x^7 +
 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*s
qrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sqrt(-772850000*sq
rt(2) + 2500000000) - 4337504500*x^2 - 3894902000*sqrt(2)*(2*x^2 - x + 3) +
 13366595500*x - 17704100000)/x^2) + 254496437500*sqrt(31)*(2828123*x^8 - 9
696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 +
```

```
 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 +
15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))
/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44
249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 1123856268*sqrt(341)*
200^(1/4)*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(-772850000*sqrt(2
) + 2500000000)*arctan(-1/7889389562500*(71300*sqrt(341)*sqrt(2*x^2 - x + 3
)*(11*200^(3/4)*(347404*x^7 - 907814*x^6 + 2112962*x^5 - 2166688*x^4 + 7873
44*x^3 + 304128*x^2 - sqrt(2)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 211723
3*x^4 + 1272680*x^3 - 1081800*x^2 - 518400*x + 1043712) - 2087424*x + 51840
0) + 5*200^(1/4)*(712757*x^7 - 10233303*x^6 + 48529768*x^5 - 94500260*x^4 +
 113086944*x^3 - 22282848*x^2 - sqrt(2)*(158647*x^7 - 2935272*x^6 + 1942874
0*x^5 - 55765712*x^4 + 78380640*x^3 - 84096000*x^2 - 37407744*x + 53208576)
 - 106417152*x + 37407744))*sqrt(-772850000*sqrt(2) + 2500000000) - 2239568
6500000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5
 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
- 539136) + 1154304*x - 456192) - sqrt(310/5711)*(sqrt(341)*sqrt(2*x^2 - x
+ 3)*(11*200^(3/4)*(1665224*x^7 - 2325796*x^6 + 7065036*x^5 - 196416*x^4 -
2176416*x^3 + 8895744*x^2 + sqrt(2)*(167914*x^7 - 195429*x^6 + 331239*x^5 +
 1685680*x^4 - 3693960*x^3 + 4195584*x^2 - 4195584*x) - 8895744*x) + 5*200^
(1/4)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4 + 1804962
24*x^3 + 376648704*x^2 - 2*sqrt(2)*(40239*x^7 - 558044*x^6 + 2804660*x^5 -
9524160*x^4 + 34843680*x^3 - 74006784*x^2 + 74006784*x) - 376648704*x))*sqr
t(-772850000*sqrt(2) + 2500000000) - 314105000*sqrt(31)*sqrt(2)*(123408*x^8
 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38223
36*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105396
0*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 14277500*sqrt
(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^
4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6
- 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(
(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) +
163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 38949
02000*sqrt(2)*(2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 2544964
37500*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 269
2*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} \cdot (5x^2 + 3x + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)), x)
```

3.98 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2} \, dx$

Optimal. Leaf size=234

$$-\frac{15101-8654x}{1035276\,(3-x+2x^2)^{3/2}}-\frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}}+\frac{4+65x}{682\,(3-x+2x^2)^{3/2}\,(2+3x+5x^2)}+\frac{625\sqrt{\frac{1}{682}\left(3\right.}}{}$$

```
[Out] 1/1035276*(-15101+8654*x)/(2*x^2-x+3)^(3/2)+1/682*(4+65*x)/(2*x^2-x+3)^(3/2
)/(5*x^2+3*x+2)+1/523849656*(-3133427-1352542*x)/(2*x^2-x+3)^(1/2)-625/4502
40032*arctanh(1/31*(203+x*(687-445*2^(1/2))-242*2^(1/2))*341^(1/2)/(-30463+
23600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-20775766+16095200*2^(1/2))^(1/2)+
625/450240032*arctan(1/31*(203+242*2^(1/2)+x*(687+445*2^(1/2)))*341^(1/2)/(
30463+23600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(20775766+16095200*2^(1/2))^(
1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {988, 1074, 1049, 1043, 212, 210}

$$\frac{625\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(30463+23600\sqrt{2}\right)}}\left(\left(687+445\sqrt{2}\right)x+242\sqrt{2}+203\right)}{\sqrt{2x^2-x+3}}\right)}{660176}-\frac{15101-8654x}{1035276\,(2x^2-x+3)^{3/2}}-\frac{1352542x+3133427}{523849656\sqrt{2x^2-x+3}}+\frac{65x+4}{682\,(2x^2-x+3)^{3/2}\,(5x^2+3x+2)}-\frac{625\sqrt{\frac{1}{682}\left(23600\sqrt{2}-30463\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(23600\sqrt{2}-30463\right)}}\left(\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203\right)}{\sqrt{2x^2-x+3}}\right)}{660176}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -1/1035276*(15101 - 8654*x)/(3 - x + 2*x^2)^(3/2) - (3133427 + 1352542*x)/(
523849656*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 +
 3*x + 5*x^2)) + (625*Sqrt[(30463 + 23600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31
*(30463 + 23600*Sqrt[2]))]*(203 + 242*Sqrt[2] + (687 + 445*Sqrt[2])*x))/Sqr
t[3 - x + 2*x^2]])/660176 - (625*Sqrt[(-30463 + 23600*Sqrt[2])/682]*ArcTanh
[(Sqrt[11/(31*(-30463 + 23600*Sqrt[2]))]*(203 - 242*Sqrt[2] + (687 - 445*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/660176
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \frac{4+65x}{682\,(3-x+2x^2)^{3/2}(2+3x+5x^2)} - \frac{\int \frac{-1738+\frac{4411x}{2}-5720x^2}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx}{7502}$$

$$= -\frac{15101-8654x}{1035276\,(3-x+2x^2)^{3/2}} + \frac{4+65x}{682\,(3-x+2x^2)^{3/2}(2+3x+5x^2)}$$

$$= -\frac{15101-8654x}{1035276\,(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{}{682\,(3-}$$

$$= -\frac{15101-8654x}{1035276\,(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{}{682\,(3-}$$

$$= -\frac{15101-8654x}{1035276\,(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{}{682\,(3-}$$

$$= -\frac{15101-8654x}{1035276\,(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{}{682\,(3-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.05, size = 416, normalized size = 1.78

$$\frac{-31010342+5712309x-84671384x^2-2879479x^3-32686812x^4-13525420x^5}{523849656\,(3-x+2x^2)^{3/2}(2+3x+5x^2)} + \frac{\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{-1376\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+106\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+95\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]}{5324} + \frac{\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{126249\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+58712\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+10095\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1^2}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\right]}{660176\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-31010342 + 5712309*x - 84671384*x^2 - 2879479*x^3 - 32686812*x^4 - 135254
20*x^5)/(523849656*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + RootSum[-56 -
 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-1376*Log[-(Sqrt[2]
*x) + Sqrt[3 - x + 2*x^2] - #1] + 106*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1]*#1 + 95*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/
(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/5324 + RootSum[-56 - 2
6*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (126249*Sqrt[2]*Log[-(
```

```
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 58712*Log[-(Sqrt[2]*x) + Sqrt[3 -
x + 2*x^2] - #1]*#1 + 10095*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
- #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/(660176*Sq
rt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5974 vs. $2(178) = 356$.
time = 0.77, size = 5975, normalized size = 25.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2253 vs. $2(178) = 356$.
time = 3.36, size = 2253, normalized size = 9.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2560433560253791411 2*(301208632500*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)
*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(30463*sqrt(2) +
47200)*arctan(1/11117215998613*(168268*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(
321084*x^7 - 1338894*x^6 + 2762802*x^5 - 4721048*x^4 + 2438224*x^3 - 131731
2*x^2 - sqrt(2)*(277258*x^7 - 994619*x^6 + 2123978*x^5 - 3198193*x^4 + 1552
680*x^3 - 621000*x^2 - 1900800*x + 1181952) - 2363904*x + 1900800) + 1829*6
962^(1/4)*sqrt(341)*(25187*x^7 - 392073*x^6 + 2114488*x^5 - 4948060*x^4 + 6
460704*x^3 - 4452768*x^2 - sqrt(2)*(20477*x^7 - 310452*x^6 + 1610140*x^5 -
3584192*x^4 + 4580640*x^3 - 2620800*x^2 - 3400704*x + 2198016) - 4396032*x
+ 3400704))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) + 3155854864122
4*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
```

36) + 1154304*x - 456192) - 2*sqrt(118/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(3
41)*(1050904*x^7 - 1523916*x^6 + 5005956*x^5 - 2572736*x^4 + 3615264*x^3 +
877824*x^2 - sqrt(2)*(1065206*x^7 - 1518091*x^6 + 4815081*x^5 - 1448880*x^4
 + 1303560*x^3 + 3131136*x^2 - 3131136*x) - 877824*x) + 1829*6962^(1/4)*sqr
t(341)*(84981*x^7 - 1100084*x^6 + 4256060*x^5 - 5639616*x^4 + 7745184*x^3 +
 2571264*x^2 - 242*sqrt(2)*(319*x^7 - 4124*x^6 + 15860*x^5 - 20160*x^4 + 24
480*x^3 + 20736*x^2 - 20736*x) - 2571264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463
*sqrt(2) + 47200) + 187549318*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1
578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*
(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x
^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 8524969*sqrt(31)*(254591*x^8 -
 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*
x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6962^(1/4)*sqrt(
341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x
+ 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)*(2*x^
2 - x + 3) + 676362371*x - 895844200)/x^2) + 358619870923*sqrt(31)*(2828123
*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493000
96*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1007
0*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94
887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*
x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 301208632500*
6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*
x^2 + 15*x + 18)*sqrt(30463*sqrt(2) + 47200)*arctan(1/11117215998613*(16826
8*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(321084*x^7 - 1338894*x^6 + 2762802*x^
5 - 4721048*x^4 + 2438224*x^3 - 1317312*x^2 - sqrt(2)*(277258*x^7 - 994619*
x^6 + 2123978*x^5 - 3198193*x^4 + 1552680*x^3 - 621000*x^2 - 1900800*x + 11
81952) - 2363904*x + 1900800) + 1829*6962^(1/4)*sqrt(341)*(25187*x^7 - 3920
73*x^6 + 2114488*x^5 - 4948060*x^4 + 6460704*x^3 - 4452768*x^2 - sqrt(2)*(2
0477*x^7 - 310452*x^6 + 1610140*x^5 - 3584192*x^4 + 4580640*x^3 - 2620800*x
^2 - 3400704*x + 2198016) - 4396032*x + 3400704))*sqrt(2*x^2 - x + 3)*sqrt(
30463*sqrt(2) + 47200) - 31558548641224*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(11
8/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(341)*(1050904*x^7 - 1523916*x^6 + 5005
956*x^5 - 2572736*x^4 + 3615264*x^3 + 877824*x^2 - sqrt(2)*(1065206*x^7 - 1
518091*x^6 + 4815081*x^5 - 1448880*x^4 + 1303560*x^3 + 3131136*x^2 - 313113
6*x) - 877824*x) + 1829*6962^(1/4)*sqrt(341)*(84981*x^7 - 1100084*x^6 + 425
6060*x^5 - 5639616*x^4 + 7745184*x^3 + 2571264*x^2 - 242*sqrt(2)*(319*x^7 -
 4124*x^6 + 15860*x^5 - 20160*x^4 + 24480*x^3 + 20736*x^2 - 20736*x) - 2571
264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) - 187549318*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762

```
88*x) - 8524969*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 9086680
8*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8
 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) +
 144820224*x))*sqrt((6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sqrt(30463*sqrt(2) + 47200) + 21
9481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x + 3) - 676362371*x + 895844200)/
x^2) - 358619870923*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14
2835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(
1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2\,x^2 - x + 3)^{5/2}\,(5\,x^2 + 3\,x + 2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2), x)
```

3.99 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=269

$$-\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}}-\frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}}+\frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}+\frac{}{1860}$$

[Out] 1/2824232928*(-12280939+19536786*x)/(2*x^2-x+3)^(3/2)+1/1364*(4+65*x)/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2+1/1860496*(46386+86885*x)/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)+1/476353953856*(-1134826571+1504660754*x)/(2*x^2-x+3)^(1/2)-35/1228254807296*arctanh(1/31*(1432939+x*(6290431-3861685*2^(1/2))-2428746*2^(1/2))*341^(1/2)/(-2243059557247+2011748500000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-1529766618042454+1372012477000000*2^(1/2))^(1/2)+35/1228254807296*arctan(1/31*(1432939+2428746*2^(1/2)+x*(6290431+3861685*2^(1/2)))*341^(1/2)/(2243059557247+2011748500000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(1529766618042454+1372012477000000*2^(1/2))^(1/2)

Rubi [A]
time = 0.38, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {988, 1074, 1049, 1043, 212, 210}

$$\frac{35\sqrt{\frac{1}{682}\left(2243059557247+2011748500000\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{11}{31\left(2243059557247+2011748500000\sqrt{2}\right)}}\left(\left(6290431+3861685\sqrt{2}\right)x+2428746\sqrt{2}+1432939\right)}{\sqrt{2x^2-x+3}}\right)}{1800960128}-\frac{1134826571-1504660754x}{476353953856\sqrt{2x^2-x+3}}+\frac{86885x+46386}{1860496\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}-\frac{12280939-19536786x}{2824232928\left(2x^2-x+3\right)^{3/2}}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}-\frac{35\sqrt{\frac{1}{682}\left(2011748500000\sqrt{2}-2243059557247\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2011748500000\sqrt{2}-2243059557247\right)}}\left(\left(6290431-3861685\sqrt{2}\right)x-2428746\sqrt{2}+1432939\right)}{\sqrt{2x^2-x+3}}\right)}{1800960128}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]

[Out] -1/2824232928*(12280939 - 19536786*x)/(3 - x + 2*x^2)^(3/2) - (1134826571 - 1504660754*x)/(476353953856*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + (46386 + 86885*x)/(1860496*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (35*Sqrt[(2243059557247 + 2011748500000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2243059557247 + 2011748500000*Sqrt[2]))]*(1432939 + 2428746*Sqrt[2] + (6290431 + 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1800960128 - (35*Sqrt[(-2243059557247 + 2011748500000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-2243059557247 + 2011748500000*Sqrt[2]))]*(1432939 - 2428746*Sqrt[2] + (6290431 - 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1800960128

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
```

```
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5687+\frac{8635x}{2}-8580x^2}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}}{15004}$$

$$= \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} + \frac{46386+868}{1860496\left(3-x+2x^2\right)^{3/}}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} + \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} +$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} +$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} +$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.20, size = 605, normalized size = 2.25

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(9739335532 + 218659985088*x + 178650961091*x^2 + 5
19223213785*x^3 + 174241614961*x^4 + 592923725931*x^5 - 12234606480*x^6 + 2
25699113100*x^7))/(6 + 7*x + 16*x^2 + x^3 + 10*x^4)^2 - 2976*RootSum[-56 -
26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-26154346*Log[-(Sqrt
```

```
[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 37230166*Sqrt[2]*Log[-(Sqrt[2]*x) + Sq
rt[3 - x + 2*x^2] - #1]*#1 - 1193705*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
 - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 2440171
2*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-364
7*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 3172*Sqrt[2]*Log[-(Sqrt[2]
*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 485*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*
x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 15*
Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & ,
 (-9138129081*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 164457
54136*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1004412885*Sqrt[2]*
Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9
*Sqrt[2]*#1^2 - 10*#1^3) & ])/5716247446272
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19013 vs. $2(209) = 418$.
time = 0.82, size = 19014, normalized size = 70.68

method	result
trager	Expression too large to display
risch	$\frac{225699113100x^7-12234606480x^6+592923725931x^5+174241614961x^4+519223213785x^3+178650961091x^2+218659985088x+973933}{1429061861568(2x^2-x+3)^{\frac{3}{2}}(5x^2+3x+2)^2}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2343 vs. $2(209) = 418$.
time = 6.36, size = 2343, normalized size = 8.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/611377875290135815296770157063555072*(2164988593398757980*129508224872072
^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 4
023497000000)*arctan(1/4525340115746282619252370338578594 39*(11475013444*sq
rt(4023497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 117686842
22*x^6 + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*
x^2 - sqrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626
609*x^4 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) -
 20220682752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(
214583731*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 576317
17152*x^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 146
88003420*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 311565
03552*x + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*
sqrt(2243059557247*sqrt(2) + 4023497000000) + 1284612678018299582239382547 7
25536472*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^
5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
 - 539136) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(402349
7)*(11*129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 +
39777303828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt
(2)*(10387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4
+ 8224291080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 12472
8407*129508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 3480
3726780*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(
2)*(367903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 27
054001440*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x
^2 - x + 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 11194868609848920
9076292438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329307
2*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051
*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 -
1036800*x) + 3276288*x) + 5088576640840418594376929*sqrt(31)*(254591*x^8 -
4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x
```

```
^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(129508224872072^(
1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*
x + 2195288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000
000) - 19211019462513817817783*x^2 - 17250711354094040489484*sqrt(2)*(2*x^2 -
 x + 3) + 59201304874277275316117*x - 78412324336791093134000)/x^2) + 1459787
13411170407072657107696083369*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560
*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*
sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3
+ 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 46612
00*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296
*x^2 - 24772608*x + 18579456)) + 2164988593398757980*129508224872072^(1/4)*
sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390
*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 40234970
00000)*arctan(1/452534011574628261925237033857859439*(11475013444*sqrt(4023
497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 11768684222*x^6
+ 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*x^2 - s
qrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626609*x^4
 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) - 202206
82752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(2145837
31*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 57631717152*x
^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 1468800342
0*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 31156503552*x
 + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*sqrt(22
43059557247*sqrt(2) + 4023497000000) - 128461267801829958223938254772553647
2*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(4023497)*(11*
129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 + 3977730
3828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt(2)*(10
387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4 + 82242
91080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 124728407*12
9508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 34803726780
*x^5 - 46915651008*x^4 + 67421983392*x^3 + 1062...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,in
finity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,infini
ty,inf
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3), x)
```

3.100 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$

Optimal. Leaf size=436

$$\frac{(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+8c^2(12abef+2a^2f^2+5b^2(e^2+2df)))\,(b}{512c^5}$$

```
[Out] 1/960*(640*c^3*d*e-105*b^3*f^2+28*b*c*f*(7*a*f+10*b*e)-8*c^2*(32*a*e*f+25*b
*(2*d*f+e^2)))*(c*x^2+b*x+a)^(3/2)/c^4+1/160*(21*b^2*f^2-4*c*f*(5*a*f+14*b*
e)+40*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(3/2)/c^3+1/20*f*(-3*b*f+8*c*e)*x^2*
(c*x^2+b*x+a)^(3/2)/c^2+1/6*f^2*x^3*(c*x^2+b*x+a)^(3/2)/c-1/1024*(-4*a*c+b^
2)*(128*c^4*d^2+21*b^4*f^2-56*b^2*c*f*(a*f+b*e)-32*c^3*(4*b*d*e+a*(2*d*f+e^
2))+8*c^2*(12*a*b*e*f+2*a^2*f^2+5*b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/512*(128*c^4*d^2+21*b^4*f^2-56*b^2*c
*f*(a*f+b*e)-32*c^3*(4*b*d*e+a*(2*d*f+e^2))+8*c^2*(12*a*b*e*f+2*a^2*f^2+5*b
^2*(2*d*f+e^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A]
time = 0.50, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 635, 212}

$$-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(8c^2(2a^2f^2+12abef+5b^2(2df+e^2))-56b^2cf(af+be)-32c^3(a(2df+e^2)+4bde)+21b^4f^2+128c^4d^2)}{1024c^{11/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\,(8c^2(2a^2f^2+12abef+5b^2(2df+e^2))-56b^2cf(af+be)-32c^3(a(2df+e^2)+4bde)+21b^4f^2+128c^4d^2)}{512c^5}+\frac{(a+bx+cx^2)^{3/2}(-8c^2(32aef+25b(2df+e^2))+28bcf(7af+10be)-105b^3f^2+640c^3de)}{960c^4}+\frac{x(a+bx+cx^2)^{3/2}(-4cf(5af+14be)+21b^2f^2+40c^2(2df+e^2))}{160c^3}+\frac{fx^2(a+bx+cx^2)^{3/2}(8ce-3bf)}{20c^2}+\frac{f^2x^3(a+bx+cx^2)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

```
[Out] ((128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(
e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(b +
2*c*x)*Sqrt[a + b*x + c*x^2])/(512*c^5) + ((640*c^3*d*e - 105*b^3*f^2 + 28*
b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(a + b*x +
c*x^2)^(3/2))/(960*c^4) + ((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(e
^2 + 2*d*f))*x*(a + b*x + c*x^2)^(3/2))/(160*c^3) + (f*(8*c*e - 3*b*f)*x^2*
(a + b*x + c*x^2)^(3/2))/(20*c^2) + (f^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c)
 - ((b^2 - 4*a*c)*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c
^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2
 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c
^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule steps

$$\int \sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)^2\,dx = \frac{f^2x^3(a+bx+cx^2)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2}\,\left(6cd^2+12cdex-3(af^2\right.}{6}$$

$$= \frac{f(8ce-3bf)x^2(a+bx+cx^2)^{3/2}}{20c^2} + \frac{f^2x^3(a+bx+cx^2)^{3/2}}{6c} + \int \sqrt{a}$$

$$= \frac{\left(21b^2f^2-4cf(14be+5af)+40c^2(e^2+2df)\right)x(a+bx+cx^2)^{3/2}}{160c^3} +$$

$$= \frac{(640c^3de-105b^3f^2+28bcf(10be+7af)-8c^2(32aef+25b(e^2+2}{960c^4}$$

$$= \frac{(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+}{512c}$$

$$= \frac{(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+}{512c}$$

$$= \frac{(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+}{512c}$$

Mathematica [A]
time = 1.98, size = 456, normalized size = 1.05

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,(315b^5f^2-210b^4cf(4e+fx)-16b^2c^2(-2af(115e+28fx)+c(120de+25e^2x+50dfx+28efx^2+9f^2x^3))+8b^3c(-210af^2+c(75e^2+70efx+3f(50d+7fx^2)))+16bc^2(113a^2f^2-2ac(65e^2+58efx+f(130d+17fx^2))+4c^2(30d^2+10dx(2e+fx)+x^2(5e^2+6efx+2f^2x^2)))-32c^3(a^2f(64e+15fx)-2ac(80de+15e^2x+30dfx+16efx^2+5f^2x^3)-4c^2x(30d^2+10dx(4e+3fx)+x^2(15e^2+24efx+10f^2x^2))))+15(b^2-4ac)\,(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+8c^2(12abef+2a^2f^2+5b^2(e^2+2df)))\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{15360c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^5*f^2 - 210*b^4*c*f*(4*e + f*x) - 1
6*b^2*c^2*(-2*a*f*(115*e + 28*f*x) + c*(120*d*e + 25*e^2*x + 50*d*f*x + 28*
e*f*x^2 + 9*f^2*x^3)) + 8*b^3*c*(-210*a*f^2 + c*(75*e^2 + 70*e*f*x + 3*f*(5
0*d + 7*f*x^2))) + 16*b*c^2*(113*a^2*f^2 - 2*a*c*(65*e^2 + 58*e*f*x + f*(13
0*d + 17*f*x^2)) + 4*c^2*(30*d^2 + 10*d*x*(2*e + f*x) + x^2*(5*e^2 + 6*e*f*
x + 2*f^2*x^2))) - 32*c^3*(a^2*f*(64*e + 15*f*x) - 2*a*c*(80*d*e + 15*e^2*x
 + 30*d*f*x + 16*e*f*x^2 + 5*f^2*x^3) - 4*c^2*x*(30*d^2 + 10*d*x*(4*e + 3*f
*x) + x^2*(15*e^2 + 24*e*f*x + 10*f^2*x^2)))) + 15*(b^2 - 4*a*c)*(128*c^4*d
^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f
)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*Log[b + 2*c*x -
2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(15360*c^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(406) = 812$.
time = 0.20, size = 1186, normalized size = 2.72

method	result
risch	$\frac{(1280f^2c^5x^5+128b\,c^4f^2x^4+3072c^5ef\,x^4+320a\,c^4f^2x^3-144b^2c^3f^2x^3+384b\,c^4ef\,x^3+3840c^5df\,x^3+1920c^5e^2x^3-544ab\,c^3f^2x^2+102}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f^2*(1/6*x^3*(c*x^2+b*x+a)^(3/2)/c-3/4*b/c*(1/5*x^2*(c*x^2+b*x+a)^(3/2)/c-7
/10*b/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2
*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
)-2/5*a/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
))-1/2*a/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-
1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x
+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
)))))+2*e*f*(1/5*x^2*(c*x^2+b*x+a)^(3/2)/c-7/10*b/c*(1/4*x*(c*x^2+b*x+a)^(3
/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*
x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-2/5*a/c*(1/3*(c*x^2+b*x+a)^(3
/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(2*d*f+e^2)*(1/4*x*(c*x^2+b*
x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c
*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+2*d*e*(1/3*(c*x^2+b*x+
a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(
3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+d^2*(1/4*(2*c*x+b)/c*(c*
x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 3.81, size = 1267, normalized size = 2.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 + 16*(5*b^4*c^2 - 24*a*b^2*c^3 +
16*a^2*c^4)*d*f + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2
 + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 - 8*(16*(b^3*c^3 - 4*a*b*c
^4)*d + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*f)*e)*sqrt(c)*log(-8*c^2*x^
2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) +
4*(1280*c^6*f^2*x^5 + 128*b*c^5*f^2*x^4 + 1920*b*c^5*d^2 + 16*(240*c^6*d*f
- (9*b^2*c^4 - 20*a*c^5)*f^2)*x^3 + 80*(15*b^3*c^3 - 52*a*b*c^4)*d*f + (315
*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(80*b*c^5*d*f + (21*b^3*c
^3 - 68*a*b*c^4)*f^2)*x^2 + 2*(1920*c^6*d^2 - 80*(5*b^2*c^4 - 12*a*c^5)*d*f
 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2)*x + 40*(48*c^6*x^3 + 8*
b*c^5*x^2 + 15*b^3*c^3 - 52*a*b*c^4 - 2*(5*b^2*c^4 - 12*a*c^5)*x)*e^2 + 8*(
384*c^6*f*x^4 + 48*b*c^5*f*x^3 + 8*(80*c^6*d - (7*b^2*c^4 - 16*a*c^5)*f)*x^
2 - 80*(3*b^2*c^4 - 8*a*c^5)*d - (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4
)*f + 2*(80*b*c^5*d + (35*b^3*c^3 - 116*a*b*c^4)*f)*x)*e)*sqrt(c*x^2 + b*x
+ a))/c^6, 1/15360*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 + 16*(5*b^4*c^2 - 24*a*
b^2*c^3 + 16*a^2*c^4)*d*f + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^
3*c^3)*f^2 + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 - 8*(16*(b^3*c^3
 - 4*a*b*c^4)*d + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*f)*e)*sqrt(-c)*ar
ctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)
) + 2*(1280*c^6*f^2*x^5 + 128*b*c^5*f^2*x^4 + 1920*b*c^5*d^2 + 16*(240*c^6*
d*f - (9*b^2*c^4 - 20*a*c^5)*f^2)*x^3 + 80*(15*b^3*c^3 - 52*a*b*c^4)*d*f +
(315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(80*b*c^5*d*f + (21*b
^3*c^3 - 68*a*b*c^4)*f^2)*x^2 + 2*(1920*c^6*d^2 - 80*(5*b^2*c^4 - 12*a*c^5)
*d*f - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2)*x + 40*(48*c^6*x^3
+ 8*b*c^5*x^2 + 15*b^3*c^3 - 52*a*b*c^4 - 2*(5*b^2*c^4 - 12*a*c^5)*x)*e^2 +
 8*(384*c^6*f*x^4 + 48*b*c^5*f*x^3 + 8*(80*c^6*d - (7*b^2*c^4 - 16*a*c^5)*f
)*x^2 - 80*(3*b^2*c^4 - 8*a*c^5)*d - (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2
*c^4)*f + 2*(80*b*c^5*d + (35*b^3*c^3 - 116*a*b*c^4)*f)*x)*e)*sqrt(c*x^2 +
b*x + a))/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2}\,\left(d + ex + fx^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)**2, x)
```

Giac [A]
time = 3.19, size = 638, normalized size = 1.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^2*x + (b*c^4*f^2 + 24*c^5*f*
e)/c^5)*x + (240*c^5*d*f - 9*b^2*c^3*f^2 + 20*a*c^4*f^2 + 24*b*c^4*f*e + 12
0*c^5*e^2)/c^5)*x + (80*b*c^4*d*f + 21*b^3*c^2*f^2 - 68*a*b*c^3*f^2 + 640*c
^5*d*e - 56*b^2*c^3*f*e + 128*a*c^4*f*e + 40*b*c^4*e^2)/c^5)*x + (1920*c^5*
d^2 - 400*b^2*c^3*d*f + 960*a*c^4*d*f - 105*b^4*c*f^2 + 448*a*b^2*c^2*f^2 -
 240*a^2*c^3*f^2 + 640*b*c^4*d*e + 280*b^3*c^2*f*e - 928*a*b*c^3*f*e - 200*
b^2*c^3*e^2 + 480*a*c^4*e^2)/c^5)*x + (1920*b*c^4*d^2 + 1200*b^3*c^2*d*f -
4160*a*b*c^3*d*f + 315*b^5*f^2 - 1680*a*b^3*c*f^2 + 1808*a^2*b*c^2*f^2 - 19
20*b^2*c^3*d*e + 5120*a*c^4*d*e - 840*b^4*c*f*e + 3680*a*b^2*c^2*f*e - 2048
*a^2*c^3*f*e + 600*b^3*c^2*e^2 - 2080*a*b*c^3*e^2)/c^5) + 1/1024*(128*b^2*c
^4*d^2 - 512*a*c^5*d^2 + 80*b^4*c^2*d*f - 384*a*b^2*c^3*d*f + 256*a^2*c^4*d
*f + 21*b^6*f^2 - 140*a*b^4*c*f^2 + 240*a^2*b^2*c^2*f^2 - 64*a^3*c^3*f^2 -
128*b^3*c^3*d*e + 512*a*b*c^4*d*e - 56*b^5*c*f*e + 320*a*b^3*c^2*f*e - 384*
a^2*b*c^3*f*e + 40*b^4*c^2*e^2 - 192*a*b^2*c^3*e^2 + 128*a^2*c^4*e^2)*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

Mupad [B]
time = 5.31, size = 1299, normalized size = 2.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2,x)
```

```
[Out] d^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e^2*x*(a + b*x + c*x^2)^(3/2
))/(4*c) + (a*f^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/
2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
 + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c)
 + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) +
 (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c) - (3*b
*f^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^
3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x
```

```
+ c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*(
(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b
*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log
((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5
/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^
2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (f^2*x^3*(a + b*
x + c*x^2)^(3/2))/(6*c) - (a*e^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) +
 (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3
/2))))/(4*c) + (d^2*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2)) - (5*b*e^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2
*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (4*a*e*f*((log((b + 2*c
*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((
8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c
) - (5*b*d*f*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(4*c) + (d*e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(12*c^2) + (d*f*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7
*b*e*f*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(5*c) + (2*e*f*x^2*(a +
 b*x + c*x^2)^(3/2))/(5*c) - (a*d*f*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2
) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c
^(3/2))))/(2*c) + (d*e*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))
*(b^3 - 4*a*b*c))/(8*c^(5/2))
```

3.101 $\int \sqrt{a+bx+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=175

$$\frac{(16c^2d-8bce+5b^2f-4acf)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\,(a+bx+cx^2)^{3/2}}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

```
[Out] 1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-1
/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c^2*
d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1675, 654, 626, 635, 212}

$$-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+2be)+5b^2f+16c^2d)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\,(-4acf+5b^2f-8bce+16c^2d)}{64c^3}+\frac{(a+bx+cx^2)^{3/2}\,(8ce-5bf)}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{fx(a+bx+cx^2)^{3/2}}{4c}+\frac{\int \left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+cx}}{4c}$$

$$= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}+\frac{(16c^2d-8bc}{}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5b}{}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5b}{}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5b}{}$$

Mathematica [A]
time = 0.63, size = 171, normalized size = 0.98

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,(15b^3f-2b^2c(12e+5fx)+4bc(-13af+2c(6d+2ex+fx^2))+8c^2(a(8e+3fx)+2cx(6d+4ex+3fx^2)))+3(b^2-4ac)\left(16c^2d+5b^2f-4c(2be+af)\right)\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) + 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e +
 a*f))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(153) = 306$.
time = 0.13, size = 343, normalized size = 1.96

method	result
default	$f\left(\frac{x(cx^2+bx+a)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{(cx^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)}{8c}\right. -$
risch	$-\frac{(-48fc^3x^3-8bc^2fx^2-64c^3ex^2-24ac^2fx+10b^2cfx-16bc^2ex-96c^3dx+52abcf-64ac^2e-15b^3f+24b^2ce-48bc^2d)\sqrt{cx^2+}}{192c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(
1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)
+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+e*(1
/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(
4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+d*(1/4*(2*
c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 4.32, size = 469, normalized size = 2.68

$$\left[\frac{3\,(16\,(b^2c^2-4\,ac^3)d+(5\,b^4-24\,ab^2c+16\,a^2c^2)f-8\,(b^3c-4\,abc^2)e)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)+4\,(48\,c^4fx^3+8\,bc^3fx^2+48\,bc^3d+(15\,b^3c-52\,abc^2)f+2\,(48\,c^4d-(5\,b^2c^2-12\,ac^3)f)x+8\,(8\,c^4x^2+2\,bc^3x-3\,b^2c^2+8\,ac^3)e)\sqrt{cx^2+bx+a}}{768\,c^4}, \frac{3\,(16\,(b^2c^2-4\,ac^3)d+(5\,b^4-24\,ab^2c+16\,a^2c^2)f-8\,(b^3c-4\,abc^2)e)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(48\,c^4fx^3+8\,bc^3fx^2+48\,bc^3d+(15\,b^3c-52\,abc^2)f+2\,(48\,c^4d-(5\,b^2c^2-12\,ac^3)f)x+8\,(8\,c^4x^2+2\,bc^3x-3\,b^2c^2+8\,ac^3)e)\sqrt{cx^2+bx+a}}{384\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f -
 8*(b^3*c - 4*a*b*c^2)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 8*b*c^3*f*
x^2 + 48*b*c^3*d + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d - (5*b^2*c^2 - 1
2*a*c^3)*f)*x + 8*(8*c^4*x^2 + 2*b*c^3*x - 3*b^2*c^2 + 8*a*c^3)*e)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d + (5*b^4 - 24*a*b^2*
c + 16*a^2*c^2)*f - 8*(b^3*c - 4*a*b*c^2)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 8*b*c^3*f*x^2 + 48*b*c^3*d + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d - (
5*b^2*c^2 - 12*a*c^3)*f)*x + 8*(8*c^4*x^2 + 2*b*c^3*x - 3*b^2*c^2 + 8*a*c^3
)*e)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A]
time = 5.10, size = 212, normalized size = 1.21

$$\frac{1}{192}\,\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{bc^2f+8\,c^3e}{c^3}\right)x+\frac{48\,c^3d-5\,b^2cf+12\,ac^2f+8\,bc^2e}{c^3}\right)x+\frac{48\,bc^2d+15\,b^3f-52\,abcf-24\,b^2ce+64\,ac^2e}{c^3}\right)+\frac{(16\,b^2c^2d-64\,ac^3d+5\,b^4f-24\,ab^2cf+16\,a^2c^2f-8\,b^3ce+32\,abc^2e)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48
*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*
```

```
f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*
log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [B]

time = 3.91, size = 320, normalized size = 1.83

$$d\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}-\frac{a\,f\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}+\frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}+\frac{e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}-\frac{5\,b\,f\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}+\frac{e\,\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}+\frac{f\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
+ (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
+ (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

3.102 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=431

$$\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\ \tan}{\sqrt{2}}$$

[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)

Rubi [A]
time = 0.71, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1003, 635, 212, 1046, 738}

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1003

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2\right.\right.}{}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4d\right.\right.\right.}{}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-\right.}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.50, size = 396, normalized size = 0.92

$$\frac{-\sqrt{c}\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)+\text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-2af\#1^2-2\sqrt{c}e\#1^3+f\#1^4\&,\frac{bcd\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)-ace\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)-2c^{3/2}d\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1+2a\sqrt{c}f\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1+ce\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1^2-bf\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1^2}{2b\sqrt{c}d-a\sqrt{c}e-4cd\#1-be\#1+2af\#1+3\sqrt{c}e\#1^2-2f\#1^3}\&\right]}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (-(Sqrt[c]*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) + RootSum[
b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 +
b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#
1 + 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*e*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - b*f*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1
- b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. 2(376) = 752.
time = 0.22, size = 1547, normalized size = 3.59

method	result	size
default	Expression too large to display	1547

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*
f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)
+1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2
)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*
(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+
1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^
(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d
*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*ln((1/2*
(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f+c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))/c^(1/
2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)
/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*
f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))
)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+
2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-%e^2>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

3.103 $\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx$

Optimal. Leaf size=488

$$-\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}-\frac{\left(f(be-4af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-}}\right.}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{ce^2-2cdf-bef+2af^2-}}$$

```
[Out] -(2*f*x+e)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)-1/2*arctanh(1/4*(
4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/
(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1
/2))^(1/2))*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)
^(3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^
(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f
+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b
*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e+(-4*d*f+e^
2)^(1/2)))/(-4*d*f+e^2)^(3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*
e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.89, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {985, 1046, 738, 212}

$$-\frac{\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(f(be-4af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] -(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) -
((f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b
*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((f*(b*e - 4*a*f) - (c
*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
 - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 985

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)} - \frac{\int \frac{\frac{1}{2}(be-4af)+(ce-bf)x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx}{-e^2+4df}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)} - \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4\right.\right.\right.}{(e^2-4}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)} + \frac{\left(2\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-\right.\right.\right.\right.}{}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)} + \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4\right.\right.\right.}{\sqrt{2}\,(e^2-4df}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.66, size = 972, normalized size = 1.99

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] (-2*e*f^2*Sqrt[a + x*(b + c*x)] - 4*f^3*x*Sqrt[a + x*(b + c*x)] - 2*c*(e^2
- 4*d*f)*(d + x*(e + f*x))*RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1
 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3
 + f*#1^4 & , (4*c*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 5*b*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*Sqrt[c]*f*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d
*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ] + (d + x*(e + f*
x))*RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4
*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (8*c^2*e^
3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 32*c^2*d*e*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 10*b*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] + 40*b*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] + b^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*
c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*b*f^3*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 4*c^(3/2)*e^2*f*Log[-(Sqrt[c]*x) +
```

```
 Sqrt[a + b*x + c*x^2] - #1]*#1 - 16*c^(3/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1 + 8*a*Sqrt[c]*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1 + 2*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
^2 - 2*b*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqr
t[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*
f*#1^3) & ])/(2*f^2*(e^2 - 4*d*f)*(d + x*(e + f*x)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4690 vs. $2(439) = 878$.
time = 0.18, size = 4691, normalized size = 9.61

method	result	size
default	Expression too large to display	4691

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2)^(1/2)+1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4
*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1
/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4
/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f
*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1
/2)*(1/2*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)/f*ln((1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f+c*(x-1/2/f*(-e
+(-4*d*f+e^2)^(1/2))))/c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-
4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*
d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2))/c^(1/2)-1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d
```

```
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*
f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(
1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1
/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2
/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))
)))-1/(4*d*f-e^2)*(-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)*f^2/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*((x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(3/2)+f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*
(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(1/2
*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e
)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2/f*(-c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)+c*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e
^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/c
^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)))+4*c/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)*f^2*(1/4*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/f*(-c*(-4*d*f+e^
2)^(1/2)+b*f-c*e))/c*((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)
^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/8*(
2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c
*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2)/c^(3/2)*ln((1/2/f*(-c*(-
4*d*f+e^2)^(1/2)+b*f-c*e)+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)))-1/(4*d*f-e^2)*(-2/(b*f*(-4*d
*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + x*e + d)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f\,x^2+e\,x+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2,x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2, x)
```

3.104 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx$

Optimal. Leaf size=564

$$-\frac{(b^2-4ac)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6bde+a(e^2+2df))+16c^2(24abef+3a^2f^2+\right.}{16384c^6}$$

```
[Out] 1/6144*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(3*a*f+4*b*e)-128*c^3*(6*b*d*e+a*
(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+14*b^2*(2*d*f+e^2)))*(2*c*x+b)*(c
*x^2+b*x+a)^(3/2)/c^5+1/13440*(5376*c^3*d*e-693*b^3*f^2+36*b*c*f*(31*a*f+56
*b*e)-32*c^2*(48*a*e*f+49*b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(5/2)/c^4+1/1344*(9
9*b^2*f^2-12*c*f*(7*a*f+24*b*e)+224*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(5/2)/
c^3+1/112*f*(-11*b*f+32*c*e)*x^2*(c*x^2+b*x+a)^(5/2)/c^2+1/8*f^2*x^3*(c*x^2
+b*x+a)^(5/2)/c+1/32768*(-4*a*c+b^2)^2*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(
3*a*f+4*b*e)-128*c^3*(6*b*d*e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+1
4*b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1
3/2)-1/16384*(-4*a*c+b^2)*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(3*a*f+4*b*e)-
128*c^3*(6*b*d*e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+14*b^2*(2*d*f+
e^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6
```

Rubi [A]
time = 0.61, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1675, 654, 626, 635, 212}

$$\frac{(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16c^2(3a^2f^2+24abef+14b^2(2df+e^2))-72b^2cf(3af+4be)-128c^3(a(2df+e^2)+6bde)+99b^4f^2+768c^4d^2\right)}{32768c^{13/2}}-\frac{(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}\left(16c^2(3a^2f^2+24abef+14b^2(2df+e^2))-72b^2cf(3af+4be)-128c^3(a(2df+e^2)+6bde)+99b^4f^2+768c^4d^2\right)}{16384c^6}+\frac{(b+2cx)(a+bx+cx^2)^{3/2}\left(16c^2(3a^2f^2+24abef+14b^2(2df+e^2))-72b^2cf(3af+4be)-128c^3(a(2df+e^2)+6bde)+99b^4f^2+768c^4d^2\right)}{6144c^5}+\frac{(a+bx+cx^2)^{5/2}\left(-32c^2(48aef+49b(2df+e^2))+36bcf(31af+56be)-693b^3f^2+5376c^3de\right)}{13440c^4}+\frac{x(a+bx+cx^2)^{5/2}\left(-12cf(7af+24be)+99b^2f^2+224c^2(2df+e^2)\right)}{1344c^3}+\frac{fx^2(a+bx+cx^2)^{5/2}(32ce-11bf)}{112c^2}+\frac{f^2x^3(a+bx+cx^2)^{5/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] -1/16384*((b^2 - 4*a*c)*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a
*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2
+ 14*b^2*(e^2 + 2*d*f)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/c^6 + ((768*c^4
*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2
+ 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*(b + 2*
c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^5) + ((5376*c^3*d*e - 693*b^3*f^2 + 3
6*b*c*f*(56*b*e + 31*a*f) - 32*c^2*(48*a*e*f + 49*b*(e^2 + 2*d*f)))*(a + b*
x + c*x^2)^(5/2))/(13440*c^4) + ((99*b^2*f^2 - 12*c*f*(24*b*e + 7*a*f) + 22
4*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(5/2))/(1344*c^3) + (f*(32*c*e - 1
1*b*f)*x^2*(a + b*x + c*x^2)^(5/2))/(112*c^2) + (f^2*x^3*(a + b*x + c*x^2)^
(5/2))/(8*c) + ((b^2 - 4*a*c)^2*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b
*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*
a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(32768*c^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)^2 \, dx = \frac{f^2x^3(a+bx+cx^2)^{5/2}}{8c} + \frac{\int (a+bx+cx^2)^{3/2} (8cd^2+16cdex-$$

$$= \frac{f(32ce-11bf)x^2(a+bx+cx^2)^{5/2}}{112c^2} + \frac{f^2x^3(a+bx+cx^2)^{5/2}}{8c} +$$

$$= \frac{(99b^2f^2-12cf(24be+7af)+224c^2(e^2+2df))\,x(a+bx+cx^2}{1344c^3}$$

$$= \frac{(5376c^3de-693b^3f^2+36bcf(56be+31af)-32c^2(48aef+49}{13440c^4}$$

$$= \frac{(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6bde+a(e^2+}{}$$

$$= -\frac{(b^2-4ac)\,(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6b}{}$$

$$= -\frac{(b^2-4ac)\,(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6b}{}$$

$$= -\frac{(b^2-4ac)\,(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6b}{}$$

<u>**Mathematica**</u> [A]
time = 5.08, size = 766, normalized size = 1.36

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-10395*b^7*f^2 + 630*b^6*c*f*(48*e + 11*f
*x) - 84*b^5*c*(-1095*a*f^2 + c*(280*e^2 + 560*d*f + 240*e*f*x + 66*f^2*x^2
)) + 8*b^4*c^2*(560*c*d*(18*e + 7*f*x) - 63*a*f*(480*e + 107*f*x) + 2*c*x*(
980*e^2 + 1008*e*f*x + 297*f^2*x^2)) - 16*b^3*c^2*(15309*a^2*f^2 - 4*a*c*(2
660*e^2 + 5320*d*f + 2184*e*f*x + 585*f^2*x^2) + 8*c^2*(630*d^2 + 28*d*x*(1
5*e + 7*f*x) + x^2*(98*e^2 + 108*e*f*x + 33*f^2*x^2))) + 96*b^2*c^3*(a^2*f*
(5488*e + 1181*f*x) + 8*c^2*x*(70*d^2 + 28*d*x*(2*e + f*x) + x^2*(14*e^2 +
16*e*f*x + 5*f^2*x^2)) - 4*a*c*(56*d*(25*e + 9*f*x) + x*(252*e^2 + 248*e*f*
x + 71*f^2*x^2))) + 64*b*c^3*(2757*a^3*f^2 - 6*a^2*c*(756*e^2 + 584*e*f*x +
 f*(1512*d + 151*f*x^2)) + 24*a*c^2*(350*d^2 + 28*d*x*(7*e + 3*f*x) + x^2*(
42*e^2 + 44*e*f*x + 13*f^2*x^2)) + 16*c^3*x^2*(630*d^2 + 28*d*x*(33*e + 26*
f*x) + x^2*(364*e^2 + 600*e*f*x + 255*f^2*x^2))) + 128*c^4*(-3*a^3*f*(512*e
```

```
 + 105*f*x) + 16*c^3*x^3*(210*d^2 + 56*d*x*(6*e + 5*f*x) + 5*x^2*(28*e^2 +
48*e*f*x + 21*f^2*x^2)) + 6*a^2*c*(56*d*(16*e + 5*f*x) + x*(140*e^2 + 128*e
*f*x + 35*f^2*x^2)) + 8*a*c^2*x*(1050*d^2 + 28*d*x*(48*e + 35*f*x) + x^2*(4
90*e^2 + 768*e*f*x + 315*f^2*x^2)))) - 105*(b^2 - 4*a*c)^2*(768*c^4*d^2 + 9
9*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)
) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*Log[b + 2*c*x -
 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3440640*c^(13/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1653 vs. $2(530) = 1060$.
time = 0.15, size = 1654, normalized size = 2.93

method	result	size
default	Expression too large to display	1654
risch	Expression too large to display	1751

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f^2*(1/8*x^3*(c*x^2+b*x+a)^(5/2)/c-11/16*b/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c
-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-
1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x
+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*
a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2
)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2
*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))))))-3/8*a/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1
/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*
(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*
x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/
8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+2*e*f
*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*
b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)
+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^
(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)
/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1
/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-
2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(
3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2
)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(2*d*f+e^2)*(1/6*
x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2
*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*
```

```
x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4
*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2)))))+2*d*e*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*
(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+
b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2)))))+d^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(
2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1086 vs. $2(536) = 1072$.
time = 4.58, size = 2175, normalized size = 3.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/6881280*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 + 64*(7*b^6*
c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d*f + 3*(33*b^8 - 336*a*
b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(7*b^6*
c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 - 96*(8*(b^5*c^3 - 8
*a*b^3*c^4 + 16*a^2*b*c^5)*d + (3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*f)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(215040*c^8*f^2*x^7 + 261120*b*c^
7*f^2*x^6 + 1280*(448*c^8*d*f + 3*(b^2*c^6 + 84*a*c^7)*f^2)*x^5 + 128*(5824
*b*c^7*d*f - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2)*x^4 + 16*(26880*c^8*d^2 + 448
*(3*b^2*c^6 + 140*a*c^7)*d*f + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)
*f^2)*x^3 - 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 - 448*(105*b^5*c^3 - 760*a*b
^3*c^4 + 1296*a^2*b*c^5)*d*f - 3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*
b^3*c^3 - 58816*a^3*b*c^4)*f^2 + 8*(80640*b*c^7*d^2 - 448*(7*b^3*c^5 - 36*a
*b*c^6)*d*f - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2)*x^2 +
```

```
2*(26880*(b^2*c^6 + 20*a*c^7)*d^2 + 448*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a
^2*c^6)*d*f + 3*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a
^3*c^5)*f^2)*x + 224*(1280*c^8*x^5 + 1664*b*c^7*x^4 - 105*b^5*c^3 + 760*a*b
^3*c^4 - 1296*a^2*b*c^5 + 16*(3*b^2*c^6 + 140*a*c^7)*x^3 - 8*(7*b^3*c^5 - 3
6*a*b*c^6)*x^2 + 2*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*x)*e^2 + 96*(
5120*c^8*f*x^6 + 6400*b*c^7*f*x^5 + 128*(56*c^8*d + (b^2*c^6 + 64*a*c^7)*f)
*x^4 + 16*(616*b*c^7*d - (9*b^3*c^5 - 44*a*b*c^6)*f)*x^3 + 8*(56*(b^2*c^6 +
 32*a*c^7)*d + (21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*x^2 + 56*(15*b
^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d + (315*b^6*c^2 - 2520*a*b^4*c^3 + 5
488*a^2*b^2*c^4 - 2048*a^3*c^5)*f - 2*(56*(5*b^3*c^5 - 28*a*b*c^6)*d + (105
*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*f)*x)*e)*sqrt(c*x^2 + b*x + a))/
c^7, -1/3440640*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 + 64*(7*
b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d*f + 3*(33*b^8 - 33
6*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(7*
b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 - 96*(8*(b^5*c^3
 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d + (3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3
 - 64*a^3*b*c^4)*f)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(215040*c^8*f^2*x^7 + 261120*b*c^7*
f^2*x^6 + 1280*(448*c^8*d*f + 3*(b^2*c^6 + 84*a*c^7)*f^2)*x^5 + 128*(5824*b
*c^7*d*f - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2)*x^4 + 16*(26880*c^8*d^2 + 448*(
3*b^2*c^6 + 140*a*c^7)*d*f + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f
^2)*x^3 - 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 - 448*(105*b^5*c^3 - 760*a*b^3
*c^4 + 1296*a^2*b*c^5)*d*f - 3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^
3*c^3 - 58816*a^3*b*c^4)*f^2 + 8*(80640*b*c^7*d^2 - 448*(7*b^3*c^5 - 36*a*b
*c^6)*d*f - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2)*x^2 + 2*
(26880*(b^2*c^6 + 20*a*c^7)*d^2 + 448*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2
*c^6)*d*f + 3*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3
*c^5)*f^2)*x + 224*(1280*c^8*x^5 + 1664*b*c^7*x^4 - 105*b^5*c^3 + 760*a*b^3
*c^4 - 1296*a^2*b*c^5 + 16*(3*b^2*c^6 + 140*a*c^7)*x^3 - 8*(7*b^3*c^5 - 36*
a*b*c^6)*x^2 + 2*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*x)*e^2 + 96*(51
20*c^8*f*x^6 + 6400*b*c^7*f*x^5 + 128*(56*c^8*d + (b^2*c^6 + 64*a*c^7)*f)*x
^4 + 16*(616*b*c^7*d - (9*b^3*c^5 - 44*a*b*c^6)*f)*x^3 + 8*(56*(b^2*c^6 + 3
2*a*c^7)*d + (21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*x^2 + 56*(15*b^4
*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d + (315*b^6*c^2 - 2520*a*b^4*c^3 + 548
8*a^2*b^2*c^4 - 2048*a^3*c^5)*f - 2*(56*(5*b^3*c^5 - 28*a*b*c^6)*d + (105*b
^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*f)*x)*e)*sqrt(c*x^2 + b*x + a))/c^
7]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1150 vs. $2(536) = 1072$.
time = 6.53, size = 1150, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f^2*x + (17*b*c^7
*f^2 + 32*c^8*f*e)/c^7)*x + (448*c^8*d*f + 3*b^2*c^6*f^2 + 252*a*c^7*f^2 +
480*b*c^7*f*e + 224*c^8*e^2)/c^7)*x + (5824*b*c^7*d*f - 33*b^3*c^5*f^2 + 15
6*a*b*c^6*f^2 + 5376*c^8*d*e + 96*b^2*c^6*f*e + 6144*a*c^7*f*e + 2912*b*c^7
*e^2)/c^7)*x + (26880*c^8*d^2 + 1344*b^2*c^6*d*f + 62720*a*c^7*d*f + 297*b^
4*c^4*f^2 - 1704*a*b^2*c^5*f^2 + 1680*a^2*c^6*f^2 + 59136*b*c^7*d*e - 864*b
^3*c^5*f*e + 4224*a*b*c^6*f*e + 672*b^2*c^6*e^2 + 31360*a*c^7*e^2)/c^7)*x +
 (80640*b*c^7*d^2 - 3136*b^3*c^5*d*f + 16128*a*b*c^6*d*f - 693*b^5*c^3*f^2
+ 4680*a*b^3*c^4*f^2 - 7248*a^2*b*c^5*f^2 + 5376*b^2*c^6*d*e + 172032*a*c^7
*d*e + 2016*b^4*c^4*f*e - 11904*a*b^2*c^5*f*e + 12288*a^2*c^6*f*e - 1568*b^
3*c^5*e^2 + 8064*a*b*c^6*e^2)/c^7)*x + (26880*b^2*c^6*d^2 + 537600*a*c^7*d^
2 + 15680*b^4*c^4*d*f - 96768*a*b^2*c^5*d*f + 107520*a^2*c^6*d*f + 3465*b^6
*c^2*f^2 - 26964*a*b^4*c^3*f^2 + 56688*a^2*b^2*c^4*f^2 - 20160*a^3*c^5*f^2
- 26880*b^3*c^5*d*e + 150528*a*b*c^6*d*e - 10080*b^5*c^3*f*e + 69888*a*b^3*
c^4*f*e - 112128*a^2*b*c^5*f*e + 7840*b^4*c^4*e^2 - 48384*a*b^2*c^5*e^2 + 5
3760*a^2*c^6*e^2)/c^7)*x - (80640*b^3*c^5*d^2 - 537600*a*b*c^6*d^2 + 47040*
b^5*c^3*d*f - 340480*a*b^3*c^4*d*f + 580608*a^2*b*c^5*d*f + 10395*b^7*c*f^2
 - 91980*a*b^5*c^2*f^2 + 244944*a^2*b^3*c^3*f^2 - 176448*a^3*b*c^4*f^2 - 80
640*b^4*c^4*d*e + 537600*a*b^2*c^5*d*e - 688128*a^2*c^6*d*e - 30240*b^6*c^2
*f*e + 241920*a*b^4*c^3*f*e - 526848*a^2*b^2*c^4*f*e + 196608*a^3*c^5*f*e +
 23520*b^5*c^3*e^2 - 170240*a*b^3*c^4*e^2 + 290304*a^2*b*c^5*e^2)/c^7) - 1/
32768*(768*b^4*c^4*d^2 - 6144*a*b^2*c^5*d^2 + 12288*a^2*c^6*d^2 + 448*b^6*c
^2*d*f - 3840*a*b^4*c^3*d*f + 9216*a^2*b^2*c^4*d*f - 4096*a^3*c^5*d*f + 99*
b^8*f^2 - 1008*a*b^6*c*f^2 + 3360*a^2*b^4*c^2*f^2 - 3840*a^3*b^2*c^3*f^2 +
768*a^4*c^4*f^2 - 768*b^5*c^3*d*e + 6144*a*b^3*c^4*d*e - 12288*a^2*b*c^5*d*
e - 288*b^7*c*f*e + 2688*a*b^5*c^2*f*e - 7680*a^2*b^3*c^3*f*e + 6144*a^3*b*
c^4*f*e + 224*b^6*c^2*e^2 - 1920*a*b^4*c^3*e^2 + 4608*a^2*b^2*c^4*e^2 - 204
8*a^3*c^5*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))
/c^(13/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,{\left(f\,x^2+e\,x+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2, x)
```

3.105 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

Optimal. Leaf size=236

$$-\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+}{192c^3}$$

```
[Out] 1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^
3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*x^2+b*x+a)^(5/2)/
c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d+7*b
^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

Rubi [A]
time = 0.14, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1675, 654, 626, 635, 212}

$$\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+3be)+7b^2f+24c^2d\right)}{1024c^{9/2}}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)+7b^2f+24c^2d\right)}{512c^4}+\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{60c^2}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/512*((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*
Sqrt[a + b*x + c*x^2])/c^4 + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx &= \frac{fx(a+bx+cx^2)^{5/2}}{6c}+\frac{\int\left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+\right.}{6c}\\
&= \frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx(a+bx+cx^2)^{5/2}}{6c}+\frac{(2c(6cd}{}\\
&= \frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}+\frac{(12c}{}\\
&= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx}}{512c^4}\\
&= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx}}{512c^4}\\
&= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx}}{512c^4}
\end{aligned}$$

Mathematica [A]
time = 1.22, size = 290, normalized size = 1.23

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\left(-105b^5f+10b^4c(18e+7fx)-8b^3c(45cd-95af+cx(15e+7fx))+48b^2c^2(-a(25e+9fx)+cx(5d+x(2e+fx)))+16bc^2(-81a^2f+6ac(25d+x(7e+3fx))+4c^2x^2(45d+x(33e+26fx)))+32c^3(3a^2(16e+5fx)+4c^2x^3(15d+2x(6e+5fx))+2acx(75d+x(48e+35fx)))\right)-15\left(b^2-4ac\right)^2\left(24c^2d+7b^2f-4c(3be+af)\right)\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{15360c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*f + 10*b^4*c*(18*e + 7*f*x) - 8*
b^3*c*(45*c*d - 95*a*f + c*x*(15*e + 7*f*x)) + 48*b^2*c^2*(-(a*(25*e + 9*f*
x)) + c*x*(5*d + x*(2*e + f*x))) + 16*b*c^2*(-81*a^2*f + 6*a*c*(25*d + x*(7
*e + 3*f*x)) + 4*c^2*x^2*(45*d + x*(33*e + 26*f*x))) + 32*c^3*(3*a^2*(16*e
+ 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e +
 35*f*x)))) - 15*(b^2 - 4*a*c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(15360*c^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(210) = 420$.
time = 0.14, size = 499, normalized size = 2.11

method	result
default	$f\left(\frac{x(cx^2+bx+a)^{\frac{5}{2}}}{6c} - \frac{7b\left(\frac{(cx^2+bx+a)^{\frac{5}{2}}}{5c} - \frac{b\left(\frac{(2cx+b)(cx^2+bx+a)^{\frac{3}{2}}}{8c} + \frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c} + \frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2} \cdots}{\sqrt{\cdots}}\right.}{\cdots}\right.}{16c}\right.}{2c}\right.}{12c}\right.$

risch	$-\frac{(-1280c^5 f x^5-1664b c^4 f x^4-1536c^5 e x^4-2240a c^4 f x^3-48b^2c^3 f x^3-2112b c^4 e x^3-1920c^5 d x^3-288ab c^3 f x^2-3072a c^4 e x^2+56b^3 c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*
(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c
*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)
/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+e*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1
/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x
^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2)))))+d*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*
(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 5.34, size = 849, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d + (7*b^6 - 60*a*b^
4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 1664*b*c^5*f*x^4 + 16*(120
*c^6*d + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d - (7*b^3*c^3 - 36*
```

```
a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d - (105*b^5*c - 760*a*b^3*c
^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d + (35*b^4*c^2 - 216*
a*b^2*c^3 + 240*a^2*c^4)*f)*x + 12*(128*c^6*x^4 + 176*b*c^5*x^3 + 15*b^4*c^
2 - 100*a*b^2*c^3 + 128*a^2*c^4 + 8*(b^2*c^4 + 32*a*c^5)*x^2 - 2*(5*b^3*c^3
 - 28*a*b*c^4)*x)*e)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*
a^3*c^3)*f - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1
280*c^6*f*x^5 + 1664*b*c^5*f*x^4 + 16*(120*c^6*d + (3*b^2*c^4 + 140*a*c^5)*
f)*x^3 + 8*(360*b*c^5*d - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3
- 20*a*b*c^4)*d - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(
b^2*c^4 + 20*a*c^5)*d + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x + 1
2*(128*c^6*x^4 + 176*b*c^5*x^3 + 15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4 +
 8*(b^2*c^4 + 32*a*c^5)*x^2 - 2*(5*b^3*c^3 - 28*a*b*c^4)*x)*e)*sqrt(c*x^2 +
 b*x + a))/c^5]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [**A**]
time = 7.86, size = 417, normalized size = 1.77

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{13\,bc^5f+12\,c^6e}{c^5}\right)x+\frac{120\,c^6d+3\,b^2c^4f+140\,ac^5f+132\,bc^5e}{c^5}\right)x+\frac{360\,bc^5d-7\,b^3c^3f+36\,abc^4f+12\,b^2c^4e+384\,ac^5e}{c^5}\right)x+\frac{120\,b^2c^4d+2400\,ac^5d+35\,b^4c^2f-216\,ab^2c^3f+240\,a^2c^4f-60\,b^3c^3e+336\,abc^4e}{c^5}\right)x-\frac{360\,b^3c^3d-2400\,abc^4d+105\,b^5cf-760\,ab^3c^2f+1296\,a^2bc^3f-180\,b^4c^2e+1200\,ab^2c^3e-1536\,a^2c^4e}{c^5}\right)-\frac{\left(24\,b^4c^2d-192\,ab^2c^3d+384\,a^2c^4d+7\,b^6f-60\,ab^4cf+144\,a^2b^2c^2f-64\,a^3c^3f-12\,b^5ce+96\,ab^3c^2e-192\,a^2bc^3e\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e
)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (
360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a
^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e +
 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^
3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.106 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=679

$$-\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2}+\frac{(3b^2f^2-12cf(be-af)+8c^2(e^2-df))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right.}{8\sqrt{c}\,f^3}$$

```
[Out] 1/8*(3*b^2*f^2-12*c*f*(-a*f+b*e)+8*c^2*(-d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^
(1/2)/(c*x^2+b*x+a)^(1/2))/f^3/c^(1/2)-1/4*(-2*c*f*x-5*b*f+4*c*e)*(c*x^2+b*
x+a)^(1/2)/f^2+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*
(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*
a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c*d*f*(-a*f+b*e)-f^2*(
-a^2*f+b^2*d)-c^2*d*(-d*f+e^2))+(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))*
(e-(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(
-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a
)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*
(-2*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2))+(-b*f+c*e)*(
f*(-2*a*f+b*e)-c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+
e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2
)
```

Rubi [A]
time = 7.73, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {991, 1090, 635, 212, 1046, 738}

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)(f(be-2af)-c(e^2-2df))-2f(-f^2(b^2d-a^2f)+2cdf(be-af)+c^2(-d)(e^2-df))\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(-2f^3(b^2d-a^2f)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)(f(be-2af)-c(e^2-2df))+4cdf^2(be-af)-2c^2df(e^2-df)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-12cf(be-af)+3b^2f^2+8c^2(e^2-df))}{8\sqrt{c}f^3}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -1/4*((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/f^2 + ((3*b^2*f^2 -
12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(8*Sqrt[c]*f^3) + (((c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]
)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2
*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]
) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])
/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((4*c*d*f^2*(b*e - a*f) - 2*f^3*(b^2*d - a^
```

```
2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*f])*(f*(b*e
- 2*a*f) - c*(e^2 - 2*d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 991

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
```

```
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}(-4bcde+5b^2df+4af(cd-2af))-\frac{1}{4}(8c^2de-4acef}{\sqrt{a}}}{}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}f(-4bcde+5b^2df+4af(cd-2af))-\frac{1}{4}d(-3b^2f^2+1}{}}{}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{(3b^2f^2-12cf(be-af)+8c^2(e^2-df))}{4f}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{(3b^2f^2-12cf(be-af)+8c^2(e^2-df))}{8\sqrt{c}\,f^3}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{(3b^2f^2-12cf(be-af)+8c^2(e^2-df))}{8\sqrt{c}\,f^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.26, size = 1150, normalized size = 1.69

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*(-4*c*e + 5*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + ((-3*b^2*f^2 + 12*c
*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)]])/Sqrt[c] + 8*RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2
*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f
*#1^4 & , (b*c^2*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c
^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - b*c^2*d^2*f*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b^2*c*d*e*f*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1] + 2*a*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] + 2*a*b*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] + b^3*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*b^2*e*f^
2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*e*f^2*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*b*f^3*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1] - 2*c^(5/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1 + 2*c^(5/2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1 + 4*b*c^(3/2)*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#
1 - 2*b^2*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 -
 4*a*c^(3/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a^
2*Sqrt[c]*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c^2*e^3*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*c^2*d*e*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*b*c*e^2*f*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*b*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1^2 + b^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1^2 + 2*a*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#
1^2 - 2*a*b*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*S
qrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 -
2*f*#1^3) & ])/(8*f^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2859 vs. $2(618) = 1236$.
time = 0.21, size = 2860, normalized size = 4.21

method	result	size
default	Expression too large to display	2860
risch	Expression too large to display	13999

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*(1/3*((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-
4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*
f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(3/2)
+1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(1/4*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/c*((x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
```

```
f+c*e^2)/f^2)^(1/2)+1/8*(2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2)
/c^(3/2)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)+c*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e
^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)))+1/2*
(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/
2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2/f*(-c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)+c*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f
*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f
*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2))/c^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/
2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4
*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))))+1/(-4*d*f+e^2)^(1/2)*(1/3*((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))
^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2
*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2)^(3/2)+1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(1/4*(2*c*(x-1/2/f*(-e+(-
4*d*f+e^2)^(1/2)))+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f)/c*((x-1/2/f*(-e+(-4*d*
f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+c*e^2)/f^2)^(1/2)+1/8*(2*c*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2/f
^2)/c^(3/2)*ln((1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f+c*(x-1/2/f*(-e+(-4*d*f
+e^2)^(1/2))))/c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)
^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)))+1/2
*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2*(1/2*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)/f*ln((1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f+c*(x-1/2/f*(
-e+(-4*d*f+e^2)^(1/2))))/c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-
4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2))/c^(1/2)-1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
```

```
e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)
^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1
/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2
))))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-%e^2>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^2+b\,x+a)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

3.107 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx$

Optimal. Leaf size=704

$$-\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\,(e^2-4df)}-\frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)}+\frac{c^{3/2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f^2}-$$

```
[Out] -(2*f*x+e)*(c*x^2+b*x+a)^(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)+c^(3/2)*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2-(-2*c*f*x-2*b*f+c*e)*(c*x^2+b
*x+a)^(1/2)/f/(-4*d*f+e^2)-1/4*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2
)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d*(-4*
d*f+e^2)+f*(2*b^2*d*f+4*a*f*(a*f+c*d)-b*e*(3*a*f+c*d)))+(-b*f+c*e)*(f*(-2*a
*f+b*e)+2*c*(-5*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^2/(-4*d*f+e^2)^(3/2)*2^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/4
*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/
2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(
-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d*(-4*d*f+e^2)+f*(2*b^2*d*f+4*a*f*(a
*f+c*d)-b*e*(3*a*f+c*d)))+(-b*f+c*e)*(f*(-2*a*f+b*e)+2*c*(-5*d*f+e^2))*(e+(
-4*d*f+e^2)^(1/2)))/f^2/(-4*d*f+e^2)^(3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a
*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A]

time = 7.55, antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {985, 1080, 1090, 635, 212, 1046, 738}

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c(e^2-5df)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d(e^2-4df)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{2\sqrt{2}f^2(e^2-4df)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)+2c(e^2-5df)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d(e^2-4df)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{2\sqrt{2}f^2(e^2-4df)^{3/2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f^2}-\frac{\sqrt{a+bx+cx^2}\,(-2bf+ce-2cfx)}{f\,(e^2-4df)}-\frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] -(((c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(f*(e^2 - 4*d*f))) - ((e
+ 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)) + (c^(3
/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f^2 - (((c*e -
b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2
*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d + 3*a*f)
)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*f^2*(e^2 - 4*d*f)^
(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f
```

```
]]) + (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^2 - 4
*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*
e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*f^2
*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*S
qrt[e^2 - 4*d*f]])
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 635</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 738</u>

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

<u>Rule 985</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

<u>Rule 1046</u>

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

<u>Rule 1080</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = -\frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)} - \frac{\int \frac{\sqrt{a+bx+cx^2}\,\left(\frac{1}{2}(3be-4af)+(3ce+bf)x+4cfx^2\right)}{d+ex+fx^2}\,dx}{-e^2+4df}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\,(e^2-4df)} - \frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)} - \frac{\int \frac{cf\,(2b^2df+\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\,(e^2-4df)} - \frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)} + \frac{c^2\int \frac{\cdots}{\sqrt{a+\cdots}}}{\cdots}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\,(e^2-4df)} - \frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)} + \frac{(2c^2)\,\mathrm{Subs}\cdots}{\cdots}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\,(e^2-4df)} - \frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)} + \frac{c^{3/2}\tanh^{-\cdots}}{\cdots}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\,(e^2-4df)} - \frac{(e+2fx)\,(a+bx+cx^2)^{3/2}}{(e^2-4df)\,(d+ex+fx^2)} + \frac{c^{3/2}\tanh^{-\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.79, size = 2416, normalized size = 3.43

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] ((-2*f^3*Sqrt[a + x*(b + c*x)]*(c*e^2*x - b*f*(2*d + e*x) + c*d*(e - 2*f*x)
 + a*f*(e + 2*f*x)))/((e^2 - 4*d*f)*(d + x*(e + f*x))) - 2*c^(3/2)*f^2*Log[
b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]] - 2*RootSum[b^2*d - a*b*e + a^
2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#
1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (4*c^3*e^3*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - 8*c^3*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] - 9*b*c^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 10*b*
c^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 6*b^2*c*e*f^2*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 6*a*c^2*e*f^2*Log[-(Sqrt[c]*
```

```
x) + Sqrt[a + b*x + c*x^2] - #1] - b^3*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] - 8*a*b*c*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
 + 2*c^(5/2)*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*c^
(5/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*b*c^(3/2)
*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*b^2*Sqrt[c]*f^
3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*a*c^(3/2)*f^3*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*c^2*e*f^2*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*b*c*f^3*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1
 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ] + RootSum[b^2*d - a*b*e + a^
2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#
1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (8*c^3*e^5*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - 48*c^3*d*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - 18*b*c^2*e^4*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 6
4*c^3*d^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 90*b*c^2*d
*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 12*b^2*c*e^3*f^2*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 14*a*c^2*e^3*f^2*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 72*b*c^2*d^2*f^3*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1] - 47*b^2*c*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1] - 58*a*c^2*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1] - 2*b^3*e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] -
 17*a*b*c*e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 6*b^3*d*
f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 70*a*b*c*d*f^4*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*b^2*e*f^4*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*e*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - 2*a^2*b*f^5*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
] + 4*c^(5/2)*e^4*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 20*
c^(5/2)*d*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 8*b*c
^(3/2)*e^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 16*c^(5/
2)*d^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 30*b*c^(3/2)
*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*b^2*Sqrt[c]*
e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 8*a*c^(3/2)*e^2
*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 12*b^2*Sqrt[c]*d*f
^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 24*a*c^(3/2)*d*f^4*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 6*a*b*Sqrt[c]*e*f^4*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 8*a^2*Sqrt[c]*f^5*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*c^2*e^3*f^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 6*c^2*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1^2 - 3*b*c*e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1^2 + 6*b*c*d*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1^2 + b^2*e*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
+ 2*a*c*e*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*a*b*f
^5*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a
*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &
]/(e^2 - 4*d*f))/(2*f^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7855 vs. $2(640) = 1280$.
time = 0.16, size = 7856, normalized size = 11.16

method	result	size
default	Expression too large to display	7856

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + x*e + d)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**2,x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,4,8,0,0,0]%%%}+%%%{%%{[%
%%{16,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{\left(f\,x^2+e\,x+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2, x)
```

3.108 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3} \, dx$

Optimal. Leaf size=671

$$-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}+\frac{3\left(4cde+4aef-b(e^2+4df)+2\left(ce^2-2bef+4af^2\right)x\right)\sqrt{a+bx+cx^2}}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

```
[Out] -1/2*(2*f*x+e)*(c*x^2+b*x+a)^(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)^2+3/4*(4*c*d*
e+4*a*e*f-b*(4*d*f+e^2)+2*(4*a*f^2-2*b*e*f+c*e^2)*x)*(c*x^2+b*x+a)^(1/2)/(-
4*d*f+e^2)^2/(f*x^2+e*x+d)-3/8*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2
)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-f*(4*b*e*(3*a*f+c
*d)-b^2*(4*d*f+e^2)-4*a*(4*a*f^2+c*e^2))+2*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e-
(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(5/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^
2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+3/8*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+
e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)
/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-f*(4*
b*e*(3*a*f+c*d)-b^2*(4*d*f+e^2)-4*a*(4*a*f^2+c*e^2))+2*(2*a*f-b*e+2*c*d)*(-
b*f+c*e)*(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(5/2)*2^(1/2)/(c*e^2-2*c*d*f-
b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A]

time = 11.09, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {985, 1027, 1046, 738, 212}

$$\frac{3\left(-2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)+4bef(3af+cd)-4af(4af^2+ce^2)+b^2(-f)(4df+e^2)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{3\left(-2\left(\sqrt{e^2-4df}+e\right)(ce-bf)(2af-be+2cd)+4bef(3af+cd)-4af(4af^2+ce^2)+b^2(-f)(4df+e^2)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{3\sqrt{a+bx+cx^2}\left(2x(4af^2-2bef+ce^2)+4aef-b(4df+e^2)+4cde\right)}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] -1/2*((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)
^2) + (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^
2)*x)*Sqrt[a + b*x + c*x^2])/(4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) + (3*(4*
b*e*f*(c*d + 3*a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*
c*d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*
(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]
*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sq
rt[a + b*x + c*x^2])]])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f
 - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (3*(4*b*e*f*(c*d + 3*
a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a
```

```
*f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 -
 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2
*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*
x^2])])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 985

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1027

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(g*b - 2*a*h - (b*h - 2*g*
c)*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1)
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
```

```
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = -\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2} + \frac{\int \frac{\left(\frac{3}{2}(be-4af)+3(ce-bf)x\right)\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx}{2(e^2-4df)}$$

$$= -\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2} + \frac{3(4cde+4aef-b(e^2+4df)+2(ce^2-2bef+}{4(e^2-4df)^2(d+ex+f}$$

$$= -\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2} + \frac{3(4cde+4aef-b(e^2+4df)+2(ce^2-2bef+}{4(e^2-4df)^2(d+ex+f}$$

$$= -\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2} + \frac{3(4cde+4aef-b(e^2+4df)+2(ce^2-2bef+}{4(e^2-4df)^2(d+ex+f}$$

$$= -\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2} + \frac{3(4cde+4aef-b(e^2+4df)+2(ce^2-2bef+}{4(e^2-4df)^2(d+ex+f}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1621 vs. $2(671) = 1342$.
time = 16.30, size = 1621, normalized size = 2.42

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*((c*d*e - 2*b*d*f + a*e*f + c*e^2*x - 2*c*d*f*x -
b*e*f*x + 2*a*f^2*x)/(2*f*(-e^2 + 4*d*f)*(d + e*x + f*x^2)^2) + (2*c*e^3 +
4*c*d*e*f - 7*b*e^2*f + 4*b*d*f^2 + 12*a*e*f^2 + 2*c*e^2*f*x + 16*c*d*f^2*x
 - 12*b*e*f^2*x + 24*a*f^3*x)/(4*f*(-e^2 + 4*d*f)^2*(d + e*x + f*x^2))))/(a
 + b*x + c*x^2) + (3*(4*c^2*d*e^2 - 2*b*c*e^3 - 8*b*c*d*e*f + 3*b^2*e^2*f +
 8*a*c*e^2*f + 4*b^2*d*f^2 - 16*a*b*e*f^2 + 16*a^2*f^3 - 4*c^2*d*e*Sqrt[e^2
 - 4*d*f] + 2*b*c*e^2*Sqrt[e^2 - 4*d*f] + 4*b*c*d*f*Sqrt[e^2 - 4*d*f] - 2*b
^2*e*f*Sqrt[e^2 - 4*d*f] - 4*a*c*e*f*Sqrt[e^2 - 4*d*f] + 4*a*b*f^2*Sqrt[e^2
```

```
 - 4*d*f])*(a + x*(b + c*x))^(3/2)*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x])/(4*
Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sq
rt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2)) + (3*(-4*
c^2*d*e^2 + 2*b*c*e^3 + 8*b*c*d*e*f - 3*b^2*e^2*f - 8*a*c*e^2*f - 4*b^2*d*f
^2 + 16*a*b*e*f^2 - 16*a^2*f^3 - 4*c^2*d*e*Sqrt[e^2 - 4*d*f] + 2*b*c*e^2*Sq
rt[e^2 - 4*d*f] + 4*b*c*d*f*Sqrt[e^2 - 4*d*f] - 2*b^2*e*f*Sqrt[e^2 - 4*d*f]
 - 4*a*c*e*f*Sqrt[e^2 - 4*d*f] + 4*a*b*f^2*Sqrt[e^2 - 4*d*f])*(a + x*(b + c
*x))^(3/2)*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/
2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqr
t[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2)) - (3*(-4*c^2*d*e^2 + 2*b*c*e^3 + 8
*b*c*d*e*f - 3*b^2*e^2*f - 8*a*c*e^2*f - 4*b^2*d*f^2 + 16*a*b*e*f^2 - 16*a^
2*f^3 - 4*c^2*d*e*Sqrt[e^2 - 4*d*f] + 2*b*c*e^2*Sqrt[e^2 - 4*d*f] + 4*b*c*d
*f*Sqrt[e^2 - 4*d*f] - 2*b^2*e*f*Sqrt[e^2 - 4*d*f] - 4*a*c*e*f*Sqrt[e^2 - 4
*d*f] + 4*a*b*f^2*Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2)*Log[b*e - 4*a*
f + b*Sqrt[e^2 - 4*d*f] + 2*c*e*x - 2*b*f*x + 2*c*Sqrt[e^2 - 4*d*f]*x - 2*S
qrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f
*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e
^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2)) - (3*(4*c^2*d*e^2 - 2*b*c*e^3 - 8*b*c
*d*e*f + 3*b^2*e^2*f + 8*a*c*e^2*f + 4*b^2*d*f^2 - 16*a*b*e*f^2 + 16*a^2*f^
3 - 4*c^2*d*e*Sqrt[e^2 - 4*d*f] + 2*b*c*e^2*Sqrt[e^2 - 4*d*f] + 4*b*c*d*f*S
qrt[e^2 - 4*d*f] - 2*b^2*e*f*Sqrt[e^2 - 4*d*f] - 4*a*c*e*f*Sqrt[e^2 - 4*d*f
] + 4*a*b*f^2*Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2)*Log[-(b*e) + 4*a*f
 + b*Sqrt[e^2 - 4*d*f] - 2*c*e*x + 2*b*f*x + 2*c*Sqrt[e^2 - 4*d*f]*x + 2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^
2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 16308 vs. $2(611) = 1222$.
time = 0.16, size = 16309, normalized size = 24.31

method	result	size
default	Expression too large to display	16309

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + x*e + d)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{\left(f\,x^2+e\,x+d\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3, x)
```

3.109 $\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=717

$$\frac{(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f(70abef+22a^2f^2+25b^2(e^2+df))-640c^4(27b}{7680c^6}$$

```
[Out] 1/1024*(1024*c^6*d^3+231*b^6*f^3-252*b^4*c*f^2*(5*a*f+3*b*e)-1536*c^5*d*(b*
d*e+a*(d*f+e^2))+840*b^2*c^2*f*(4*a*b*e*f+2*a^2*f^2+b^2*(d*f+e^2))+384*c^4*
(3*b^2*d*(d*f+e^2)+3*a^2*f*(d*f+e^2)+2*a*b*e*(6*d*f+e^2))-320*c^3*(9*a^2*b*
e*f^2+a^3*f^3+9*a*b^2*f*(d*f+e^2)+b^3*(6*d*e*f+e^3)))*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)+1/7680*(23040*c^5*d^2*e-3465*b^5*f^3
+420*b^3*c*f^2*(34*a*f+27*b*e)-504*b*c^2*f*(70*a*b*e*f+22*a^2*f^2+25*b^2*(d
*f+e^2))-640*c^4*(27*b*d*(d*f+e^2)+8*a*e*(6*d*f+e^2))+96*c^3*(128*a^2*e*f^2
+275*a*b*f*(d*f+e^2)+50*b^2*(6*d*e*f+e^3)))*(c*x^2+b*x+a)^(1/2)/c^6+1/3840*
(1155*b^4*f^3-252*b^2*c*f^2*(14*a*f+15*b*e)+5760*c^4*d*(d*f+e^2)+24*c^2*f*(
322*a*b*e*f+50*a^2*f^2+175*b^2*(d*f+e^2))-160*c^3*(27*a*f*(d*f+e^2)+10*b*(6
*d*e*f+e^3)))*x*(c*x^2+b*x+a)^(1/2)/c^5-1/960*(231*b^3*f^3-36*b*c*f^2*(13*a
*f+21*b*e)-320*c^3*(6*d*e*f+e^3)+24*c^2*f*(32*a*e*f+35*b*(d*f+e^2)))*x^2*(c
*x^2+b*x+a)^(1/2)/c^4+1/480*f*(99*b^2*f^2-4*c*f*(25*a*f+81*b*e)+360*c^2*(d*
f+e^2))*x^3*(c*x^2+b*x+a)^(1/2)/c^3+1/60*f^2*(-11*b*f+36*c*e)*x^4*(c*x^2+b*
x+a)^(1/2)/c^2+1/6*f^3*x^5*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A]
time = 1.71, antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1675, 654, 635, 212}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((23040*c^5*d^2*e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*
c^2*f*(70*a*b*e*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2
 + d*f) + 8*a*e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d
*f) + 50*b^2*(e^3 + 6*d*e*f)))*Sqrt[a + b*x + c*x^2])/(7680*c^6) + ((1155*b
^4*f^3 - 252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*
f*(322*a*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 +
 d*f) + 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(3840*c^5) - ((231*
b^3*f^3 - 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f
*(32*a*e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/(960*c^4) + (f*(
99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c^2*(e^2 + d*f))*x^3*Sqrt[a + b*
```

```
x + c*x^2])/(480*c^3) + (f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a + b*x + c*x^2])/(
60*c^2) + (f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((1024*c^6*d^3 + 231*b^6*
f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) +
840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*
(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2
*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{6cd^3+18cd^2ex+18cd(e^2+df)x^2+6ce(e^2+6df)x^3-f(5af^2-18c(e^2+df))x^4+\frac{1}{2}\cdots}{\sqrt{a+bx+cx^2}}}{6c}$$

$$= \frac{f^2(36ce-11bf)x^4\sqrt{a+bx+cx^2}}{60c^2} + \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int 30c^2d^3+90c^2d^2ex+90c^2d(\cdots}{\cdots}$$

$$= \frac{f(99b^2f^2-4cf(81be+25af)+360c^2(e^2+df))\,x^3\sqrt{a+bx+cx^2}}{480c^3} + \frac{f^2(36ce-11\cdots}{\cdots}$$

$$= -\frac{(231b^3f^3-36bcf^2(21be+13af)-320c^3(e^3+6def)+24c^2f(32aef+35b(e^2+\cdots}{960c^4}$$

$$= \frac{(1155b^4f^3-252b^2cf^2(15be+14af)+5760c^4d(e^2+df)+24c^2f(322abef+50a^2f^2\cdots}{3840\cdots}$$

$$= \frac{(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f(70abef+22a^2f^2+2\cdots}{\cdots}$$

$$= \frac{(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f(70abef+22a^2f^2+2\cdots}{\cdots}$$

$$= \frac{(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f(70abef+22a^2f^2+2\cdots}{\cdots}$$

Mathematica [A]
time = 2.79, size = 615, normalized size = 0.86

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^5*f^3 + 210*b^3*c*f^2*(54*b*e + 6
8*a*f + 11*b*f*x) - 168*b*c^2*f*(66*a^2*f^2 + 42*a*b*f*(5*e + f*x) + b^2*(7
5*e^2 + 75*d*f + 45*e*f*x + 11*f^2*x^2)) + 128*c^5*(90*d^2*(2*e + f*x) + 15
*d*x*(6*e^2 + 8*e*f*x + 3*f^2*x^2) + x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^
2 + 10*f^3*x^3)) + 48*c^3*(2*a^2*f^2*(128*e + 25*f*x) + b^2*(100*e^3 + 175*
e^2*f*x + 6*e*f*(100*d + 21*f*x^2) + f^2*x*(175*d + 33*f*x^2)) + 2*a*b*f*(2
75*e^2 + 161*e*f*x + f*(275*d + 39*f*x^2))) - 64*c^4*(a*(80*e^3 + 135*e^2*f
*x + 96*e*f*(5*d + f*x^2) + 5*f^2*x*(27*d + 5*f*x^2)) + b*(270*d^2*f + 15*d
*(18*e^2 + 20*e*f*x + 7*f^2*x^2) + x*(50*e^3 + 105*e^2*f*x + 81*e*f^2*x^2 +
 22*f^3*x^3)))) - 15*(1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3*b*e + 5
```

```
*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*e*f + 2*a
^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*f*(e^2 + d
*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f
*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(
b + c*x)]])/(15360*c^(13/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(683) = 1366$.
time = 0.15, size = 2207, normalized size = 3.08

method	result	size
risch	Expression too large to display	1387
default	Expression too large to display	2207

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f^3*(1/6*x^5/c*(c*x^2+b*x+a)^(1/2)-11/12*b/c*(1/5*x^4/c*(c*x^2+b*x+a)^(1/2)
-9/10*b/c*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(
1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-
1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)
-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x
/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))))-4/5*a/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/
2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-5/6*a/c*(1/4*x^3/c*(c*x^2+b*x
+a)^(1/2)-7/8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*
x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*
(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+3*e*f
^2*(1/5*x^4/c*(c*x^2+b*x+a)^(1/2)-9/10*b/c*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7
/8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-
3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2
+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/
2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-4/5*a/c*(1/3*x^2/
```

```
c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*
x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c
*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
)))+(d*f^2+2*e^2*f+f*(2*d*f+e^2))*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/8*b/c*(1
/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(
1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*
(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))))-3/4*a/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(
1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(4*d*e*f+e*(2*d*f+e^2))*(1
/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(
1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*
(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))))+(d*(2*d*f+e^2)+2*d*e^2+f*d^2)*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/
c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+3*d^2
*e*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2)))+d^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 3.70, size = 1569, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(1024*c^6*d^3 + 384*(3*b^2*c^4 - 4*a*c^5)*d^2*f + 24*(35*b^4*c
^2 - 120*a*b^2*c^3 + 48*a^2*c^4)*d*f^2 + (231*b^6 - 1260*a*b^4*c + 1680*a^2
*b^2*c^2 - 320*a^3*c^3)*f^3 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + 24*(16*(3*b
```

```
^2*c^4 - 4*a*c^5)*d + (35*b^4*c^2 - 120*a*b^2*c^3 + 48*a^2*c^4)*f)*e^2 - 12
*(128*b*c^5*d^2 + 32*(5*b^3*c^3 - 12*a*b*c^4)*d*f + (63*b^5*c - 280*a*b^3*c
^2 + 240*a^2*b*c^3)*f^2)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*c^6*f^3*x^5 - 1408
*b*c^5*f^3*x^4 - 17280*b*c^5*d^2*f - 600*(21*b^3*c^3 - 44*a*b*c^4)*d*f^2 -
21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*b*c^3)*f^3 + 16*(360*c^6*d*f^2 + (9
9*b^2*c^4 - 100*a*c^5)*f^3)*x^3 - 24*(280*b*c^5*d*f^2 + (77*b^3*c^3 - 156*a
*b*c^4)*f^3)*x^2 + 6*(1920*c^6*d^2*f + 40*(35*b^2*c^4 - 36*a*c^5)*d*f^2 + (
385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3)*x + 320*(8*c^6*x^2 - 10*b*
c^5*x + 15*b^2*c^4 - 16*a*c^5)*e^3 + 120*(48*c^6*f*x^3 - 56*b*c^5*f*x^2 - 1
44*b*c^5*d - 5*(21*b^3*c^3 - 44*a*b*c^4)*f + 2*(48*c^6*d + (35*b^2*c^4 - 36
*a*c^5)*f)*x)*e^2 + 12*(384*c^6*f^2*x^4 - 432*b*c^5*f^2*x^3 + 1920*c^6*d^2
+ 160*(15*b^2*c^4 - 16*a*c^5)*d*f + (945*b^4*c^2 - 2940*a*b^2*c^3 + 1024*a^
2*c^4)*f^2 + 8*(160*c^6*d*f + (63*b^2*c^4 - 64*a*c^5)*f^2)*x^2 - 2*(800*b*c
^5*d*f + 7*(45*b^3*c^3 - 92*a*b*c^4)*f^2)*x)*e)*sqrt(c*x^2 + b*x + a))/c^7,
 -1/15360*(15*(1024*c^6*d^3 + 384*(3*b^2*c^4 - 4*a*c^5)*d^2*f + 24*(35*b^4*
c^2 - 120*a*b^2*c^3 + 48*a^2*c^4)*d*f^2 + (231*b^6 - 1260*a*b^4*c + 1680*a^
2*b^2*c^2 - 320*a^3*c^3)*f^3 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + 24*(16*(3*
b^2*c^4 - 4*a*c^5)*d + (35*b^4*c^2 - 120*a*b^2*c^3 + 48*a^2*c^4)*f)*e^2 - 1
2*(128*b*c^5*d^2 + 32*(5*b^3*c^3 - 12*a*b*c^4)*d*f + (63*b^5*c - 280*a*b^3*
c^2 + 240*a^2*b*c^3)*f^2)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c
*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*f^3*x^5 - 1408*b*c^
5*f^3*x^4 - 17280*b*c^5*d^2*f - 600*(21*b^3*c^3 - 44*a*b*c^4)*d*f^2 - 21*(1
65*b^5*c - 680*a*b^3*c^2 + 528*a^2*b*c^3)*f^3 + 16*(360*c^6*d*f^2 + (99*b^2
*c^4 - 100*a*c^5)*f^3)*x^3 - 24*(280*b*c^5*d*f^2 + (77*b^3*c^3 - 156*a*b*c^
4)*f^3)*x^2 + 6*(1920*c^6*d^2*f + 40*(35*b^2*c^4 - 36*a*c^5)*d*f^2 + (385*b
^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3)*x + 320*(8*c^6*x^2 - 10*b*c^5*x
 + 15*b^2*c^4 - 16*a*c^5)*e^3 + 120*(48*c^6*f*x^3 - 56*b*c^5*f*x^2 - 144*b*
c^5*d - 5*(21*b^3*c^3 - 44*a*b*c^4)*f + 2*(48*c^6*d + (35*b^2*c^4 - 36*a*c^
5)*f)*x)*e^2 + 12*(384*c^6*f^2*x^4 - 432*b*c^5*f^2*x^3 + 1920*c^6*d^2 + 160
*(15*b^2*c^4 - 16*a*c^5)*d*f + (945*b^4*c^2 - 2940*a*b^2*c^3 + 1024*a^2*c^4
)*f^2 + 8*(160*c^6*d*f + (63*b^2*c^4 - 64*a*c^5)*f^2)*x^2 - 2*(800*b*c^5*d*
f + 7*(45*b^3*c^3 - 92*a*b*c^4)*f^2)*x)*e)*sqrt(c*x^2 + b*x + a))/c^7]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**3/sqrt(a + b*x + c*x**2), x)
```

Giac [A]

time = 3.67, size = 824, normalized size = 1.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^3*x/c - (11*b*c^4*f^3 - 36*c
^5*f^2*e)/c^6)*x + (360*c^5*d*f^2 + 99*b^2*c^3*f^3 - 100*a*c^4*f^3 - 324*b*
c^4*f^2*e + 360*c^5*f*e^2)/c^6)*x - (840*b*c^4*d*f^2 + 231*b^3*c^2*f^3 - 46
8*a*b*c^3*f^3 - 1920*c^5*d*f*e - 756*b^2*c^3*f^2*e + 768*a*c^4*f^2*e + 840*
b*c^4*f*e^2 - 320*c^5*e^3)/c^6)*x + (5760*c^5*d^2*f + 4200*b^2*c^3*d*f^2 -
4320*a*c^4*d*f^2 + 1155*b^4*c*f^3 - 3528*a*b^2*c^2*f^3 + 1200*a^2*c^3*f^3 -
 9600*b*c^4*d*f*e - 3780*b^3*c^2*f^2*e + 7728*a*b*c^3*f^2*e + 5760*c^5*d*e^
2 + 4200*b^2*c^3*f*e^2 - 4320*a*c^4*f*e^2 - 1600*b*c^4*e^3)/c^6)*x - (17280
*b*c^4*d^2*f + 12600*b^3*c^2*d*f^2 - 26400*a*b*c^3*d*f^2 + 3465*b^5*f^3 - 1
4280*a*b^3*c*f^3 + 11088*a^2*b*c^2*f^3 - 23040*c^5*d^2*e - 28800*b^2*c^3*d*
f*e + 30720*a*c^4*d*f*e - 11340*b^4*c*f^2*e + 35280*a*b^2*c^2*f^2*e - 12288
*a^2*c^3*f^2*e + 17280*b*c^4*d*e^2 + 12600*b^3*c^2*f*e^2 - 26400*a*b*c^3*f*
e^2 - 4800*b^2*c^3*e^3 + 5120*a*c^4*e^3)/c^6) - 1/1024*(1024*c^6*d^3 + 1152
*b^2*c^4*d^2*f - 1536*a*c^5*d^2*f + 840*b^4*c^2*d*f^2 - 2880*a*b^2*c^3*d*f^
2 + 1152*a^2*c^4*d*f^2 + 231*b^6*f^3 - 1260*a*b^4*c*f^3 + 1680*a^2*b^2*c^2*
f^3 - 320*a^3*c^3*f^3 - 1536*b*c^5*d^2*e - 1920*b^3*c^3*d*f*e + 4608*a*b*c^
4*d*f*e - 756*b^5*c*f^2*e + 3360*a*b^3*c^2*f^2*e - 2880*a^2*b*c^3*f^2*e + 1
152*b^2*c^4*d*e^2 - 1536*a*c^5*d*e^2 + 840*b^4*c^2*f*e^2 - 2880*a*b^2*c^3*f
*e^2 + 1152*a^2*c^4*f*e^2 - 320*b^3*c^3*e^3 + 768*a*b*c^4*e^3)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x^2+e\,x+d)^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2), x)
```

3.110 $\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=316

$$\frac{(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2(16aef+9b(e^2+2df)))\sqrt{a+bx+cx^2}}{192c^4}+\frac{(35b^2f^2-4cf(\cdots$$

```
[Out] 1/128*(128*c^4*d^2+35*b^4*f^2-40*b^2*c*f*(3*a*f+2*b*e)-64*c^3*(2*b*d*e+a*(2
*d*f+e^2))+48*c^2*(4*a*b*e*f+a^2*f^2+b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+1/192*(384*c^3*d*e-105*b^3*f^2+20*b*
c*f*(11*a*f+12*b*e)-16*c^2*(16*a*e*f+9*b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(1/2)/
c^4+1/96*(35*b^2*f^2-4*c*f*(9*a*f+20*b*e)+48*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+
a)^(1/2)/c^3+1/24*f*(-7*b*f+16*c*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*f^2*x^3
*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A]
time = 0.40, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1675, 654, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2(a^2f^2+4abef+b^2(2df+e^2))-40b^2cf(3af+2be)-64c^3(a(2df+e^2)+2bde)+35b^4f^2+128c^4d^2\right)}{128c^{9/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16c^2(16aef+9b(2df+e^2))+20bcf(11af+12be)-105b^3f^2+384c^3de\right)}{192c^4}+\frac{x\sqrt{a+bx+cx^2}\left(-4cf(9af+20be)+35b^2f^2+48c^2(2df+e^2)\right)}{96c^3}+\frac{fx^2\sqrt{a+bx+cx^2}(16ce-7bf)}{24c^2}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((384*c^3*d*e - 105*b^3*f^2 + 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f
 + 9*b*(e^2 + 2*d*f)))*Sqrt[a + b*x + c*x^2])/(192*c^4) + ((35*b^2*f^2 - 4*
c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(96*c
^3) + (f*(16*c*e - 7*b*f)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (f^2*x^3*Sq
rt[a + b*x + c*x^2])/(4*c) + ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e
 + 3*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^
2 + b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(128*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{4cd^2+8cdex-\left(3af^2-4c\left(e^2+2df\right)\right)x^2+\frac{1}{2}f(16ce-7bf)x^3}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{f(16ce-7bf)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{12c^2d^2+\left(24c^2de-16acef+7a\right.}{}}{}$$

$$= \frac{\left(35b^2f^2-4cf(20be+9af)+48c^2\left(e^2+2df\right)\right)x\sqrt{a+bx+cx^2}}{96c^3} + \frac{f(16ce-7bf)x^2}{24c}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx}}{192c^4}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx}}{192c^4}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx}}{192c^4}$$

Mathematica [A]
time = 0.86, size = 251, normalized size = 0.79

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(-105b^3f^2+10bcf(24be+22af+7bfx)+16c^3\left(12d(2e+fx)+x\left(6e^2+8efx+3f^2x^2\right)\right)-8c^2\left(af(32e+9fx)+b\left(18e^2+36df+20efx+7f^2x^2\right)\right)\right)-3\left(128c^4d^2+35b^4f^2-40b^2cf(2be+3af)-64c^3\left(2bde+a\left(e^2+2df\right)\right)+48c^2\left(4abef+a^2f^2+b^2\left(e^2+2df\right)\right)\right)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{384c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f^2 + 10*b*c*f*(24*b*e + 22*a*f
+ 7*b*f*x) + 16*c^3*(12*d*(2*e + f*x) + x*(6*e^2 + 8*e*f*x + 3*f^2*x^2)) -
8*c^2*(a*f*(32*e + 9*f*x) + b*(18*e^2 + 36*d*f + 20*e*f*x + 7*f^2*x^2))) -
3*(128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*e
+ a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*Log[
b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(290) = 580$.
time = 0.15, size = 714, normalized size = 2.26

method	result
risch	$\frac{(48f^2c^3x^3-56b\,c^2f^2x^2+128c^3ef\,x^2-72a\,c^2f^2x+70b^2c\,f^2x-160b\,c^2ef x+192c^3df x+96c^3e^2x+220abc\,f^2-256a\,c^2ef-105b^3f^2+240}{192c^4}$

default	$f^2\left(\frac{x^3\sqrt{c\,x^2+bx+a}}{4c}-\frac{7b\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+c}{\sqrt{c}}\right.}{}\right.}{4c}\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f^2*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5
/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/
c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x/c*(c*
x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))))+2*e*f*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c
*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(2*d*f+e^2)*(1/2*x/c*(c*x^2+b*x+a)^(1/
```

```
2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
)))+2*d*e*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2)))+d^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

<u>Fricas</u> [A]
time = 4.09, size = 629, normalized size = 1.99

$$\left[-\frac{3\,{\left(128\,c^{4} d^{2} + 32\,{\left(3\,b^{2} c^{2} - 4\,a c^{3}\right)} d f + {\left(35\,b^{4} - 120\,a b^{2} c + 48\,a^{2} c^{2}\right)} f^{2} + 16\,{\left(3\,b^{2} c^{2} - 4\,a c^{3}\right)} e^{2} - 16\,{\left(8\,b c^{3} d + {\left(5\,b^{3} c - 12\,a b c^{2}\right)} f\right)} e\right)} \sqrt{c} \log\left(-8\,c^{2} x^{2} - 8\,b c x - b^{2} + 4\,\sqrt{c x^{2} + b x + a} {\left(2\,c x + b\right)} \sqrt{c} - 4\,a c\right) - 4\,{\left(48\,c^{4} f^{2} x^{3} - 56\,b c^{3} f^{2} x^{2} - 288\,b c^{3} d f - 5\,{\left(21\,b^{3} c - 44\,a b c^{2}\right)} f^{2} + 2\,{\left(96\,c^{4} d f + {\left(35\,b^{2} c^{2} - 36\,a c^{3}\right)} f^{2}\right)} x + 48\,{\left(2\,c^{4} x - 3\,b c^{3}\right)} e^{2} + 16\,{\left(8\,c^{4} f x^{2} - 10\,b c^{3} f x + 24\,c^{4} d + {\left(15\,b^{2} c^{2} - 16\,a c^{3}\right)} f\right)} e\right)} \sqrt{c x^{2} + b x + a}}{768\,c^{5}}, -\frac{3\,{\left(128\,c^{4} d^{2} + 32\,{\left(3\,b^{2} c^{2} - 4\,a c^{3}\right)} d f + {\left(35\,b^{4} - 120\,a b^{2} c + 48\,a^{2} c^{2}\right)} f^{2} + 16\,{\left(3\,b^{2} c^{2} - 4\,a c^{3}\right)} e^{2} - 16\,{\left(8\,b c^{3} d + {\left(5\,b^{3} c - 12\,a b c^{2}\right)} f\right)} e\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{2} + b x + a} {\left(2\,c x + b\right)} \sqrt{-c}}{2\,{\left(c^{2} x^{2} + b c x + a c\right)}}\right) - 2\,{\left(48\,c^{4} f^{2} x^{3} - 56\,b c^{3} f^{2} x^{2} - 288\,b c^{3} d f - 5\,{\left(21\,b^{3} c - 44\,a b c^{2}\right)} f^{2} + 2\,{\left(96\,c^{4} d f + {\left(35\,b^{2} c^{2} - 36\,a c^{3}\right)} f^{2}\right)} x + 48\,{\left(2\,c^{4} x - 3\,b c^{3}\right)} e^{2} + 16\,{\left(8\,c^{4} f x^{2} - 10\,b c^{3} f x + 24\,c^{4} d + {\left(15\,b^{2} c^{2} - 16\,a c^{3}\right)} f\right)} e\right)} \sqrt{c x^{2} + b x + a}}{384\,c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(128*c^4*d^2 + 32*(3*b^2*c^2 - 4*a*c^3)*d*f + (35*b^4 - 120*a*b^
2*c + 48*a^2*c^2)*f^2 + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 - 16*(8*b*c^3*d + (5*b
^3*c - 12*a*b*c^2)*f)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*
x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(48*c^4*f^2*x^3 - 56*b*c^3*
f^2*x^2 - 288*b*c^3*d*f - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 2*(96*c^4*d*f + (
35*b^2*c^2 - 36*a*c^3)*f^2)*x + 48*(2*c^4*x - 3*b*c^3)*e^2 + 16*(8*c^4*f*x^
2 - 10*b*c^3*f*x + 24*c^4*d + (15*b^2*c^2 - 16*a*c^3)*f)*e)*sqrt(c*x^2 + b*
x + a))/c^5, -1/384*(3*(128*c^4*d^2 + 32*(3*b^2*c^2 - 4*a*c^3)*d*f + (35*b^
4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 - 16*(8*b*
c^3*d + (5*b^3*c - 12*a*b*c^2)*f)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f^2*x^3 - 56*
b*c^3*f^2*x^2 - 288*b*c^3*d*f - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 2*(96*c^4*d
*f + (35*b^2*c^2 - 36*a*c^3)*f^2)*x + 48*(2*c^4*x - 3*b*c^3)*e^2 + 16*(8*c^
4*f*x^2 - 10*b*c^3*f*x + 24*c^4*d + (15*b^2*c^2 - 16*a*c^3)*f)*e)*sqrt(c*x^
2 + b*x + a))/c^5]
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + fx^{2}\right)^{2}}{\sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 6.71, size = 304, normalized size = 0.96

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6f^2x}{c}-\frac{7bc^2f^2-16c^3fe}{c^4}\right)x+\frac{96c^3df+35b^2cf^2-36ac^2f^2-80bc^2fe+48c^3e^2}{c^4}\right)x-\frac{288bc^2df+105b^3f^2-220abcf^2-384c^3de-240b^2cfe+256ac^2fe+144bc^2e^2}{c^4}\right)-\frac{(128c^4d^2+96b^2c^2df-128ac^3df+35b^4f^2-120ab^2cf^2+48a^2c^2f^2-128bc^3de-80b^3cfe+192abc^2fe+48b^2c^2e^2-64ac^3e^2)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f^2*x/c - (7*b*c^2*f^2 - 16*c^3*f*e)/c
^4)*x + (96*c^3*d*f + 35*b^2*c*f^2 - 36*a*c^2*f^2 - 80*b*c^2*f*e + 48*c^3*e
^2)/c^4)*x - (288*b*c^2*d*f + 105*b^3*f^2 - 220*a*b*c*f^2 - 384*c^3*d*e - 2
40*b^2*c*f*e + 256*a*c^2*f*e + 144*b*c^2*e^2)/c^4) - 1/128*(128*c^4*d^2 + 9
6*b^2*c^2*d*f - 128*a*c^3*d*f + 35*b^4*f^2 - 120*a*b^2*c*f^2 + 48*a^2*c^2*f
^2 - 128*b*c^3*d*e - 80*b^3*c*f*e + 192*a*b*c^2*f*e + 48*b^2*c^2*e^2 - 64*a
*c^3*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9
/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x^2+e\,x+d)^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2), x)
```

3.111 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=116

$$\frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}+\frac{(8c^2d+3b^2f-4c(be+af))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)+1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c*x
^2+b*x+a)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1675, 654, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf\right)}{4c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf\right)}{}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{(8c^2d+3b^2f-4c(be+af))\,\mathrm{ta}}{8c^5}$$

Mathematica [A]
time = 0.44, size = 99, normalized size = 0.85

$$\frac{2\sqrt{c}\,(4ce-3bf+2cfx)\sqrt{a+x(b+cx)}+(-8c^2d-3b^2f+4c(be+af))\log\left(c^2\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*(4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (-8*c^2*d - 3*
b^2*f + 4*c*(b*e + a*f))*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])/(8*c^(5/2))
```

Maple [A]
time = 0.13, size = 188, normalized size = 1.62

method	result

risch	$-\frac{(-2cfx+3bf-4ce)\sqrt{c\,x^2+bx+a}}{4c^2} - \frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)af}{2c^{\frac{3}{2}}} + \frac{3\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)b^2 f}{8c^{\frac{5}{2}}}$
default	$f\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c} - \frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c} - \frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c} - \frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+b}\right.}{2c^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] f*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+e*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more deta

Fricas [A]
time = 3.59, size = 231, normalized size = 1.99

$$\left[-\frac{(8c^2d-4bce+(3b^2-4ac)f)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2+4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)-4(2c^2fx-3bcf+4c^2e)\sqrt{cx^2+bx+a}}{16c^3}, -\frac{(8c^2d-4bce+(3b^2-4ac)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right)-2(2c^2fx-3bcf+4c^2e)\sqrt{cx^2+bx+a}}{8c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c

```
^2*f*x - 3*b*c*f + 4*c^2*e)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x - 3*b*c*f + 4*c^2*e)
*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 5.55, size = 98, normalized size = 0.84

$$\frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}-\frac{3\,bf-4\,ce}{c^2}\right)-\frac{(8\,c^2d+3\,b^2f-4\,acf-4\,bce)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}\,-b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.112 $\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$

Optimal. Leaf size=374

$$-\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}+\frac{\sqrt{2}\,f\tanh^{-1}}{}$$

```
[Out] -f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^
(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)
*(-4*d*f+e^2)^(1/2))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+f*arctanh(1/4*(4*a*f-b*(e+(-4
*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*2^
(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {997, 738, 212}

$$\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 997

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[1/((
b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[2*(c/q), Int[1/((b + q
 + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b
^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-\cdots\right.}}}{\sqrt{\cdots}}$$

$$= -\frac{(4f)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx,\cdots\right.}{\sqrt{e^2-4df}\cdots}$$

$$= -\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-\cdots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-\cdots}}}\right.}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.39, size = 211, normalized size = 0.56

$$-\text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}\,d\#1+2a\sqrt{c}\,e\#1+4cd\#1^2+be\#1^2-2af\#1^2-2\sqrt{c}\,e\#1^3+f\#1^4\&,\frac{b\log\left(-\sqrt{c}\,x+\sqrt{a+bx+cx^2}-\#1\right)-2\sqrt{c}\,\log\left(-\sqrt{c}\,x+\sqrt{a+bx+cx^2}-\#1\right)\#1}{2b\sqrt{c}\,d-a\sqrt{c}\,e-4cd\#1-be\#1+2af\#1+3\sqrt{c}\,e\#1^2-2f\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*
d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*Log[-(Sqr
```

```
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1
+ 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(330) = 660$.
time = 0.16, size = 761, normalized size = 2.03

method	result
default	$\frac{\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2af^2-bef-2cdf+ce^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+bf-ce\right)\left(x+\frac{e+\sqrt{-4df+e}}{2f}\right.}{f}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e
^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2
)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-%e^2>0)', see `assume?` for m
ore det
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11127 vs. $2(326) = 652$.
time = 15.62, size = 11127, normalized size = 29.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((2*c*d*f - 2*a*f^2 + b*f*e - c*e^2 + (4*c^2*d^3*f + 4*a^2*
d*f^3 + 4*(b^2 - 2*a*c)*d^2*f^2 - a*c*e^4 + (b*c*d + a*b*f)*e^3 - (c^2*d^2
+ a^2*f^2 + (b^2 - 6*a*c)*d*f)*e^2 - 4*(b*c*d^2*f + a*b*d*f^2)*e)*sqrt(-(b^
2*f^2 - 2*b*c*f*e + c^2*e^2)/(4*c^4*d^5*f + 4*a^4*d*f^5 + 8*(b^2*c^2 - 2*a*
c^3)*d^4*f^2 + 4*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 + 8*(a^2*b^2 - 2*a^3
*c)*d^2*f^4 - a^2*c^2*e^6 + 2*(a*b*c^2*d + a^2*b*c*f)*e^5 - ((b^2*c^2 + 2*a
*c^3)*d^2 + 4*(a*b^2*c - 2*a^2*c^2)*d*f + (a^2*b^2 + 2*a^3*c)*f^2)*e^4 + 2*
(b*c^3*d^3 + a^3*b*f^3 + (b^3*c - 5*a*b*c^2)*d^2*f + (a*b^3 - 5*a^2*b*c)*d*
f^2)*e^3 - (c^4*d^4 + a^4*f^4 - 2*(b^2*c^2 + 6*a*c^3)*d^3*f + (b^4 - 20*a*b
^2*c + 22*a^2*c^2)*d^2*f^2 - 2*(a^2*b^2 + 6*a^3*c)*d*f^3)*e^2 - 8*(b*c^3*d^
4*f + a^3*b*d*f^4 + (b^3*c - a*b*c^2)*d^3*f^2 + (a*b^3 - a^2*b*c)*d^2*f^3)*
e)))/(4*c^2*d^3*f + 4*a^2*d*f^3 + 4*(b^2 - 2*a*c)*d^2*f^2 - a*c*e^4 + (b*c*
d + a*b*f)*e^3 - (c^2*d^2 + a^2*f^2 + (b^2 - 6*a*c)*d*f)*e^2 - 4*(b*c*d^2*f
 + a*b*d*f^2)*e))*log(-(4*b*c*d*f^2*x + 2*b^2*d*f^2 + sqrt(2)*(4*b*c*d^2*f^
2 - 4*a*b*d*f^3 + (c^2*d - a*c*f)*e^3 - (b*c*d*f - a*b*f^2)*e^2 - 4*(c^2*d^
2*f - a*c*d*f^2)*e - (8*b*c^2*d^4*f^2 + 8*a^2*b*d^2*f^4 + 8*(b^3 - 2*a*b*c)
*d^3*f^3 + (a*c^2*d + a^2*c*f)*e^5 - (b*c^2*d^2 + 4*a*b*c*d*f + a^2*b*f^2)*
e^4 + (c^3*d^3 + a^3*f^3 + (3*b^2*c - 5*a*c^2)*d^2*f + (3*a*b^2 - 5*a^2*c)*
d*f^2)*e^3 + 2*(b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 10*a*b*c)*d^2*f^2)*e^2 -
 4*(c^3*d^4*f + a^3*d*f^4 + (3*b^2*c - a*c^2)*d^3*f^2 + (3*a*b^2 - a^2*c)*d
^2*f^3)*e)*sqrt(-(b^2*f^2 - 2*b*c*f*e + c^2*e^2)/(4*c^4*d^5*f + 4*a^4*d*f^5
 + 8*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + 4*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3
+ 8*(a^2*b^2 - 2*a^3*c)*d^2*f^4 - a^2*c^2*e^6 + 2*(a*b*c^2*d + a^2*b*c*f)*e
^5 - ((b^2*c^2 + 2*a*c^3)*d^2 + 4*(a*b^2*c - 2*a^2*c^2)*d*f + (a^2*b^2 + 2*
a^3*c)*f^2)*e^4 + 2*(b*c^3*d^3 + a^3*b*f^3 + (b^3*c - 5*a*b*c^2)*d^2*f + (a
*b^3 - 5*a^2*b*c)*d*f^2)*e^3 - (c^4*d^4 + a^4*f^4 - 2*(b^2*c^2 + 6*a*c^3)*d
^3*f + (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*f^2 - 2*(a^2*b^2 + 6*a^3*c)*d*f^
3)*e^2 - 8*(b*c^3*d^4*f + a^3*b*d*f^4 + (b^3*c - a*b*c^2)*d^3*f^2 + (a*b^3
- a^2*b*c)*d^2*f^3)*e)))*sqrt(c*x^2 + b*x + a)*sqrt((2*c*d*f - 2*a*f^2 + b*
f*e - c*e^2 + (4*c^2*d^3*f + 4*a^2*d*f^3 + 4*(b^2 - 2*a*c)*d^2*f^2 - a*c*e^
```

```
4 + (b*c*d + a*b*f)*e^3 - (c^2*d^2 + a^2*f^2 + (b^2 - 6*a*c)*d*f)*e^2 - 4*(
b*c*d^2*f + a*b*d*f^2)*e)*sqrt(-(b^2*f^2 - 2*b*c*f*e + c^2*e^2)/(4*c^4*d^5*
f + 4*a^4*d*f^5 + 8*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + 4*(b^4 - 4*a*b^2*c + 6*a^
2*c^2)*d^3*f^3 + 8*(a^2*b^2 - 2*a^3*c)*d^2*f^4 - a^2*c^2*e^6 + 2*(a*b*c^2*d
 + a^2*b*c*f)*e^5 - ((b^2*c^2 + 2*a*c^3)*d^2 + 4*(a*b^2*c - 2*a^2*c^2)*d*f
+ (a^2*b^2 + 2*a^3*c)*f^2)*e^4 + 2*(b*c^3*d^3 + a^3*b*f^3 + (b^3*c - 5*a*b*
c^2)*d^2*f + (a*b^3 - 5*a^2*b*c)*d*f^2)*e^3 - (c^4*d^4 + a^4*f^4 - 2*(b^2*c
^2 + 6*a*c^3)*d^3*f + (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*f^2 - 2*(a^2*b^2
+ 6*a^3*c)*d*f^3)*e^2 - 8*(b*c^3*d^4*f + a^3*b*d*f^4 + (b^3*c - a*b*c^2)*d^
3*f^2 + (a*b^3 - a^2*b*c)*d^2*f^3)*e)))/(4*c^2*d^3*f + 4*a^2*d*f^3 + 4*(b^2
 - 2*a*c)*d^2*f^2 - a*c*e^4 + (b*c*d + a*b*f)*e^3 - (c^2*d^2 + a^2*f^2 + (b
^2 - 6*a*c)*d*f)*e^2 - 4*(b*c*d^2*f + a*b*d*f^2)*e)) + (b*c*f*x + 2*a*c*f)*
e^2 - (2*b*c*d*f + 2*a*b*f^2 + (4*c^2*d*f + b^2*f^2)*x)*e - (8*a*c^2*d^3*f^
2 + 8*a^3*d*f^4 + 8*(a*b^2 - 2*a^2*c)*d^2*f^3 + 4*(b*c^2*d^3*f^2 + a^2*b*d*
f^4 + (b^3 - 2*a*b*c)*d^2*f^3)*x - (a*b*c*f*x + 2*a^2*c*f)*e^4 + (2*a*b*c*d
*f + 2*a^2*b*f^2 + (b^2*c*d*f + a*b^2*f^2)*x)*e^3 - (2*a*c^2*d^2*f + 2*a^3*
f^3 + 2*(a*b^2 - 6*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 + (b^3 - 6*a*b*c
)*d*f^2)*x)*e^2 - 4*(2*a*b*c*d^2*f^2 + 2*a^2*b*d*f^3 + (b^2*c*d^2*f^2 + a*b
^2*d*f^3)*x)*e)*sqrt(-(b^2*f^2 - 2*b*c*f*e + c^2*e^2)/(4*c^4*d^5*f + 4*a^4*
d*f^5 + 8*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + 4*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
*f^3 + 8*(a^2*b^2 - 2*a^3*c)*d^2*f^4 - a^2*c^2*e^6 + 2*(a*b*c^2*d + a^2*b*c
*f)*e^5 - ((b^2*c^2 + 2*a*c^3)*d^2 + 4*(a*b^2*c - 2*a^2*c^2)*d*f + (a^2*b^2
 + 2*a^3*c)*f^2)*e^4 + 2*(b*c^3*d^3 + a^3*b*f^3 + (b^3*c - 5*a*b*c^2)*d^2*f
 + (a*b^3 - 5*a^2*b*c)*d*f^2)*e^3 - (c^4*d^4 + a^4*f^4 - 2*(b^2*c^2 + 6*a*c
^3)*d^3*f + (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*f^2 - 2*(a^2*b^2 + 6*a^3*c)
*d*f^3)*e^2 - 8*(b*c^3*d^4*f + a^3*b*d*f^4 + (b^3*c - a*b*c^2)*d^3*f^2 + (a
*b^3 - a^2*b*c)*d^2*f^3)*e)))/x) - 1/4*sqrt(2)*sqrt((2*c*d*f - 2*a*f^2 + b*
f*e - c*e^2 + (4*c^2*d^3*f + 4*a^2*d*f^3 + 4*(b^2 - 2*a*c)*d^2*f^2 - a*c*e^
4 + (b*c*d + a*b*f)*e^3 - (c^2*d^2 + a^2*f^2 + (b^2 - 6*a*c)*d*f)*e^2 - 4*(
b*c*d^2*f + a*b*d*f^2)*e)*sqrt(-(b^2*f^2 - 2*b*c*f*e + c^2*e^2)/(4*c^4*d^5*
f + 4*a^4*d*f^5 + 8*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + 4*(b^4 - 4*a*b^2*c + 6*a^
2*c^2)*d^3*f^3 + 8*(a^2*b^2 - 2*a^3*c)*d^2*f^4 - a^2*c^2*e^6 + 2*(a*b*c^2*d
 + a^2*b*c*f)*e^5 - ((b^2*c^2 + 2*a*c^3)*d^2 + 4*(a*b^2*c - 2*a^2*c^2)*d*f
+ (a^2*b^2 + 2*a^3*c)*f^2)*e^4 + 2*(b*c^3*d^3 + a^3*b*f^3 + (b^3*c - 5*a*b*
c^2)*d^2*f + (a*b^3 - 5*a^2*b*c)*d*f^2)*e^3 - (c^4*d^4 + a^4*f^4 - 2*(b^2*c
^2 + 6*a*c^3)*d^3*f + (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*f^2 - 2*(a^2*b^2
+ 6*a^3*c)*d*f^3)*e^2 - 8*(b*c^3*d^4*f + a^3*b*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.113 $\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx$

Optimal. Leaf size=789

$$\frac{\left(f(be^2-2bdf-aef)-c(e^3-3def)+f(f(be-2af)-c(e^2-2df))x\right)\sqrt{a+bx+cx^2}}{(e^2-4df)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(d+ex+fx^2)}+\frac{\Big(f(2cd-be+2\ldots}{\ldots}$$

```
[Out] (f*(-a*e*f-2*b*d*f+b*e^2)-c*(-3*d*e*f+e^3)+f*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2)
)*x)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/
(f*x^2+e*x+d)+1/4*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(
e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a
*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d*(-4*d*f+e^2)+f*(3
*a*b*e*f-4*a^2*f^2+b^2*(-6*d*f+e^2))-c*(4*a*f*(-3*d*f+e^2)+b*(-5*d*e*f+e^3)
))+f*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(3/2
)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2
-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/4*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e
^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/
(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2
*c^2*d*(-4*d*f+e^2)+f*(3*a*b*e*f-4*a^2*f^2+b^2*(-6*d*f+e^2))-c*(4*a*f*(-3*d
*f+e^2)+b*(-5*d*e*f+e^3)))+f*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e+(-4*d*f+e^2)^(
1/2)))/(-4*d*f+e^2)^(3/2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(c*e
^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A]
time = 7.68, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {988, 1046, 738, 212}

$$\frac{\left(f\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2f(-4a^2f^3+3abef^2-4acf(e^2-3df)+b^2f(e^2-6df)-bc(e^3-5def)+2c^2d(e^2-4df))\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2\sqrt{2}(e^2-4df)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(f\left(\sqrt{e^2-4df}+e\right)(ce-bf)(2af-be+2cd)-2f(-4a^2f^3+3abef^2-4acf(e^2-3df)+b^2f(e^2-6df)-bc(e^3-5def)+2c^2d(e^2-4df))\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2\sqrt{2}(e^2-4df)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\sqrt{a+bx+cx^2}\left(fx(f(be-2af)-c(e^2-2df))+f(-aef-2bdf+be^2)-c(e^3-3def)\right)}{(e^2-4df)(d+ex+fx^2)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] ((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c*
(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*(2*c*d - b*e + 2*a*f)*(c*e
- b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2
- 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e
*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*(e^2 - 4*d*f)^(3
```

```
/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((f*(2*c*d - b*e + 2*a*f)*(c*
e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^
2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d
*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*(e^2 - 4*d*f)^
(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
```

```
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \frac{(f(be^2-2bdf-aef)-c(e^3-3def)+f(f(be-2af)-c(e^2-2\ldots}{(e^2-4df)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(d+e\ldots}$$

$$= \frac{(f(be^2-2bdf-aef)-c(e^3-3def)+f(f(be-2af)-c(e^2-2\ldots}{(e^2-4df)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(d+e\ldots}$$

$$= \frac{(f(be^2-2bdf-aef)-c(e^3-3def)+f(f(be-2af)-c(e^2-2\ldots}{(e^2-4df)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(d+e\ldots}$$

$$= \frac{(f(be^2-2bdf-aef)-c(e^3-3def)+f(f(be-2af)-c(e^2-2\ldots}{(e^2-4df)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(d+e\ldots}$$

Mathematica [A]
time = 12.95, size = 1369, normalized size = 1.74

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] ((-4*Sqrt[a + x*(b + c*x)]*(c*(e^3 - 3*d*e*f + e^2*f*x - 2*d*f^2*x) + f*(a*
f*(e + 2*f*x) - b*(e^2 - 2*d*f + e*f*x))))/((e^2 - 4*d*f)*(d + x*(e + f*x))
) - (Sqrt[2]*f*(8*a^2*f^3 + 2*a*b*f^2*(-4*e + Sqrt[e^2 - 4*d*f]) - b^2*f*(e
^2 - 12*d*f + e*Sqrt[e^2 - 4*d*f]) - 2*c^2*d*(e^2 - 8*d*f + e*Sqrt[e^2 - 4*
d*f]) - 2*a*c*f*(-5*e^2 + 12*d*f + e*Sqrt[e^2 - 4*d*f]) + b*c*(e^3 - 12*d*e
*f + e^2*Sqrt[e^2 - 4*d*f] + 2*d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqrt[e^2 -
4*d*f] - 2*f*x])/((e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*f*(8*a^2*f^3 -
2*a*b*f^2*(4*e + Sqrt[e^2 - 4*d*f]) + 2*a*c*f*(5*e^2 - 12*d*f + e*Sqrt[e^2
- 4*d*f]) + 2*c^2*d*(-e^2 + 8*d*f + e*Sqrt[e^2 - 4*d*f]) + b^2*f*(-e^2 + 12
*d*f + e*Sqrt[e^2 - 4*d*f]) + b*c*(e^3 - 12*d*e*f - e^2*Sqrt[e^2 - 4*d*f] -
 2*d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x])/((e^2 - 4*d*
```

```
f)^(3/2)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqr
t[e^2 - 4*d*f]))]) + (Sqrt[2]*f*(-8*a^2*f^3 + 2*a*b*f^2*(4*e + Sqrt[e^2 - 4
*d*f]) + b^2*f*(e^2 - 12*d*f - e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f*(-5*e^2 + 12*
d*f - e*Sqrt[e^2 - 4*d*f]) - 2*c^2*d*(-e^2 + 8*d*f + e*Sqrt[e^2 - 4*d*f]) +
 b*c*(-e^3 + 12*d*e*f + e^2*Sqrt[e^2 - 4*d*f] + 2*d*f*Sqrt[e^2 - 4*d*f]))*L
og[-4*a*f + 2*c*e*x + 2*c*Sqrt[e^2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] -
2*f*x) - 2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f -
b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]])/((e^2 - 4*d*f)^(3/2)*Sq
rt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d
*f]))]) + (Sqrt[2]*f*(8*a^2*f^3 + 2*a*b*f^2*(-4*e + Sqrt[e^2 - 4*d*f]) - b^
2*f*(e^2 - 12*d*f + e*Sqrt[e^2 - 4*d*f]) - 2*c^2*d*(e^2 - 8*d*f + e*Sqrt[e^
2 - 4*d*f]) - 2*a*c*f*(-5*e^2 + 12*d*f + e*Sqrt[e^2 - 4*d*f]) + b*c*(e^3 -
12*d*e*f + e^2*Sqrt[e^2 - 4*d*f] + 2*d*f*Sqrt[e^2 - 4*d*f]))*Log[b*(-e + Sq
rt[e^2 - 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d*f]*x + Sqrt[
2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[
e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/((e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]))/(4
*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2107 vs. $2(735) = 1470$.

time = 0.16, size = 2108, normalized size = 2.67

method	result	size
default	Expression too large to display	2108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+
e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((
-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)
/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f))-2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d
*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)
))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(
-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d
*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
```

```
))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-1/(4*d*f-e^2)*(-2/(b*f*(-4*d*f+e^2)^(
1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/(x-1/2/f*(-e+(
-4*d*f+e^2)^(1/2)))*((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^
(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1
/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+(c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*f/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/
2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e
^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2)))))-1/(4*d*f-e^2)*(-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*((x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(
-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)
*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + x*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,{\left(f\,x^2+e\,x+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2), x)
```

3.114 $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=649

$$\frac{2(3ab^4cef^2-ab^5f^3+ab^3cf(5af^2-3c(e^2+df))-bc^2(c^3d^3+5a^3f^3+3ac^2d(e^2+df)-9a^2cf(e^2+df))-}{}$$

```
[Out] 3/128*(105*b^4*f^3-280*b^2*c*f^2*(a*f+b*e)+128*c^4*d*(d*f+e^2)+80*c^2*f*(6*
a*b*e*f+a^2*f^2+3*b^2*(d*f+e^2))-64*c^3*(3*a*f*(d*f+e^2)+b*(6*d*e*f+e^3)))*
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+2*(3*a*b^4*c*e*
f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^3*d^3+5*a^3*f^3+3*
a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a*f^2-c*(6*d*f+e^2))
+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*c*f+b^2*f-b*c*e+2*c^
2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*d*f-3*a*c^3*e^2+b^4*f
^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*d^2)*x)/c^5/(-4*a*c+b^
2)/(c*x^2+b*x+a)^(1/2)-1/64*(187*b^3*f^3-4*b*c*f^2*(73*a*f+114*b*e)-64*c^3*
(6*d*e*f+e^3)+16*c^2*f*(20*a*e*f+21*b*(d*f+e^2)))*(c*x^2+b*x+a)^(1/2)/c^5+1
/32*f*(41*b^2*f^2-4*c*f*(7*a*f+22*b*e)+48*c^2*(d*f+e^2))*x*(c*x^2+b*x+a)^(1
/2)/c^4+1/8*f^2*(-5*b*f+8*c*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1/4*f^3*x^3*(c*x
^2+b*x+a)^(1/2)/c^2
```

Rubi [A]
time = 1.27, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1674, 1675, 654, 635, 212}

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(80c^2f(a^2f^2+6abef+3b^2(df+e^2))-280b^2cf^2(af+be)-64c^3(3af(df+e^2)+b(6def+e^3))+105b^4f^3+128c^4d(df+e^2)\right)}{128c^{11/2}}+\text{[illegible]}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(c^5*(b^2 -
 4*a*c)*Sqrt[a + b*x + c*x^2]) - ((187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*
f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*Sqrt[
a + b*x + c*x^2])/(64*c^5) + (f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) + 48*c
^2*(e^2 + d*f))*x*Sqrt[a + b*x + c*x^2])/(32*c^4) + (f^2*(8*c*e - 5*b*f)*x^
2*Sqrt[a + b*x + c*x^2])/(8*c^3) + (f^3*x^3*Sqrt[a + b*x + c*x^2])/(4*c^2)
```

```
+ (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*
c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f)
 + b*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(128*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

Mathematica [A]
time = 4.24, size = 768, normalized size = 1.18

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*(315*b^6*f^3*x + 105*b^5*f^2*(3*a*f + c*x*(-8*e + f*x)) - 2*b^4*
c*f*(105*a*f*(4*e + 9*f*x) + c*x*(-360*e^2 + 140*e*f*x + 3*f*(-120*d + 7*f*
x^2))) + 8*b^3*c*(-210*a^2*f^3 + a*c*f*(90*e^2 + 530*e*f*x + f*(90*d - 77*f
*x^2)) + c^2*x*(-24*e^3 + 30*e^2*f*x + 3*f^2*x*(10*d + f*x^2) + 2*e*f*(-72*
d + 7*f*x^2))) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3 + 1
86*e^2*f*x + 2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*x*(-
24*d^2*f + 6*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4*e*f^
2*x^2 + f^3*x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) + a^2*c*
```

(16*e^3 + 36*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2)) + 2*
a*c^2*(-12*d^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2*(4*e^3
+ 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8*c^3*d^2*
(d - 3*e*x) + a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^2)) + 2*a*
c^2*(12*d^2*f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3 + 30*e^2*f*
x + 14*e*f^2*x^2 + 3*f^3*x^3)))) + 3*(b^2 - 4*a*c)*(105*b^4*f^3 - 280*b^2*c
f^2(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6*a*b*e*f + a^2*f^2 +
3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^3 + 6*d*e*f)))*Sqrt[a
 + x*(b + c*x)]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(128*c^(1
1/2)*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)])

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2298 vs. $2(621) = 1242$.
time = 0.16, size = 2299, normalized size = 3.54

method	result	size
default	Expression too large to display	2299
risch	Expression too large to display	2715

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] f^3*(1/4*x^5/c/(c*x^2+b*x+a)^(1/2)-9/8*b/c*(1/3*x^4/c/(c*x^2+b*x+a)^(1/2)-7
/6*b/c*(1/2*x^3/c/(c*x^2+b*x+a)^(1/2)-5/4*b/c*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/
2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*
x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)))-3/2*a/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x
^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-4/3*a/c*(x^2/c/(c*x^2+b*x+a)^(1
/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c
*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2))))-5/4*a/c*(1/2*x^3/c/(c*x^2+b*x+a)^(1/2)-5/4*b/
c*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/
c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^
(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))-3/2*a/c*(-x/c/(c*x^2+
b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c
*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
+3*e*f^2*(1/3*x^4/c/(c*x^2+b*x+a)^(1/2)-7/6*b/c*(1/2*x^3/c/(c*x^2+b*x+a)^(1
/2)-5/4*b/c*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/
2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x
^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))-3/2*a/c*(-x

```
/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))))-4/3*a/c*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1
/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+
a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/
c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))))+(d*f
^2+2*e^2*f+f*(2*d*f+e^2))*(1/2*x^3/c/(c*x^2+b*x+a)^(1/2)-5/4*b/c*(x^2/c/(c*
x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x
+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2
*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))-3/2*a/c*(-x/c/(c*x^2+b*x+a)^(1/2)
-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(4*d*e*f+e*(
2*d*f+e^2))*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/
2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x
^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))+(d*(2*d*f+e
^2)+2*d*e^2+f*d^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1
/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+3*d^2*e*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+2*d^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1553 vs. $2(619) = 1238$.
time = 4.19, size = 3109, normalized size = 4.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

[Out] [1/256*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d^2*f + 48*(5*a*b^4*c^2 - 24*a^2*b^2
*c^3 + 16*a^3*c^4)*d*f^2 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 -
64*a^4*c^3)*f^3 + (128*(b^2*c^5 - 4*a*c^6)*d^2*f + 48*(5*b^4*c^3 - 24*a*b^2
*c^4 + 16*a^2*c^5)*d*f^2 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 -
64*a^3*c^4)*f^3)*x^2 + (128*(b^3*c^4 - 4*a*b*c^5)*d^2*f + 48*(5*b^5*c^2 - 2
4*a*b^3*c^3 + 16*a^2*b*c^4)*d*f^2 + 5*(21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c
^2 - 64*a^3*b*c^3)*f^3)*x - 64*(a*b^3*c^3 - 4*a^2*b*c^4 + (b^3*c^4 - 4*a*b*
c^5)*x^2 + (b^4*c^3 - 4*a*b^2*c^4)*x)*e^3 + 16*((8*(b^2*c^5 - 4*a*c^6)*d +
3*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*x^2 + 8*(a*b^2*c^4 - 4*a^2*c^5
)*d + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*f + (8*(b^3*c^4 - 4*a*b
*c^5)*d + 3*(5*b^5*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x)*e^2 - 8*(48*(a*
b^3*c^3 - 4*a^2*b*c^4)*d*f + 5*(7*a*b^5*c - 40*a^2*b^3*c^2 + 48*a^3*b*c^3)*
f^2 + (48*(b^3*c^4 - 4*a*b*c^5)*d*f + 5*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*
b*c^4)*f^2)*x^2 + (48*(b^4*c^3 - 4*a*b^2*c^4)*d*f + 5*(7*b^6*c - 40*a*b^4*c
^2 + 48*a^2*b^2*c^3)*f^2)*x)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(128*b*c^6*d^3 + 384
*a*b*c^5*d^2*f - 16*(b^2*c^5 - 4*a*c^6)*f^3*x^5 + 24*(b^3*c^4 - 4*a*b*c^5)*
f^3*x^4 + 48*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d*f^2 + (315*a*b^5*c - 1680*a^2*
b^3*c^2 + 1808*a^3*b*c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*d*f^2 + (21*b^4*c
^3 - 104*a*b^2*c^4 + 80*a^2*c^5)*f^3)*x^3 + (240*(b^3*c^4 - 4*a*b*c^5)*d*f^
2 + 7*(15*b^5*c^2 - 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3)*x^2 + (256*c^7*d^3 +
 384*(b^2*c^5 - 2*a*c^6)*d^2*f + 48*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5
)*d*f^2 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 - 480*a^3*c^4)*f^3
)*x - 64*(3*a*b^2*c^4 - 8*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (3*b^3*c^4 -
10*a*b*c^5)*x)*e^3 + 48*(8*a*b*c^5*d - 2*(b^2*c^5 - 4*a*c^6)*f*x^3 + 5*(b^3
*c^4 - 4*a*b*c^5)*f*x^2 + (15*a*b^3*c^3 - 52*a^2*b*c^4)*f + (8*(b^2*c^5 - 2
*a*c^6)*d + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*f)*x)*e^2 - 8*(96*a*c^
6*d^2 + 8*(b^2*c^5 - 4*a*c^6)*f^2*x^4 - 14*(b^3*c^4 - 4*a*b*c^5)*f^2*x^3 +
48*(3*a*b^2*c^4 - 8*a^2*c^5)*d*f + (105*a*b^4*c^2 - 460*a^2*b^2*c^3 + 256*a
^3*c^4)*f^2 + (48*(b^2*c^5 - 4*a*c^6)*d*f + (35*b^4*c^3 - 172*a*b^2*c^4 + 1
28*a^2*c^5)*f^2)*x^2 + (48*b*c^6*d^2 + 48*(3*b^3*c^4 - 10*a*b*c^5)*d*f + (1
05*b^5*c^2 - 530*a*b^3*c^3 + 488*a^2*b*c^4)*f^2)*x)*e)*sqrt(c*x^2 + b*x + a
))/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 - 4*a*c^8)*x^2 + (b^3*c^6 - 4*a*b*c^7)
x), -1/128(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d^2*f + 48*(5*a*b^4*c^2 - 24*a^
2*b^2*c^3 + 16*a^3*c^4)*d*f^2 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c
^2 - 64*a^4*c^3)*f^3 + (128*(b^2*c^5 - 4*a*c^6)*d^2*f + 48*(5*b^4*c^3 - 24*
a*b^2*c^4 + 16*a^2*c^5)*d*f^2 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c
^3 - 64*a^3*c^4)*f^3)*x^2 + (128*(b^3*c^4 - 4*a*b*c^5)*d^2*f + 48*(5*b^5*c^
2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*d*f^2 + 5*(21*b^7 - 140*a*b^5*c + 240*a^2*
b^3*c^2 - 64*a^3*b*c^3)*f^3)*x - 64*(a*b^3*c^3 - 4*a^2*b*c^4 + (b^3*c^4 - 4
*a*b*c^5)*x^2 + (b^4*c^3 - 4*a*b^2*c^4)*x)*e^3 + 16*((8*(b^2*c^5 - 4*a*c^6)
d + 3(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*x^2 + 8*(a*b^2*c^4 - 4*a^
2*c^5)*d + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*f + (8*(b^3*c^4 -
4*a*b*c^5)*d + 3*(5*b^5*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x)*e^2 - 8*(4
8*(a*b^3*c^3 - 4*a^2*b*c^4)*d*f + 5*(7*a*b^5*c - 40*a^2*b^3*c^2 + 48*a^3*b*

```
c^3)*f^2 + (48*(b^3*c^4 - 4*a*b*c^5)*d*f + 5*(7*b^5*c^2 - 40*a*b^3*c^3 + 48
*a^2*b*c^4)*f^2)*x^2 + (48*(b^4*c^3 - 4*a*b^2*c^4)*d*f + 5*(7*b^6*c - 40*a*
b^4*c^2 + 48*a^2*b^2*c^3)*f^2)*x)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(128*b*c^6*d^3 + 384*
a*b*c^5*d^2*f - 16*(b^2*c^5 - 4*a*c^6)*f^3*x^5 + 24*(b^3*c^4 - 4*a*b*c^5)*f
^3*x^4 + 48*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d*f^2 + (315*a*b^5*c - 1680*a^2*b
^3*c^2 + 1808*a^3*b*c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*d*f^2 + (21*b^4*c^
3 - 104*a*b^2*c^4 + 80*a^2*c^5)*f^3)*x^3 + (240*(b^3*c^4 - 4*a*b*c^5)*d*f^2
 + 7*(15*b^5*c^2 - 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3)*x^2 + (256*c^7*d^3 +
384*(b^2*c^5 - 2*a*c^6)*d^2*f + 48*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)
*d*f^2 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 - 480*a^3*c^4)*f^3)
*x - 64*(3*a*b^2*c^4 - 8*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (3*b^3*c^4 - 1
0*a*b*c^5)*x)*e^3 + 48*(8*a*b*c^5*d - 2*(b^2*c^5 - 4*a*c^6)*f*x^3 + 5*(b^3*
c^4 - 4*a*b*c^5)*f*x^2 + (15*a*b^3*c^3 - 52*a^2*b*c^4)*f + (8*(b^2*c^5 - 2*
a*c^6)*d + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*f)*x)*e^2 - 8*(96*a*c^6
*d^2 + 8*(b^2*c^5 - 4*a*c^6)*f^2*x^4 - 14*(b^3*c^4 - 4*a*b*c^5)*f^2*x^3 + 4
8*(3*a*b^2*c^4 - 8*a^2*c^5)*d*f + (105*a*b^4*c^2 - 460*a^2*b^2*c^3 + 256*a^
3*c^4)*f^2 + (48*(b^2*c^5 - 4*a*c^6)*d*f + (35*b^4*c^3 - 172*a*b^2*c^4 + 12
8*a^2*c^5)*f^2)*x^2 + (48*b*c^6*d^2 + 48*(3*b^3*c^4 - 10*a*b*c^5)*d*f + (10
5*b^5*c^2 - 530*a*b^3*c^3 + 488*a^2*b*c^4)*f^2)*x)*e)*sqrt(c*x^2 + b*x + a)
)/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 - 4*a*c^8)*x^2 + (b^3*c^6 - 4*a*b*c^7)*
x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**3/(a + b*x + c*x**2)**(3/2), x)
```

Giac [**A**]
time = 3.94, size = 1099, normalized size = 1.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(((2*(4*(2*(b^2*c^4*f^3 - 4*a*c^5*f^3)*x/(b^2*c^5 - 4*a*c^6) - (3*b^3*
c^3*f^3 - 12*a*b*c^4*f^3 - 8*b^2*c^4*f^2*e + 32*a*c^5*f^2*e)/(b^2*c^5 - 4*a
*c^6))*x + (48*b^2*c^4*d*f^2 - 192*a*c^5*d*f^2 + 21*b^4*c^2*f^3 - 104*a*b^2
```

```
*c^3*f^3 + 80*a^2*c^4*f^3 - 56*b^3*c^3*f^2*e + 224*a*b*c^4*f^2*e + 48*b^2*c
^4*f*e^2 - 192*a*c^5*f*e^2)/(b^2*c^5 - 4*a*c^6))*x - (240*b^3*c^3*d*f^2 - 9
60*a*b*c^4*d*f^2 + 105*b^5*c*f^3 - 616*a*b^3*c^2*f^3 + 784*a^2*b*c^3*f^3 -
384*b^2*c^4*d*f*e + 1536*a*c^5*d*f*e - 280*b^4*c^2*f^2*e + 1376*a*b^2*c^3*f
^2*e - 1024*a^2*c^4*f^2*e + 240*b^3*c^3*f*e^2 - 960*a*b*c^4*f*e^2 - 64*b^2*
c^4*e^3 + 256*a*c^5*e^3)/(b^2*c^5 - 4*a*c^6))*x - (256*c^6*d^3 + 384*b^2*c^
4*d^2*f - 768*a*c^5*d^2*f + 720*b^4*c^2*d*f^2 - 2976*a*b^2*c^3*d*f^2 + 1152
*a^2*c^4*d*f^2 + 315*b^6*f^3 - 1890*a*b^4*c*f^3 + 2704*a^2*b^2*c^2*f^3 - 48
0*a^3*c^3*f^3 - 384*b*c^5*d^2*e - 1152*b^3*c^3*d*f*e + 3840*a*b*c^4*d*f*e -
 840*b^5*c*f^2*e + 4240*a*b^3*c^2*f^2*e - 3904*a^2*b*c^3*f^2*e + 384*b^2*c^
4*d*e^2 - 768*a*c^5*d*e^2 + 720*b^4*c^2*f*e^2 - 2976*a*b^2*c^3*f*e^2 + 1152
*a^2*c^4*f*e^2 - 192*b^3*c^3*e^3 + 640*a*b*c^4*e^3)/(b^2*c^5 - 4*a*c^6))*x
- (128*b*c^5*d^3 + 384*a*b*c^4*d^2*f + 720*a*b^3*c^2*d*f^2 - 2496*a^2*b*c^3
*d*f^2 + 315*a*b^5*f^3 - 1680*a^2*b^3*c*f^3 + 1808*a^3*b*c^2*f^3 - 768*a*c^
5*d^2*e - 1152*a*b^2*c^3*d*f*e + 3072*a^2*c^4*d*f*e - 840*a*b^4*c*f^2*e + 3
680*a^2*b^2*c^2*f^2*e - 2048*a^3*c^3*f^2*e + 384*a*b*c^4*d*e^2 + 720*a*b^3*
c^2*f*e^2 - 2496*a^2*b*c^3*f*e^2 - 192*a*b^2*c^3*e^3 + 512*a^2*c^4*e^3)/(b^
2*c^5 - 4*a*c^6))/sqrt(c*x^2 + b*x + a) - 3/128*(128*c^4*d^2*f + 240*b^2*c^
2*d*f^2 - 192*a*c^3*d*f^2 + 105*b^4*f^3 - 280*a*b^2*c*f^3 + 80*a^2*c^2*f^3
- 384*b*c^3*d*f*e - 280*b^3*c*f^2*e + 480*a*b*c^2*f^2*e + 128*c^4*d*e^2 + 2
40*b^2*c^2*f*e^2 - 192*a*c^3*f*e^2 - 64*b*c^3*e^3)*log(abs(-2*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x^2+e\,x+d)^3}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2), x)
```

3.115 $\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=309

$$\frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd-af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2+2df)) - (2c^4d^2 + b^4f^2 - 2b^2cf(be+2af)}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] 1/8*(15*b^2*f^2-12*c*f*(a*f+2*b*e)+8*c^2*(2*d*f+e^2))*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+2*(2*a*b^2*c*e*f-a*b^3*f^2+4*a*c^2*e*
(-a*f+c*d)-b*c*(c^2*d^2-3*a^2*f^2+a*c*(2*d*f+e^2))-(2*c^4*d^2+b^4*f^2-2*b^2
*c*f*(2*a*f+b*e)-2*c^3*(b*d*e+a*(2*d*f+e^2))+c^2*(6*a*b*e*f+2*a^2*f^2+b^2*(
2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+1/4*f*(-7*b*f+8*c*e)*(
c*x^2+b*x+a)^(1/2)/c^3+1/2*f^2*x*(c*x^2+b*x+a)^(1/2)/c^2
```

Rubi [A]

time = 0.27, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1674, 1675, 654, 635, 212}

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a^2f^2+ac(2df+e^2)+c^2d^2)-ab^3f^2+2ab^2cef+4ac^2e(cd-af))}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-12cf(af+2be)+15b^2f^2+8c^2(2df+e^2))}{8c^{7/2}}+\frac{f\sqrt{a+bx+cx^2}(8ce-7bf)}{4c^3}+\frac{f^2x\sqrt{a+bx+cx^2}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*(8*c*e - 7*b*
f)*Sqrt[a + b*x + c*x^2])/(4*c^3) + (f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) +
 ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeqQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2 \cdots}{c^3\,(b^2- \cdots}$$

$$= \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2 \cdots}{c^3\,(b^2- \cdots}$$

$$= \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2 \cdots}{c^3\,(b^2- \cdots}$$

$$= \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2 \cdots}{c^3\,(b^2- \cdots}$$

$$= \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2 \cdots}{c^3\,(b^2- \cdots}$$

Mathematica [A]
time = 1.50, size = 291, normalized size = 0.94

$$\frac{-\frac{2\sqrt{c}\,(15b^4f^2x+b^3f(15af+cx(-24e+5fx))+4bc(-13a^2f^2+2c^2d(d-2ex)+ac(2e^2+4df+20efx-5f^2x^2))-2b^2c(af(12e+31fx)+cx(-4e^2-8df+4efx+f^2x^2))+8c^2(2c^2d^2x+a^2f(8e+3fx)+ac(-4d(e+fx)+x(-2e^2+4efx+f^2x^2)))))}{(b^2-4ac)\sqrt{a+x(b+cx)}}+(-15b^2f^2+12cf(2be+af)-8c^2(e^2+2df))\log\left(c^3\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((-2*Sqrt[c]*(15*b^4*f^2*x + b^3*f*(15*a*f + c*x*(-24*e + 5*f*x)) + 4*b*c*(
-13*a^2*f^2 + 2*c^2*d*(d - 2*e*x) + a*c*(2*e^2 + 4*d*f + 20*e*f*x - 5*f^2*x
^2)) - 2*b^2*c*(a*f*(12*e + 31*f*x) + c*x*(-4*e^2 - 8*d*f + 4*e*f*x + f^2*x
^2)) + 8*c^2*(2*c^2*d^2*x + a^2*f*(8*e + 3*f*x) + a*c*(-4*d*(e + f*x) + x*(
-2*e^2 + 4*e*f*x + f^2*x^2)))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (-1
5*b^2*f^2 + 12*c*f*(2*b*e + a*f) - 8*c^2*(e^2 + 2*d*f))*Log[c^3*(b + 2*c*x
- 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])])/(8*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(289) = 578$.
time = 0.16, size = 749, normalized size = 2.42

method	result

default	$f^2\left(\frac{x^3}{2c\sqrt{cx^2+bx+a}}-\frac{5b\left(\frac{x^2}{c\sqrt{cx^2+bx+a}}-\frac{3b\left(-\frac{x}{c\sqrt{cx^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{cx^2+bx+a}}-\frac{\cdots}{c\left(4ac-b^2\cdots\right.}\right.}{2c}\cdots\right.}{2c}\cdots\right.}{\cdots}\cdots\right.$
risch	$\frac{2b^2xdf}{c(4ac-b^2)\sqrt{cx^2+bx+a}}+\frac{b^2xaf^2}{c^2(4ac-b^2)\sqrt{cx^2+bx+a}}-\frac{4bxde}{(4ac-b^2)\sqrt{cx^2+bx+a}}+\frac{b^4xf^2}{8c^3(4ac-b^2)\sqrt{cx^2\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f^2*(1/2*x^3/c/(c*x^2+b*x+a)^(1/2)-5/4*b/c*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b
/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b
)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c
*x^2+b*x+a)^(1/2)))-3/2*a/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+
b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+2*e*f*(x^2/c/(c*x^2+b*x+a)^(1/2)-3
/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c
*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)))+(2*d*f+e^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c
/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+2*d*e*(-1/c/(c*x^2+b*x+a)^(
1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+2*d^2*(2*c*x+b)/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(289) = 578$.
time = 3.19, size = 1277, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((16*(a*b^2*c^2 - 4*a^2*c^3)*d*f + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3
*c^2)*f^2 + (16*(b^2*c^3 - 4*a*c^4)*d*f + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^
2*c^3)*f^2)*x^2 + (16*(b^3*c^2 - 4*a*b*c^3)*d*f + 3*(5*b^5 - 24*a*b^3*c + 1
6*a^2*b*c^2)*f^2)*x + 8*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 +
(b^3*c^2 - 4*a*b*c^3)*x)*e^2 - 24*((b^3*c^2 - 4*a*b*c^3)*f*x^2 + (b^4*c - 4
*a*b^2*c^2)*f*x + (a*b^3*c - 4*a^2*b*c^2)*f)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(8*b
*c^4*d^2 + 16*a*b*c^3*d*f - 2*(b^2*c^3 - 4*a*c^4)*f^2*x^3 + 5*(b^3*c^2 - 4*
a*b*c^3)*f^2*x^2 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 + (16*c^5*d^2 + 16*(b^2*
c^3 - 2*a*c^4)*d*f + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2)*x + 8*(a*b
*c^3 + (b^2*c^3 - 2*a*c^4)*x)*e^2 - 8*(4*a*c^4*d + (b^2*c^3 - 4*a*c^4)*f*x^
2 + (3*a*b^2*c^2 - 8*a^2*c^3)*f + (2*b*c^4*d + (3*b^3*c^2 - 10*a*b*c^3)*f)*
x)*e)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x
^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((16*(a*b^2*c^2 - 4*a^2*c^3)*d*f + 3*(5
*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (16*(b^2*c^3 - 4*a*c^4)*d*f + 3*(
5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2)*x^2 + (16*(b^3*c^2 - 4*a*b*c^3)*d
*f + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2)*x + 8*(a*b^2*c^2 - 4*a^2*c^
3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)*e^2 - 24*((b^3*c^2 -
 4*a*b*c^3)*f*x^2 + (b^4*c - 4*a*b^2*c^2)*f*x + (a*b^3*c - 4*a^2*b*c^2)*f)*
e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
+ b*c*x + a*c)) + 2*(8*b*c^4*d^2 + 16*a*b*c^3*d*f - 2*(b^2*c^3 - 4*a*c^4)*f
^2*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*f^2*x^2 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2
+ (16*c^5*d^2 + 16*(b^2*c^3 - 2*a*c^4)*d*f + (15*b^4*c - 62*a*b^2*c^2 + 24*
a^2*c^3)*f^2)*x + 8*(a*b*c^3 + (b^2*c^3 - 2*a*c^4)*x)*e^2 - 8*(4*a*c^4*d +
(b^2*c^3 - 4*a*c^4)*f*x^2 + (3*a*b^2*c^2 - 8*a^2*c^3)*f + (2*b*c^4*d + (3*b
^3*c^2 - 10*a*b*c^3)*f)*x)*e)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5
 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A]
time = 6.40, size = 407, normalized size = 1.32

$$\frac{\left(\left(\frac{2\,(b^2c^2f^2-4\,ac^3f^2)x}{b^2c^3-4\,ac^4}-\frac{5\,b^3cf^2-20\,abc^2f^2-8\,b^2c^2fe+32\,ac^3fe}{b^2c^3-4\,ac^4}\right)x-\frac{16\,c^4d^2+16\,b^2c^2df-32\,ac^3df+15\,b^4f^2-62\,ab^2cf^2+24\,a^2c^2f^2-16\,bc^3de-24\,b^3cfe+80\,abc^2fe+8\,b^2c^2e^2-16\,ac^3e^2}{b^2c^3-4\,ac^4}\right)x-\frac{8\,bc^3d^2+16\,abc^2df+15\,ab^3f^2-52\,a^2bcf^2-32\,ac^3de-24\,ab^2cfe+64\,a^2c^2fe+8\,abc^2e^2}{b^2c^3-4\,ac^4}}{4\,\sqrt{cx^2+bx+a}}-\frac{(16\,c^2df+15\,b^2f^2-12\,acf^2-24\,bcfe+8\,c^2e^2)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{8\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f^2 - 4*a*c^3*f^2)*x/(b^2*c^3 - 4*a*c^4) - (5*b^3*c*f^2 -
 20*a*b*c^2*f^2 - 8*b^2*c^2*f*e + 32*a*c^3*f*e)/(b^2*c^3 - 4*a*c^4))*x - (1
6*c^4*d^2 + 16*b^2*c^2*d*f - 32*a*c^3*d*f + 15*b^4*f^2 - 62*a*b^2*c*f^2 + 2
4*a^2*c^2*f^2 - 16*b*c^3*d*e - 24*b^3*c*f*e + 80*a*b*c^2*f*e + 8*b^2*c^2*e^
2 - 16*a*c^3*e^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^2 + 16*a*b*c^2*d*f +
15*a*b^3*f^2 - 52*a^2*b*c*f^2 - 32*a*c^3*d*e - 24*a*b^2*c*f*e + 64*a^2*c^2*
f*e + 8*a*b*c^2*e^2)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 1/8*(16*c
^2*d*f + 15*b^2*f^2 - 12*a*c*f^2 - 24*b*c*f*e + 8*c^2*e^2)*log(abs(-2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f\,x^2+e\,x+d\right)}^2}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x)
```

3.116 $\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)+2*(c*(2*a*e-b*
(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1674, 12, 635, 212}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{a+bx+cx^2}}\,d}{b^2-4ac}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(2f)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x\right.}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{c^{3/2}}$$

Mathematica [A]
time = 0.55, size = 114, normalized size = 1.03

$$\frac{\frac{2\sqrt{c}\left(abf+2c^2dx+b^2fx+bc(d-ex)-2ac(e+fx)\right)}{\sqrt{a+x(b+cx)}}+\left(b^2-4ac\right)f\log\left(c\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}\left(-b^2+4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x))
)/Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*f*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)]])])/(c^(3/2)*(-b^2 + 4*a*c))
```

Maple [A]
time = 0.13, size = 201, normalized size = 1.81

method	result
default	$f\left(-\frac{x}{c\sqrt{c\,x^{2}+b x+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+b x+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+b x+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right.}{c^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2)))+e*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2))+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(101) = 202$.
time = 3.01, size = 435, normalized size = 3.92

$$\left[\frac{\left((b^{2}c-4\,ac^{2})fx^{2}+(b^{3}-4\,abc)fx+(ab^{2}-4\,a^{2}c)f\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{2}-8\,bcx-b^{2}-4\,\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)-4\,\left(bc^{2}d+abcf+(2\,c^{3}d+(b^{2}c-2\,ac^{2})f)x-(bc^{2}x+2\,ac^{2})e\right)\sqrt{cx^{2}+bx+a}}{2\,\left(ab^{2}c^{2}-4\,a^{2}c^{3}+(b^{2}c^{3}-4\,ac^{4})x^{2}+(b^{3}c^{2}-4\,abc^{3})x\right)},\ -\frac{\left((b^{2}c-4\,ac^{2})fx^{2}+(b^{3}-4\,abc)fx+(ab^{2}-4\,a^{2}c)f\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{2}+bcx+ac)}\right)+2\,\left(bc^{2}d+abcf+(2\,c^{3}d+(b^{2}c-2\,ac^{2})f)x-(bc^{2}x+2\,ac^{2})e\right)\sqrt{cx^{2}+bx+a}}{ab^{2}c^{2}-4\,a^{2}c^{3}+(b^{2}c^{3}-4\,ac^{4})x^{2}+(b^{3}c^{2}-4\,abc^{3})x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d + a*b*c*f + (2*c^3*d + (b^2*c - 2*a*c^2)*f
)*x - (b*c^2*x + 2*a*c^2)*e)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3
+ (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*
f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*
d + a*b*c*f + (2*c^3*d + (b^2*c - 2*a*c^2)*f)*x - (b*c^2*x + 2*a*c^2)*e)*sq
```

```
rt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^
3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A]
time = 6.44, size = 122, normalized size = 1.10

$$-\frac{2\left(\frac{(2c^2d+b^2f-2acf-bce)x}{b^2c-4ac^2}+\frac{bcd+abf-2ace}{b^2c-4ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{f\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*
f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

Mupad [B]
time = 3.73, size = 143, normalized size = 1.29

$$\frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}-\frac{e\,(4\,a+2\,b\,x)}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}}+\frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c - b
^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c -
b^2/4)*(a + b*x + c*x^2)^(1/2))
```

3.117 $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=666

$$\frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2acf)\,x)}{(b^2-4ac)\,((cd-af)^2-(bd-ae)(ce-bf))\,\sqrt{a+bx+cx^2}}-\frac{f\left(c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\right.}{\sqrt{2}\,\sqrt{e^2-4df}\,((}$$

[Out] 2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d)-c*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)-1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)

Rubi [A]
time = 1.15, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {988, 1046, 738, 212}

$$\frac{2(-cx(-2acf+b^2f-bce+2c^2d)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,((cd-af)^2-(bd-ae)(ce-bf))}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e

```
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2ac}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2ac}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2ac}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2ac}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.06, size = 692, normalized size = 1.04

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x +
 a*(e - f*x))) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a*b*e
+ a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a
*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1]) + b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] + b^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*e*f*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*b*f^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1 - 2*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1 - 2*b*Sqrt[c]*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
*#1 + 2*a*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c
*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b*f^2*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4
*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])/((b^2 - 4*a
*c)*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqr
t[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1905 vs. $2(609) = 1218$.
time = 0.16, size = 1906, normalized size = 2.86

method	result	size
default	Expression too large to display	1906

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*(2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-
c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2
)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e
^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-b*
f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^
2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/(-4*d*f+e
^2)^(1/2)*(2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2
)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*(c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*f/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+(c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)/f)/(2*c*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2/f^2)/((x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2
/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((b*f*(-4*d*f+e^
2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(
((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
```

```
)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/
2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*
f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^
2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-%e^2>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.118 $\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=891

$$\frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df) - 9a^2cf(e^2 + df)) -}{}$$

```
[Out] 2/3*(3*a*b^4*c*e*f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^3
*d^3+5*a^3*f^3+3*a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a*f
^2-c*(6*d*f+e^2))+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*c*f
+b^2*f-b*c*e+2*c^2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*d*f-
3*a*c^3*e^2+b^4*f^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*d^2)*
x)/c^5/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+1/8*f*(35*b^2*f^2-20*c*f*(a*f+3*b*e
)+24*c^2*(d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9
/2)-2/3*(3*b^6*c*e*f^2-b^7*f^3+3*b^5*c*f*(6*a*f^2-c*(d*f+e^2))-3*b^3*c^2*(2
9*a^2*f^3+c^2*d*(d*f+e^2)-10*a*c*f*(d*f+e^2))-4*b*c^3*(2*c^3*d^3-29*a^3*f^3
+3*a*c^2*d*(d*f+e^2)+24*a^2*c*f*(d*f+e^2))-24*a^2*c^4*e*(6*a*f^2-c*(6*d*f+e
^2))-b^4*c^2*e*(42*a*f^2-c*(6*d*f+e^2))+6*b^2*c^3*e*(2*c^2*d^2+28*a^2*f^2-a
*c*(6*d*f+e^2))-c*(16*c^6*d^3-10*b^6*f^3+3*b^4*c*f^2*(26*a*f+7*b*e)-24*c^5*
d*(b*d*e-a*(d*f+e^2))-6*b^2*c^2*f*(25*a*b*e*f+27*a^2*f^2+2*b^2*(d*f+e^2))+6
*c^4*(b^2*d*(d*f+e^2)-16*a^2*f*(d*f+e^2)-2*a*b*e*(6*d*f+e^2))+c^3*(240*a^2*
b*e*f^2+56*a^3*f^3+84*a*b^2*f*(d*f+e^2)+b^3*(6*d*e*f+e^3)))*x)/c^5/(-4*a*c+
b^2)^2/(c*x^2+b*x+a)^(1/2)+1/4*f^2*(-11*b*f+12*c*e)*(c*x^2+b*x+a)^(1/2)/c^4
+1/2*f^3*x*(c*x^2+b*x+a)^(1/2)/c^3
```

Rubi [A]
time = 1.10, antiderivative size = 891, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1674, 1675, 654, 635, 212}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(3*c^5*(b^2
 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*c*e*f^2 - b^7*f^3 + 3*b^5*c*
```

```
f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 + d*f) - 1
0*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*d*(e^2 + d
*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 + 6*d*f)) -
b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2 + 28*a^2*f^
2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*f^2*(7*b*e +
26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*a*b*e*f + 27*a
^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*a^2*f*(e^2 + d*
f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3*f^3 + 84*a*b^2*
f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x))/(3*c^5*(b^2 - 4*a*c)^2*Sqrt[a + b
*x + c*x^2]) + (f^2*(12*c*e - 11*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^4) + (f^3
*x*Sqrt[a + b*x + c*x^2])/(2*c^3) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) +
 24*c^2*(e^2 + d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
```

```
c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2}{}$$

Mathematica [A]
time = 10.90, size = 872, normalized size = 0.98

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-105*b^7*f^3*x^2 - 10*b^6*f^2*x*(21*a*f + 2*c*x*(-9*e + 7*f*x)) + 6*b^4*c*
f*(5*a^2*f*(6*e + 53*f*x) - 6*a*c*x*(4*e^2 + 4*d*f + 30*e*f*x - 31*f^2*x^2)
 + c^2*x^3*(-16*e^2 - 16*d*f + 6*e*f*x + f^2*x^2)) - 3*b^5*f*(35*a^2*f^2 -
10*a*c*f*x*(12*e + 23*f*x) + c^2*x^2*(24*e^2 + 24*d*f - 80*e*f*x + 7*f^2*x^
2)) - 48*b*c^2*(27*a^4*f^3 - 4*c^4*d^2*x^2*(d - e*x) + a^2*c^2*(-4*d^2*f +
```

```
4*e^3*x - 64*e*f^2*x^3 + 7*f^3*x^4 - 4*d*e*(e - 6*f*x)) - 2*a*c^3*(d^3 - e^
3*x^3 + 3*d*e*x^2*(e - 2*f*x) + 3*d^2*x*(-e + f*x)) - 2*a^3*c*f*(5*e^2 + 39
*e*f*x + f*(5*d - 14*f*x^2))) - 8*b^3*c*(-95*a^3*f^3 + c^3*(d^3 - e^3*x^3 +
 9*d^2*x*(e - f*x) - 3*d*e*x^2*(3*e + 2*f*x)) - 3*a*c^2*f*x^2*(18*e^2 - 74*
e*f*x + f*(18*d + 7*f*x^2)) + 3*a^2*c*f*(3*e^2 + 105*e*f*x + f*(3*d + 29*f*
x^2))) + 32*c^3*(4*c^4*d^3*x^3 + 3*a^4*f^2*(16*e + 5*f*x) + 6*a*c^3*d*x*(d^
2 + e^2*x^2 + d*f*x^2) - 2*a^3*c*(2*e^3 + 9*e^2*f*x + f^2*x*(9*d - 10*f*x^2
) + 12*e*f*(d - 3*f*x^2)) - 3*a^2*c^2*(2*d^2*e + 4*d*f*x^2*(3*e + 2*f*x) +
x^2*(2*e^3 + 8*e^2*f*x - 6*e*f^2*x^2 - f^3*x^3))) - 48*b^2*c^2*(a^3*f^2*(25
*e + 63*f*x) - c^3*d*x*(d^2 + e^2*x^2 + d*x*(-6*e + f*x)) + a^2*c*f*x*(-21*
e^2 - 12*e*f*x + 7*f*(-3*d + 7*f*x^2)) + a*c^2*(d^2*(e - 6*f*x) - 2*d*x*(3*
e^2 - 3*e*f*x + 7*f^2*x^2) + x^2*(e^3 - 14*e^2*f*x + 6*e*f^2*x^2 + f^3*x^3)
)))/(12*c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f*(35*b^2*f^2 - 20*
c*f*(3*b*e + a*f) + 24*c^2*(e^2 + d*f))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a +
x*(b + c*x)]])/(8*c^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3250 vs. $2(865) = 1730$.
time = 0.20, size = 3251, normalized size = 3.65

method	result	size
default	Expression too large to display	3251
risch	Expression too large to display	19191

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] f^3*(1/2*x^5/c/(c*x^2+b*x+a)^(3/2)-7/4*b/c*(x^4/c/(c*x^2+b*x+a)^(3/2)-5/2*b
/c*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*
b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b
/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c
*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/
(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+
16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))+1/c*(-x/c/(c*x^2+b*x+
a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-4*a/
c*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c
*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+
a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c
*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b
)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x
+a)^(1/2)))))-5/2*a/c*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/(c*x^
2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2
```

```
+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c
/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*
c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/
2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(
c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))+1/
c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2)))))+3*e*f^2*(x^4/c/(c*x^2+b*x+a)^(3/2)-5/2*b/c*(-1/3*x^3/c/(c*x
^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^
2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(
4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)
^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a
*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)
-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^
2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))+1/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-
1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^
(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-4*a/c*(-x^2/c/(c*x^2+b*
x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x
+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*
a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^
2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))
+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^
2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))))+(d*f^
2+2*e^2*f+f*(2*d*f+e^2))*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/(c
*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*
x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/
3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^
(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2
)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))
+1/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))))+(4*d*e*f+e*(2*d*f+e^2))*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b
/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/
c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*
x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)
^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(
c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1
6/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))+(d*(2*d*f+e^2)+2*d*e^2
+f*d^2)*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1
/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*
(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b
*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+3*d^2*e*(-
1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+d^3*(2/3*(2*c*x+
```

```
b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*
x+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. 2(855) = 1710.
time = 9.65, size = 4005, normalized size = 4.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d*f^2 + 5*(7*b^6*c^2 - 6
0*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3)*x^4 + 24*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d*f^2 + 5*(7*a^2*b^6 - 60*a^3*b^4*c + 144*a^4*b^2
*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*f^2
 + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3 - 64*a^3*b*c^4)*f^3)*x^3 + (
24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d*f^2 + 5*(7*b^8 - 46*a*b^6*c + 24*
a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3)*x^2 + 2*(24*(a*b^5*c^2 -
8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d*f^2 + 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3*b
^3*c^2 - 64*a^4*b*c^3)*f^3)*x + 24*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f*
x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f*x^3 + (b^6*c^2 - 6*a*b^4*c
^3 + 32*a^3*c^5)*f*x^2 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*f*x +
 (a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*f)*e^2 - 60*((b^5*c^3 - 8*a*b^3
*c^4 + 16*a^2*b*c^5)*f^2*x^4 + 2*(b^6*c^2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*f
^2*x^3 + (b^7*c - 6*a*b^5*c^2 + 32*a^3*b*c^4)*f^2*x^2 + 2*(a*b^6*c - 8*a^2*
b^4*c^2 + 16*a^3*b^2*c^3)*f^2*x + (a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*f^2)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(192*a^2*b*c^5*d^2*f + 6*(b^4*c^4 - 8*a*b^
2*c^5 + 16*a^2*c^6)*f^3*x^5 - 21*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f^3
*x^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 24*(3*a^2*b^3*c^3 - 20*a^3*b*c^4)*d*f
^2 - (105*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3
 + 12*(b^2*c^6 + 4*a*c^7)*d^2*f - 24*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d*
```

```
f^2 - (35*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3)*x^3
 + 3*(64*b*c^7*d^3 + 24*(b^3*c^5 + 4*a*b*c^6)*d^2*f - 24*(b^5*c^3 - 6*a*b^3
*c^4)*d*f^2 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^3 + 448*a^3*b*c^4)*
f^3)*x^2 + 6*(48*a*b^2*c^5*d^2*f + 8*(b^2*c^6 + 4*a*c^7)*d^3 - 24*(a*b^4*c^
3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d*f^2 - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*
a^3*b^2*c^3 - 80*a^4*c^4)*f^3)*x - 8*(24*a^2*b*c^5*x + 16*a^3*c^5 - (b^3*c^
5 - 12*a*b*c^6)*x^3 + 6*(a*b^2*c^5 + 4*a^2*c^6)*x^2)*e^3 + 24*(8*a^2*b*c^5*
d + 2*((b^2*c^6 + 4*a*c^7)*d - 2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*f)*x^3
 + 3*((b^3*c^5 + 4*a*b*c^6)*d - (b^5*c^3 - 6*a*b^3*c^4)*f)*x^2 - (3*a^2*b^3
*c^3 - 20*a^3*b*c^4)*f + 6*(2*a*b^2*c^5*d - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*
a^3*c^5)*f)*x)*e^2 - 12*(64*a^3*c^5*d*f - 3*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2
*c^6)*f^2*x^4 + 4*(4*b*c^7*d^2 - (b^3*c^5 - 12*a*b*c^6)*d*f - (5*b^5*c^3 -
37*a*b^3*c^4 + 64*a^2*b*c^5)*f^2)*x^3 + 4*(a*b^2*c^5 + 4*a^2*c^6)*d^2 - (15
*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*f^2 + 3*(8*b^2*c^6*d^2 + 8*(a
*b^2*c^5 + 4*a^2*c^6)*d*f - (5*b^6*c^2 - 30*a*b^4*c^3 + 16*a^2*b^2*c^4 + 64
*a^3*c^5)*f^2)*x^2 + 6*(16*a^2*b*c^5*d*f + (b^3*c^5 + 4*a*b*c^6)*d^2 - (5*a
*b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*f^2)*x)*e)*sqrt(c*x^2 + b*x + a))
/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^
2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*x^3 + (b^6*c^5 - 6*a*
b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c^6 + 16*a^3*b*c^7)*x)
, -1/24*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d*f^2 + 5*(7*b^6*c^2 -
 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3)*x^4 + 24*(a^2*b^4*c^2 -
8*a^3*b^2*c^3 + 16*a^4*c^4)*d*f^2 + 5*(7*a^2*b^6 - 60*a^3*b^4*c + 144*a^4*b
^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*f
^2 + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3 - 64*a^3*b*c^4)*f^3)*x^3 +
 (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d*f^2 + 5*(7*b^8 - 46*a*b^6*c + 2
4*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3)*x^2 + 2*(24*(a*b^5*c^2
- 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d*f^2 + 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3
*b^3*c^2 - 64*a^4*b*c^3)*f^3)*x + 24*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*
f*x^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f*x^3 + (b^6*c^2 - 6*a*b^4
*c^3 + 32*a^3*c^5)*f*x^2 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*f*x
 + (a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*f)*e^2 - 60*((b^5*c^3 - 8*a*b
^3*c^4 + 16*a^2*b*c^5)*f^2*x^4 + 2*(b^6*c^2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)
*f^2*x^3 + (b^7*c - 6*a*b^5*c^2 + 32*a^3*b*c^4)*f^2*x^2 + 2*(a*b^6*c - 8*a^
2*b^4*c^2 + 16*a^3*b^2*c^3)*f^2*x + (a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c
^3)*f^2)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/
(c^2*x^2 + b*c*x + a*c)) - 2*(192*a^2*b*c^5*d^2*f + 6*(b^4*c^4 - 8*a*b^2*c^
5 + 16*a^2*c^6)*f^3*x^5 - 21*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f^3*x^4
 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 24*(3*a^2*b^3*c^3 - 20*a^3*b*c^4)*d*f^2 -
 (105*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3 + 1
2*(b^2*c^6 + 4*a*c^7)*d^2*f - 24*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d*f^2
- (35*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3)*x^3 + 3
*(64*b*c^7*d^3 + 24*(b^3*c^5 + 4*a*b*c^6)*d^2*f - 24*(b^5*c^3 - 6*a*b^3*c^4
)*d*f^2 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^3 + 448*a^3*b*c^4)*f^3)
*x^2 + 6*(48*a*b^2*c^5*d^2*f + 8*(b^2*c^6 + 4*a*c^7)*d^3 - 24*(a*b^4*c^3 -
```

```
7*a^2*b^2*c^4 + 4*a^3*c^5)*d*f^2 - (35*a*b^6*c ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(5/2),x)
[Out] Timed out
```

Giac [**A**]
time = 1.97, size = 1401, normalized size = 1.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
[Out] 1/12*((((3*(2*(b^4*c^3*f^3 - 8*a*b^2*c^4*f^3 + 16*a^2*c^5*f^3)*x/(b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6) - (7*b^5*c^2*f^3 - 56*a*b^3*c^3*f^3 + 112*a^2*b*
c^4*f^3 - 12*b^4*c^3*f^2*e + 96*a*b^2*c^4*f^2*e - 192*a^2*c^5*f^2*e)/(b^4*c
^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 4*(32*c^7*d^3 + 12*b^2*c^5*d^2*f + 48*a
*c^6*d^2*f - 24*b^4*c^3*d*f^2 + 168*a*b^2*c^4*d*f^2 - 192*a^2*c^5*d*f^2 - 3
5*b^6*c*f^3 + 279*a*b^4*c^2*f^3 - 588*a^2*b^2*c^3*f^3 + 160*a^3*c^4*f^3 - 4
8*b*c^6*d^2*e + 12*b^3*c^4*d*f*e - 144*a*b*c^5*d*f*e + 60*b^5*c^2*f^2*e - 4
44*a*b^3*c^3*f^2*e + 768*a^2*b*c^4*f^2*e + 12*b^2*c^5*d*e^2 + 48*a*c^6*d*e^
2 - 24*b^4*c^3*f*e^2 + 168*a*b^2*c^4*f*e^2 - 192*a^2*c^5*f*e^2 + 2*b^3*c^4*
e^3 - 24*a*b*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 3*(64*b*c^6
*d^3 + 24*b^3*c^4*d^2*f + 96*a*b*c^5*d^2*f - 24*b^5*c^2*d*f^2 + 144*a*b^3*c
^3*d*f^2 - 35*b^7*f^3 + 230*a*b^5*c*f^3 - 232*a^2*b^3*c^2*f^3 - 448*a^3*b*c
^3*f^3 - 96*b^2*c^5*d^2*e - 96*a*b^2*c^4*d*f*e - 384*a^2*c^5*d*f*e + 60*b^6
*c*f^2*e - 360*a*b^4*c^2*f^2*e + 192*a^2*b^2*c^3*f^2*e + 768*a^3*c^4*f^2*e
+ 24*b^3*c^4*d*e^2 + 96*a*b*c^5*d*e^2 - 24*b^5*c^2*f*e^2 + 144*a*b^3*c^3*f*
e^2 - 16*a*b^2*c^4*e^3 - 64*a^2*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^
6))*x + 6*(8*b^2*c^5*d^3 + 32*a*c^6*d^3 + 48*a*b^2*c^4*d^2*f - 24*a*b^4*c^2
*d*f^2 + 168*a^2*b^2*c^3*d*f^2 - 96*a^3*c^4*d*f^2 - 35*a*b^6*f^3 + 265*a^2*
b^4*c*f^3 - 504*a^3*b^2*c^2*f^3 + 80*a^4*c^3*f^3 - 12*b^3*c^4*d^2*e - 48*a*
b*c^5*d^2*e - 192*a^2*b*c^4*d*f*e + 60*a*b^5*c*f^2*e - 420*a^2*b^3*c^2*f^2*
e + 624*a^3*b*c^3*f^2*e + 48*a*b^2*c^4*d*e^2 - 24*a*b^4*c^2*f*e^2 + 168*a^2
*b^2*c^3*f*e^2 - 96*a^3*c^4*f*e^2 - 32*a^2*b*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^
5 + 16*a^2*c^6))*x - (8*b^3*c^4*d^3 - 96*a*b*c^5*d^3 - 192*a^2*b*c^4*d^2*f
+ 72*a^2*b^3*c^2*d*f^2 - 480*a^3*b*c^3*d*f^2 + 105*a^2*b^5*f^3 - 760*a^3*b^
3*c*f^3 + 1296*a^4*b*c^2*f^3 + 48*a*b^2*c^4*d^2*e + 192*a^2*c^5*d^2*e + 768
*a^3*c^4*d*f*e - 180*a^2*b^4*c*f^2*e + 1200*a^3*b^2*c^2*f^2*e - 1536*a^4*c^
```

```
3*f^2*e - 192*a^2*b*c^4*d*e^2 + 72*a^2*b^3*c^2*f*e^2 - 480*a^3*b*c^3*f*e^2
+ 128*a^3*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/(c*x^2 + b*x + a)^
(3/2) - 1/8*(24*c^2*d*f^2 + 35*b^2*f^3 - 20*a*c*f^3 - 60*b*c*f^2*e + 24*c^2
*f*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2
)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f\,x^2+e\,x+d\right)}^3}{{\left(c\,x^2+b\,x+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2), x)
```

3.119 $\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=444

$$\frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd-af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2+2df)) - (2c^4d^2 + b^4f^2 - 2b^2cf(be+2af) -}{3c^3\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}}$$

```
[Out] 2/3*(2*a*b^2*c*e*f-a*b^3*f^2+4*a*c^2*e*(-a*f+c*d)-b*c*(c^2*d^2-3*a^2*f^2+a*
c*(2*d*f+e^2))-(2*c^4*d^2+b^4*f^2-2*b^2*c*f*(2*a*f+b*e)-2*c^3*(b*d*e+a*(2*d
*f+e^2))+c^2*(6*a*b*e*f+2*a^2*f^2+b^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)/(c*
x^2+b*x+a)^(3/2)+f^2*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
5/2)-2/3*(2*b^4*c*e*f+48*a^2*c^3*e*f-b^5*f^2+4*b^2*c^2*e*(-3*a*f+2*c*d)+b^3
*c*(10*a*f^2-c*(2*d*f+e^2))-4*b*c^2*(2*c^2*d^2+8*a^2*f^2+a*c*(2*d*f+e^2))-2
*c*(8*c^4*d^2-2*b^4*f^2+b^2*c*f*(14*a*f+b*e)-c^3*(8*b*d*e-4*a*(2*d*f+e^2))-
c^2*(12*a*b*e*f+16*a^2*f^2-b^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)^2/(c*x^2+b
*x+a)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1674, 12, 635, 212}

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a^2f^2+ac(2df+e^2)+c^2d^2)-ab^3f^2+2ab^2cef+4ac^2e(cd-af))}{3c^3\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}}-\frac{2(-2cx(-c^2(16a^2f^2+12abef-(b^2(2df+e^2)))+b^2cf(14af+be)-c^3(8bde-4a(2df+e^2))-2b^4f^2+8c^4d^2)-4bc^2(8a^2f^2+ac(2df+e^2)+2c^2d^2)+48a^2c^3ef+b^3c(10af^2-c(2df+e^2))+4b^2c^2e(2cd-3af)+b^5(-f^2)+2b^4cef)}{3c^3\,(b^2-4ac)^2\sqrt{a+bx+cx^2}}+\frac{f^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(2*b^4*c*
e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*(10*a*
f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 + 2*d*f)
) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*d*e - 4*
a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f)))*x))/(
3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (f^2*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2}{3c^3\,(b^2-}$$

$$= \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2}{3c^3\,(b^2-}$$

$$= \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2}{3c^3\,(b^2-}$$

$$= \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2}{3c^3\,(b^2-}$$

$$= \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2}{3c^3\,(b^2-}$$

Mathematica [A]

time = 2.36, size = 392, normalized size = 0.88

$$\frac{2(-3b^5f^2x^2-2b^4f^2x(3a+2cx^2)+4bc(5a^3f^2+2c^3dx^2(3d-2ex)+2a^2c(e^2+2df-6efx)+3ac^2(d-ex)(d+x(-e+2fx)))+b^3(-3a^2f^2+18acf^2x^2+c^2(-d^2+6dx(-e+fx)+ex^2(3e+2fx)))+8c^2(2c^3d^2x^3-a^3f(4e+3fx)+ac^2x(3d^2+e^2x^2+2dfx^2)-2a^2c(de+fx^2(3e+2fx)))+2b^2c(21a^2f^2x+c^2x(3d^2+e^2x^2+2dx(-6e+fx))-2ac(d(e-6fx)+x(-3e^2+3efx-7f^2x^2))))}{3c^2\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}-\frac{f^2\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-3*b^5*f^2*x^2 - 2*b^4*f^2*x*(3*a + 2*c*x^2) + 4*b*c*(5*a^3*f^2 + 2*c^3
*d*x^2*(3*d - 2*e*x) + 2*a^2*c*(e^2 + 2*d*f - 6*e*f*x) + 3*a*c^2*(d - e*x)*
(d + x*(-e + 2*f*x))) + b^3*(-3*a^2*f^2 + 18*a*c*f^2*x^2 + c^2*(-d^2 + 6*d*
x*(-e + f*x) + e*x^2*(3*e + 2*f*x))) + 8*c^2*(2*c^3*d^2*x^3 - a^3*f*(4*e +
3*f*x) + a*c^2*x*(3*d^2 + e^2*x^2 + 2*d*f*x^2) - 2*a^2*c*(d*e + f*x^2*(3*e
+ 2*f*x))) + 2*b^2*c*(21*a^2*f^2*x + c^2*x*(3*d^2 + e^2*x^2 + 2*d*x*(-6*e +
 f*x)) - 2*a*c*(d*(e - 6*f*x) + x*(-3*e^2 + 3*e*f*x - 7*f^2*x^2))))))/(3*c^2
*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) - (f^2*Log[c^2*(b + 2*c*x - 2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(428) = 856$.
time = 0.14, size = 1129, normalized size = 2.54

method	result	size
default	Expression too large to display	1129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] f^2*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2
*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*
b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*
c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+
a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c
/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)
+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))+1/c*(-x/c/(c*x^2+b*x
+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+2*e
*f*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/
c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x
+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3
*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(
c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+
b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*
x+a)^(1/2))))+(2*d*f+e^2)*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*
x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/
3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4
```

```
*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^
(1/2)))+2*d*e*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2
)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+
d^2*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*
c*x+b)/(c*x^2+b*x+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 7.99, size = 1595, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2 + 2*(a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2)*f^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(16*a^2*b*c^3*d*f + 4*(4*c^6*d^2 + (b^2*c^
4 + 4*a*c^5)*d*f - (b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2)*x^3 - (b^3*c^3
- 12*a*b*c^4)*d^2 - (3*a^2*b^3*c - 20*a^3*b*c^2)*f^2 + 3*(8*b*c^5*d^2 + 2*(
b^3*c^3 + 4*a*b*c^4)*d*f - (b^5*c - 6*a*b^3*c^2)*f^2)*x^2 + 6*(4*a*b^2*c^3*
d*f + (b^2*c^4 + 4*a*c^5)*d^2 - (a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*f^2)*
x + (12*a*b^2*c^3*x + 8*a^2*b*c^3 + 2*(b^2*c^4 + 4*a*c^5)*x^3 + 3*(b^3*c^3
+ 4*a*b*c^4)*x^2)*e^2 - 2*(16*a^3*c^3*f + (8*b*c^5*d - (b^3*c^3 - 12*a*b*c^
4)*f)*x^3 + 6*(2*b^2*c^4*d + (a*b^2*c^3 + 4*a^2*c^4)*f)*x^2 + 2*(a*b^2*c^3
+ 4*a^2*c^4)*d + 3*(8*a^2*b*c^3*f + (b^3*c^3 + 4*a*b*c^4)*d)*x)*e)*sqrt(c*x
^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (
b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16
*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3
)*f^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3
```

```
*b^2*c + 16*a^4*c^2)*f^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(16*a^2*b*c^3*d*f + 4*(4*c^6*d^2
 + (b^2*c^4 + 4*a*c^5)*d*f - (b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2)*x^3 -
 (b^3*c^3 - 12*a*b*c^4)*d^2 - (3*a^2*b^3*c - 20*a^3*b*c^2)*f^2 + 3*(8*b*c^5
*d^2 + 2*(b^3*c^3 + 4*a*b*c^4)*d*f - (b^5*c - 6*a*b^3*c^2)*f^2)*x^2 + 6*(4*
a*b^2*c^3*d*f + (b^2*c^4 + 4*a*c^5)*d^2 - (a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*
c^3)*f^2)*x + (12*a*b^2*c^3*x + 8*a^2*b*c^3 + 2*(b^2*c^4 + 4*a*c^5)*x^3 + 3
*(b^3*c^3 + 4*a*b*c^4)*x^2)*e^2 - 2*(16*a^3*c^3*f + (8*b*c^5*d - (b^3*c^3 -
 12*a*b*c^4)*f)*x^3 + 6*(2*b^2*c^4*d + (a*b^2*c^3 + 4*a^2*c^4)*f)*x^2 + 2*(
a*b^2*c^3 + 4*a^2*c^4)*d + 3*(8*a^2*b*c^3*f + (b^3*c^3 + 4*a*b*c^4)*d)*x)*e
)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c
^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^
6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^
3*c^4 + 16*a^3*b*c^5)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.52, size = 587, normalized size = 1.32

$$-\frac{f^{2} \log\left(\left|-2 \left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right) \sqrt{c} - b\right|\right)}{c^{\frac{5}{2}}} + \frac{2 \, {\left({\left({\left(\frac{2 \, {\left(8 \, c^{5} d^{2} + 2 \, b^{2} c^{3} d f + 8 \, a c^{4} d f - 2 \, b^{4} c f^{2} + 14 \, a b^{2} c^{2} f^{2} - 16 \, a^{2} c^{3} f^{2} - 8 \, b c^{4} d e + b^{3} c^{2} f e - 12 \, a b c^{3} f e + b^{2} c^{3} e^{2} + 4 \, a c^{4} e^{2}\right)} x}{b^{4} c^{2} - 8 \, a b^{2} c^{3} + 16 \, a^{2} c^{4}} + \frac{3 \, {\left(8 \, b c^{4} d^{2} + 2 \, b^{3} c^{2} d f + 8 \, a b c^{3} d f - b^{5} f^{2} + 6 \, a b^{3} c f^{2} - 8 \, b^{2} c^{3} d e - 4 \, a b^{2} c^{2} f e - 16 \, a^{2} c^{3} f e + b^{3} c^{2} e^{2} + 4 \, a b c^{3} e^{2}\right)}}{b^{4} c^{2} - 8 \, a b^{2} c^{3} + 16 \, a^{2} c^{4}}\right)} x + \frac{6 \, {\left(b^{2} c^{3} d^{2} + 4 \, a c^{4} d^{2} + 4 \, a b^{2} c^{2} d f - a b^{4} f^{2} + 7 \, a^{2} b^{2} c f^{2} - 4 \, a^{3} c^{2} f^{2} - b^{3} c^{2} d e - 4 \, a b c^{3} d e - 8 \, a^{2} b c^{2} f e + 2 \, a b^{2} c^{2} e^{2}\right)}}{b^{4} c^{2} - 8 \, a b^{2} c^{3} + 16 \, a^{2} c^{4}}\right)} x - \frac{b^{3} c^{2} d^{2} - 12 \, a b c^{3} d^{2} - 16 \, a^{2} b c^{2} d f + 3 \, a^{2} b^{3} f^{2} - 20 \, a^{3} b c f^{2} + 4 \, a b^{2} c^{2} d e + 16 \, a^{2} c^{3} d e + 32 \, a^{3} c^{2} f e - 8 \, a^{2} b c^{2} e^{2}}{b^{4} c^{2} - 8 \, a b^{2} c^{3} + 16 \, a^{2} c^{4}}\right)}}{3 \, {\left(c x^{2} + b x + a\right)}^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -f^2*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) +
 2/3*(((2*(8*c^5*d^2 + 2*b^2*c^3*d*f + 8*a*c^4*d*f - 2*b^4*c*f^2 + 14*a*b^2
*c^2*f^2 - 16*a^2*c^3*f^2 - 8*b*c^4*d*e + b^3*c^2*f*e - 12*a*b*c^3*f*e + b^
2*c^3*e^2 + 4*a*c^4*e^2)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^
4*d^2 + 2*b^3*c^2*d*f + 8*a*b*c^3*d*f - b^5*f^2 + 6*a*b^3*c*f^2 - 8*b^2*c^3
*d*e - 4*a*b^2*c^2*f*e - 16*a^2*c^3*f*e + b^3*c^2*e^2 + 4*a*b*c^3*e^2)/(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(b^2*c^3*d^2 + 4*a*c^4*d^2 + 4*a*b^
2*c^2*d*f - a*b^4*f^2 + 7*a^2*b^2*c*f^2 - 4*a^3*c^2*f^2 - b^3*c^2*d*e - 4*a
*b*c^3*d*e - 8*a^2*b*c^2*f*e + 2*a*b^2*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4))*x - (b^3*c^2*d^2 - 12*a*b*c^3*d^2 - 16*a^2*b*c^2*d*f + 3*a^2*b^3
*f^2 - 20*a^3*b*c*f^2 + 4*a*b^2*c^2*d*e + 16*a^2*c^3*d*e + 32*a^3*c^2*f*e -
 8*a^2*b*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(
3/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^2}{\left(c\,x^2+b\,x+a\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x)
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2), x)
```

3.120 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2\left(8cd-4be+4af+\frac{b^2f}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*(c*(2*a*e-b*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)
/(c*x^2+b*x+a)^(3/2)+2/3*(8*c*d-4*b*e+4*a*f+b^2*f/c)*(2*c*x+b)/(-4*a*c+b^2)
^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1674, 12, 627}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f + (b
^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
```

```
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{8cd-4be+4af+\frac{b^2f}{c}}{2(a+bx+cx^2)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{\left(8cd-4be+4af+\frac{b^2f}{c}\right.}{3\left(b^2-\right.}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\left(8cd-4be+4af+\frac{b^2}{}\right.}{3\left(b^2-4ac\right)^2\sqrt{a+}}$$

Mathematica [A]
time = 0.88, size = 147, normalized size = 1.12

$$\frac{-2b^3(d+3x(e-fx))+16c(-a^2e+2c^2dx^3+acx(3d+fx^2))-4b^2(a(e-6fx)-cx(3d-6ex+fx^2))+8b(2a^2f-2c^2x^2(-3d+ex)+3ac(d-ex+fx^2))}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*b^3*(d + 3*x*(e - f*x)) + 16*c*(-(a^2*e) + 2*c^2*d*x^3 + a*c*x*(3*d + f
*x^2)) - 4*b^2*(a*(e - 6*f*x) - c*x*(3*d - 6*e*x + f*x^2)) + 8*b*(2*a^2*f -
 2*c^2*x^2*(-3*d + e*x) + 3*a*c*(d - e*x + f*x^2)))/(3*(b^2 - 4*a*c)^2*(a +
 x*(b + c*x))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(123) = 246.
time = 0.13, size = 351, normalized size = 2.68

method	result
trager	$\frac{\frac{16}{3}a\,c^2f\,x^3+\frac{4}{3}b^2cf\,x^3-\frac{16}{3}b\,c^2e\,x^3+\frac{32}{3}c^3d\,x^3+8abcf\,x^2+2b^3f\,x^2-8b^2ce\,x^2+16b\,c^2d\,x^2+8a\,b^2fx-8abcex+16a\,c^2dx-2b^3ex+4b^2cdx}{(4ac-b^2)^2(c\,x^2+bx+a)^{\frac{3}{2}}}$
gosper	$\frac{\frac{16}{3}a\,c^2f\,x^3+\frac{4}{3}b^2cf\,x^3-\frac{16}{3}b\,c^2e\,x^3+\frac{32}{3}c^3d\,x^3+8abcf\,x^2+2b^3f\,x^2-8b^2ce\,x^2+16b\,c^2d\,x^2+8a\,b^2fx-8abcex+16a\,c^2dx-2b^3ex+4b^2cdx}{(c\,x^2+bx+a)^{\frac{3}{2}}(16a^2c^2-8a\,b^2c+b^4)}$

default	$f\left(-\frac{x}{2c(cx^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c(cx^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{(4ac-b^2)(cx^2+bx+a)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3(4ac-b^2)^2\sqrt{cx^2+bx+a}}\right)}{2c}\right)}{4c}+\frac{a\left(\frac{}{(4ac-b}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c
*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x
+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+e*(-1/3/c/(c*x^2
+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c
/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+d*(2/3*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(124) = 248$.
time = 6.66, size = 291, normalized size = 2.22

$$\frac{2\left(8a^2bf+2\left(8c^3d+(b^2c+4ac^2)f\right)x^3+3\left(8bc^2d+(b^3+4abc)f\right)x^2-(b^3-12abc)d+6\left(2ab^2f+(b^2c+4ac^2)d\right)x-\left(8bc^2x^3+12b^2cx^2+2ab^2+8a^2c+3(b^3+4abc)x\right)e\right)\sqrt{cx^2+bx+a}}{3\left(a^2b^4-8a^3b^2c+16a^4c^2+(b^4c^2-8ab^2c^3+16a^2c^4)x^4+2(b^5c-8ab^3c^2+16a^2bc^3)x^3+(b^6-6ab^4c+32a^3c^3)x^2+2(ab^5-8a^2b^3c+16a^3bc^2)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*a^2*b*f + 2*(8*c^3*d + (b^2*c + 4*a*c^2)*f)*x^3 + 3*(8*b*c^2*d + (b^
3 + 4*a*b*c)*f)*x^2 - (b^3 - 12*a*b*c)*d + 6*(2*a*b^2*f + (b^2*c + 4*a*c^2)
```

```
*d)*x - (8*b*c^2*x^3 + 12*b^2*c*x^2 + 2*a*b^2 + 8*a^2*c + 3*(b^3 + 4*a*b*c)
*x)*e)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b
*c^2)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.74, size = 240, normalized size = 1.83

$$\frac{2\left(\left(\left(\frac{2\,(8\,c^3d+b^2cf+4\,ac^2f-4\,bc^2e)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,(8\,bc^2d+b^3f+4\,abcf-4\,b^2ce)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{3\,(2\,b^2cd+8\,ac^2d+4\,ab^2f-b^3e-4\,abce)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x-\frac{b^3d-12\,abcd-8\,a^2bf+2\,ab^2e+8\,a^2ce}{b^4-8\,ab^2c+16\,a^2c^2}\right)}{3\,(cx^2+bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(((2*(8*c^3*d + b^2*c*f + 4*a*c^2*f - 4*b*c^2*e)*x/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2) + 3*(8*b*c^2*d + b^3*f + 4*a*b*c*f - 4*b^2*c*e)/(b^4 - 8*a*b^2*c
 + 16*a^2*c^2))*x + 3*(2*b^2*c*d + 8*a*c^2*d + 4*a*b^2*f - b^3*e - 4*a*b*c*
e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x - (b^3*d - 12*a*b*c*d - 8*a^2*b*f + 2*
a*b^2*e + 8*a^2*c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)^(3/2
)
```

Mupad [**B**]
time = 3.71, size = 175, normalized size = 1.34

$$\frac{2\,(8\,f\,a^2\,b-8\,e\,a^2\,c+12\,f\,a\,b^2\,x-2\,e\,a\,b^2+12\,f\,a\,b\,c\,x^2-12\,e\,a\,b\,c\,x+12\,d\,a\,b\,c+8\,f\,a\,c^2\,x^3+24\,d\,a\,c^2\,x+3\,f\,b^3\,x^2-3\,e\,b^3\,x-d\,b^3+2\,f\,b^2\,c\,x^3-12\,e\,b^2\,c\,x^2+6\,d\,b^2\,c\,x-8\,e\,b\,c^2\,x^3+24\,d\,b\,c^2\,x^2+16\,d\,c^3\,x^3)}{3\,(4\,a\,c-b^2)^2\,(c\,x^2+b\,x+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] (2*(16*c^3*d*x^3 - b^3*d + 3*b^3*f*x^2 - 2*a*b^2*e + 8*a^2*b*f - 8*a^2*c*e
- 3*b^3*e*x + 24*a*c^2*d*x + 12*a*b^2*f*x + 6*b^2*c*d*x + 24*b*c^2*d*x^2 -
12*b^2*c*e*x^2 + 8*a*c^2*f*x^3 - 8*b*c^2*e*x^3 + 2*b^2*c*f*x^3 + 12*a*b*c*d
 - 12*a*b*c*e*x + 12*a*b*c*f*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/
2))
```

3.121 $\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx$

Optimal. Leaf size=51

$$\frac{1}{10}\tan^{-1}\left(\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right)$$

```
[Out] 1/10*arctan(5/2*(2+x)/(5*x^2+2*x-7)^(1/2))+1/5*arctanh(5*(1+x)/(5*x^2+2*x-7
)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1000, 1043, 209, 213}

$$\frac{1}{10}\text{ArcTan}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

```
[Out] ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sqr
t[-7 + 2*x + 5*x^2]]/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx &= -\left(\frac{1}{50}\int \frac{-100-50x}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx\right)+\frac{1}{50}\int \frac{}{\sqrt{-7}}\\ &= 400\text{Subst}\left(\int \frac{1}{160000+100x^2}\,dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}}\right)+160\\ &= \frac{1}{10}\tan^{-1}\left(\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right.\end{aligned}$$

Mathematica [A]
time = 0.16, size = 53, normalized size = 1.04

$$\frac{1}{10}\tan^{-1}\left(\frac{5+\frac{5x}{2}}{\sqrt{-7+2x+5x^2}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5+5x}{\sqrt{-7+2x+5x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
[Out] ArcTan[(5 + (5*x)/2)/Sqrt[-7 + 2*x + 5*x^2]]/10 + ArcTanh[(5 + 5*x)/Sqrt[-7
 + 2*x + 5*x^2]]/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(41) = 82$.
time = 0.39, size = 144, normalized size = 2.82

method	result
default	$-\frac{\sqrt{-\frac{4(x+2)^2}{(-1-x)^2}+9}\left(2\,\text{arctanh}\left(\frac{\sqrt{-\frac{4(x+2)^2}{(-1-x)^2}+9}}{5}\right)+\arctan\left(\frac{5\sqrt{-\frac{4(x+2)^2}{(-1-x)^2}+9}\,(x+2)}{2\left(\frac{4(x+2)^2}{(-1-x)^2}-9\right)(-1-x)}\right)\right)}{10\sqrt{-\frac{\frac{4(x+2)^2}{(-1-x)^2}-9}{\left(1+\frac{x+2}{-1-x}\right)^2}}\left(1+\frac{x+2}{-1-x}\right)}$

trager	$\mathrm{RootOf}\left(80\_Z^2-16\_Z+1\right)\ln\left(\frac{129600\,\mathrm{RootOf}\left(80\_Z^2-16\_Z+1\right)^2x-8750\sqrt{5x^2+2x-7}\ \mathrm{RootOf}\left(80\_Z^2\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(-4*(x+2)^2/(-1-x)^2+9)^(1/2)*(2*arctanh(1/5*(-4*(x+2)^2/(-1-x)^2+9)^
(1/2))+arctan(5/2*(-4*(x+2)^2/(-1-x)^2+9)^(1/2)/(4*(x+2)^2/(-1-x)^2-9)*(x+2
)/(-1-x)))/(-(4*(x+2)^2/(-1-x)^2-9)/(1+(x+2)/(-1-x))^2)^(1/2)/(1+(x+2)/(-1-
x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 12*x + 8)*sqrt(5*x^2 + 2*x - 7)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(41) = 82$.
time = 3.34, size = 154, normalized size = 3.02

$$\frac{1}{20}\arctan\left(\frac{27x^2+20\sqrt{5x^2+2x-7}\,(x+2)+36x}{31x^2+16x-56}\right)+\frac{1}{20}\arctan\left(-\frac{27x^2-20\sqrt{5x^2+2x-7}\,(x+2)+36x}{31x^2+16x-56}\right)+\frac{1}{20}\log\left(\frac{15x^2+5\sqrt{5x^2+2x-7}\,(x+1)+26x+9}{x^2}\right)-\frac{1}{20}\log\left(\frac{15x^2-5\sqrt{5x^2+2x-7}\,(x+1)+26x+9}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*arctan((27*x^2 + 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)/(31*x^2 + 16
*x - 56)) + 1/20*arctan(-(27*x^2 - 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)
/(31*x^2 + 16*x - 56)) + 1/20*log((15*x^2 + 5*sqrt(5*x^2 + 2*x - 7)*(x + 1)
 + 26*x + 9)/x^2) - 1/20*log((15*x^2 - 5*sqrt(5*x^2 + 2*x - 7)*(x + 1) + 26
*x + 9)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(x-1)\,(5x+7)}\,\left(5x^2+12x+8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+12*x+8)/(5*x**2+2*x-7)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(5*x + 7))*(5*x**2 + 12*x + 8)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(41) = 82$.
time = 4.96, size = 205, normalized size = 4.02

$$-\frac{1}{10}\arctan\left(-\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5x^2+2x-7}+5}{2\left(\sqrt{5}+5\right)}\right)-\frac{1}{10}\arctan\left(\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5x^2+2x-7}-5}{2\left(\sqrt{5}-5\right)}\right)+\frac{1}{10}\log\left(5\left(\sqrt{5}x-\sqrt{5x^2+2x-7}\right)^2+2\left(\sqrt{5}x-\sqrt{5x^2+2x-7}\right)\left(6\sqrt{5}+5\right)+20\sqrt{5}+65\right)-\frac{1}{10}\log\left(5\left(\sqrt{5}x-\sqrt{5x^2+2x-7}\right)^2+2\left(\sqrt{5}x-\sqrt{5x^2+2x-7}\right)\left(6\sqrt{5}-5\right)-20\sqrt{5}+65\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*arctan(-1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2*x - 7) + 5)/(
sqrt(5) + 5)) - 1/10*arctan(1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2
*x - 7) - 5)/(sqrt(5) - 5)) + 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7)
)^2 + 2*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) + 5) + 20*sqrt(5) +
65) - 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))^2 + 2*(sqrt(5)*x - sqr
t(5*x^2 + 2*x - 7))*(6*sqrt(5) - 5) - 20*sqrt(5) + 65)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{5\,x^2+2\,x-7}\,\left(5\,x^2+12\,x+8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)),x)
```

```
[Out] int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)), x)
```

3.122 $\int \frac{1}{\sqrt{a+bx+cx^2}\,\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=1432

$$\sqrt[4]{b^2d+b\left(\sqrt{b^2-4ac}\,d-ae\right)-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\,\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{\ldots}\right.}$$

$$-\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

```
[Out] -(cos(2*arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^
(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1
/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2
))^(1/2)))^2)^(1/2)/cos(2*arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-
4*a*c+b^2)^(1/2))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*(-a*e
+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b+2*c*x
+(-4*a*c+b^2)^(1/2))^(1/2)))*EllipticF(sin(2*arctan((2*c^2*d-b*c*e+b^2*f-2*
a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/
2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2))),1/2*(2+(2*a*f-b*e+2*c*d)*(b
+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*
(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c^2*d+b*f*(b+(-4*a*c+b^2)^(1/2))-c*(b*e+2*a*f
+e*(-4*a*c+b^2)^(1/2)))^(1/2))^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2))^(3/2)*(b
^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1
/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((f*x^2+e*x+d)*(4*a*c-(b+(-4*a*c+b
^2)^(1/2))^2)^2/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f-2*a*e*(b+(-4*a*c+b^
2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2))^2))^(1/2)*(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2
)))*(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*
c*x+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f
+e*(-4*a*c+b^2)^(1/2)))^(1/2))*((1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^2*(4*c^2*
d-2*c*e*(b+(-4*a*c+b^2)^(1/2))+f*(b+(-4*a*c+b^2)^(1/2))^2)/(b+2*c*x+(-4*a*c
+b^2)^(1/2))^2/(4*a^2*f-2*a*e*(b+(-4*a*c+b^2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2
))^2)-(2*a*f-b*e+2*c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)
))/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c
*d-2*a*f+e*(-4*a*c+b^2)^(1/2))))/(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d
-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c*x+(-4*a*c+
```

```
b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b
^2)^(1/2)))^(1/2))^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c
+b^2)^(1/2))^(1/4)/(4*a*c-(b+(-4*a*c+b^2)^(1/2))^2)/(c*x^2+b*x+a)^(1/2)/(f*
x^2+e*x+d)^(1/2)/(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^2*(4*c^2*d-2*c*e*(b+(-4*
a*c+b^2)^(1/2))+f*(b+(-4*a*c+b^2)^(1/2))^2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/
(4*a^2*f-2*a*e*(b+(-4*a*c+b^2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2))^2)-(2*a*f-b*
e+2*c*d)*(b+(-4*a*c+b^2)^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))/(b+2*c*x+(-4
*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*
a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]

time = 3.90, antiderivative size = 1432, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1006, 949, 1117}

$$-\frac{\sqrt[4]{db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2-4ac}\right)^2\right)^2(fx^2+ex+d)}{\left(4fa^2-2\left(b+\sqrt{b^2-4ac}\right)ea+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4ac}\right)^2}}\left(\frac{\sqrt{fb^2-ceb+2c^2d-2acf-\sqrt{b^2-4ac}\,(ce-bf)}\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\sqrt{db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\left(b+2cx+\sqrt{b^2-4ac}\right)}+1\right)\sqrt{\frac{\frac{\left(4dc^2-2\left(b+\sqrt{b^2-4ac}\right)ec+\left(b+\sqrt{b^2-4ac}\right)^2f\right)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)^2}{\left(4fa^2-2\left(b+\sqrt{b^2-4ac}\right)ea+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4ac}\right)^2}-\frac{\left(b+\sqrt{b^2-4ac}\right)(2cd-be+2af)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)\right)\left(b+2cx+\sqrt{b^2-4ac}\right)}+1}{\left(\frac{\sqrt{fb^2-ceb+2c^2d-2acf-\sqrt{b^2-4ac}\,(ce-bf)}\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\sqrt{db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\left(b+2cx+\sqrt{b^2-4ac}\right)}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{fb^2-ceb+2c^2d-2acf-\sqrt{b^2-4ac}\,(ce-bf)}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}}{\sqrt[4]{db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\sqrt{b+2cx+\sqrt{b^2-4ac}}}\right)|\frac{1}{4}\left(\frac{\left(b+\sqrt{b^2-4ac}\right)(2cd-be+2af)}{\sqrt{db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\sqrt{2dc^2-\left(be+\sqrt{b^2-4ac}\,e+2af\right)c+b\left(b+\sqrt{b^2-4ac}\right)f}}+2\right)\right)}{\left(4ac-\left(b+\sqrt{b^2-4ac}\right)^2\right)\sqrt[4]{fb^2-ceb+2c^2d-2acf-\sqrt{b^2-4ac}\,(ce-bf)}\sqrt{cx^2+bx+a}\sqrt{fx^2+ex+d}\sqrt{\frac{\left(4dc^2-2\left(b+\sqrt{b^2-4ac}\right)ec+\left(b+\sqrt{b^2-4ac}\right)^2f\right)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)^2}{\left(4fa^2-2\left(b+\sqrt{b^2-4ac}\right)ea+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4ac}\right)^2}-\frac{\left(b+\sqrt{b^2-4ac}\right)(2cd-be+2af)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)\right)\left(b+2cx+\sqrt{b^2-4ac}\right)}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] -(((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e
- 2*a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[
b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^2*(d + e*x + f*x
^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*
f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f -
 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)
)/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c
]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - ((b + Sqrt[b^2 -
4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/((b^2*d +
b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e
+ (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b +
Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - S
qrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*
d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)
]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[((2*c^2*d - b*c*e
 + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[2*a + (b + S
qrt[b^2 - 4*a*c])*x])/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d +
Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])],
(2 + ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f))/(Sqrt[b^2*d + b*(Sqrt[
b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[2*c^2
*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)]))
/4])/((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - b*c*e + b^2*f - 2*a*c*
f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x
```

```
 + f*x^2]*Sqrt[1 - ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b
 + Sqrt[b^2 - 4*a*c])*x))/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*
d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^
2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (
b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[
b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]))
```

Rule 949

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2])), Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*e
*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 - b
*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 1006

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b + r + 2
*c*x]*(Sqrt[2*a + (b + r)*x]/Sqrt[a + b*x + c*x^2]), Int[1/(Sqrt[b + r + 2*
c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}+2cx}\,\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\right)\int \frac{}{\sqrt{b+}}}{\sqrt{a+}}$$

$$= -\frac{\left(2\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\,\sqrt{\frac{}{(}}\right.}{\sqrt[4]{b^2d+b\left(\sqrt{b^2-4ac}\,d-ae\right)-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}}$$

$$= -\frac{}{}$$

Mathematica [A]

time = 3.92, size = 670, normalized size = 0.47

$$-\frac{\left(-b+\sqrt{b^2-4ac}-2cx\right)\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\sqrt{-\frac{c\left(4af+\sqrt{b^2-4ac}\sqrt{e^2-4df}-2\sqrt{b^2-4ac}fx+2c\sqrt{e^2-4df}x-e\left(\sqrt{b^2-4ac}+2cx\right)+b\left(-e+\sqrt{e^2-4df}+2fx\right)\right)}{\left(\left(b+\sqrt{b^2-4ac}\right)f+c\left(-e+\sqrt{e^2-4df}\right)\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\left(\left(-b+\sqrt{b^2-4ac}\right)f+c\left(e-\sqrt{e^2-4df}\right)\right)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(\left(b+\sqrt{b^2-4ac}\right)f+c\left(-e+\sqrt{e^2-4df}\right)\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\right)\middle|\frac{2cd-be+2af-\sqrt{b^2-4ac}\sqrt{e^2-4df}}{2cd-be+2af+\sqrt{b^2-4ac}\sqrt{e^2-4df}}\right)}{\left(\left(-b+\sqrt{b^2-4ac}\right)f+c\left(e-\sqrt{e^2-4df}\right)\right)\sqrt{\frac{c\sqrt{b^2-4ac}\left(-e+\sqrt{e^2-4df}-2fx\right)}{\left(\left(b+\sqrt{b^2-4ac}\right)f+c\left(-e+\sqrt{e^2-4df}\right)\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\sqrt{a+x(b+cx)}\sqrt{d+x(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] -(((-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[-(
(c*Sqrt[b^2 - 4*a*c]*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(((b + Sqrt[b^2 - 4*a
*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*Sqr
```

```
t[-((c*(4*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 - 4*d*f] - 2*Sqrt[b^2 - 4*a*c]*f
*x + 2*c*Sqrt[e^2 - 4*d*f]*x - e*(Sqrt[b^2 - 4*a*c] + 2*c*x) + b*(-e + Sqrt
[e^2 - 4*d*f] + 2*f*x)))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4
*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*EllipticF[ArcSin[Sqrt[(((-b + S
qrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt
[b^2 - 4*a*c] - 2*c*x))]], (2*c*d - b*e + 2*a*f - Sqrt[b^2 - 4*a*c]*Sqrt[e^
2 - 4*d*f])/(2*c*d - b*e + 2*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 - 4*d*f])])/(
((-b + Sqrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*Sqrt[(c*Sqrt[b^2 -
 4*a*c]*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(
-e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*Sqrt[a + x*(b +
 c*x)]*Sqrt[d + x*(e + f*x)])
```

Maple [A]
time = 0.29, size = 906, normalized size = 0.63

method	result
elliptic	$\frac{2\sqrt{(f\,x^2+ex+d)\,(c\,x^2+bx+a)}\left(\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{-e+\sqrt{-4df+e^2}}{2f}\right)\sqrt{\frac{\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{e+\sqrt{\cdots}}{\cdots}\right.}{\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-e+\cdots}{\cdots}\right.}}\cdots}{\cdots}$
default	$-\frac{8\left(-2b\,f^2x^2+2cef\,x^2-2cf\,x^2\sqrt{-4df+e^2}-2\sqrt{-4ac+b^2}\,f^2x^2-2bef x-2bf x\sqrt{-4df+e^2}+8cdf x-2\sqrt{-4ac+\cdots}\right.\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8*(-2*b*f^2*x^2+2*c*e*f*x^2-2*c*f*x^2*(-4*d*f+e^2)^(1/2)-2*(-4*a*c+b^2)^(1
/2)*f^2*x^2-2*b*e*f*x-2*b*f*x*(-4*d*f+e^2)^(1/2)+8*c*d*f*x-2*(-4*a*c+b^2)^(
1/2)*e*f*x-2*f*x*(-4*a*c+b^2)^(1/2)*(-4*d*f+e^2)^(1/2)+2*b*d*f-e^2*b-b*e*(-
4*d*f+e^2)^(1/2)+2*c*d*e+2*c*d*(-4*d*f+e^2)^(1/2)+2*(-4*a*c+b^2)^(1/2)*d*f-
(-4*a*c+b^2)^(1/2)*e^2-e*(-4*a*c+b^2)^(1/2)*(-4*d*f+e^2)^(1/2))*EllipticF((
-(f*(-4*a*c+b^2)^(1/2)-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(-2*f*x+(-4*d*f+e^2)^(
1/2)-e)/(f*(-4*a*c+b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(2*f*x+(-4*d*f+
e^2)^(1/2)+e))^(1/2),((f*(-4*a*c+b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)-b*f+c*e)*(
f*(-4*a*c+b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(f*(-4*a*c+b^2)^(1/2)-c*
(-4*d*f+e^2)^(1/2)-b*f+c*e)/(f*(-4*a*c+b^2)^(1/2)-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e))^(1/2))*((-4*d*f+e^2)^(1/2)*(b+2*c*x+(-4*a*c+b^2)^(1/2))*f/(f*(-4*a*c+
```

```
b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^(1/2
)*((-4*d*f+e^2)^(1/2)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))*f/(f*(-4*a*c+b^2)^(1/2)
-c*(-4*d*f+e^2)^(1/2)-b*f+c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^(1/2)*(-(f*(-4
*a*c+b^2)^(1/2)-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(-2*f*x+(-4*d*f+e^2)^(1/2)-e)
/(f*(-4*a*c+b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1
/2)+e))^(1/2)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^(1/2)/(1/c/f*(-2*f*x+(-4*d*
f+e^2)^(1/2)-e)*(2*f*x+(-4*d*f+e^2)^(1/2)+e)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))*
(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(-4*a*c+b^2)^(1/2
)-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/((f*x^2+e*x+d)*(c*x^2+b*x+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + x*e + d)/(c*f*x^4 + b*f*x^3 + b
*d*x + (c*d + a*f)*x^2 + a*d + (c*x^3 + b*x^2 + a*x)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx + cx^2}\,\sqrt{d + ex + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)), x)
```

3.123 $\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx$

Optimal. Leaf size=652

$$\sqrt{\frac{23}{11}}\left(1-i\sqrt{23}-4x\right)\sqrt{-1+i\sqrt{23}+4x}\,\sqrt{6-\left(1-i\sqrt{23}\right)x}\,\sqrt{\frac{\left(11i-\sqrt{23}\right)(2+3x+5x^2)}{\left(7i+\sqrt{23}\right)\left(1-i\sqrt{23}-4x\right)^2}}\ \Bigg($$

$$\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}$$

```
[Out] 1/11*(cos(2*arctan(((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)*(6-x*(1-I*23^(1/2
)))^(1/2)/(-1+4*x+I*23^(1/2))^(1/2)))^2)^(1/2)/cos(2*arctan(((-3*I+23^(1/2)
)/(7*I+23^(1/2)))^(1/4)*(6-x*(1-I*23^(1/2)))^(1/2)/(-1+4*x+I*23^(1/2))^(1/2
)))*EllipticF(sin(2*arctan(((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)*(6-x*(1-I
*23^(1/2)))^(1/2)/(-1+4*x+I*23^(1/2))^(1/2))),1/22*11^(1/2)*((66*I-22*23^(1
/2)+41*(-23*(3*I-23^(1/2))/(7*I+23^(1/2)))^(1/2)+41*I*((-3*I+23^(1/2))/(7*I
+23^(1/2)))^(1/2))/(3*I-23^(1/2)))^(1/2))*253^(1/2)*(1-4*x-I*23^(1/2))*(6-x
*(1-I*23^(1/2)))^(1/2)*(-1+4*x+I*23^(1/2))^(1/2)*(1-(6-x*(1-I*23^(1/2)))*((
-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/2)/(1-4*x-I*23^(1/2)))*((5*x^2+3*x+2)*(11
*I-23^(1/2))/(1-4*x-I*23^(1/2))^2/(7*I+23^(1/2)))^(1/2)*((11-11*(6-x*(1-I*2
3^(1/2)))^2*(3*I-23^(1/2))/(1-4*x-I*23^(1/2))^2/(7*I+23^(1/2))-41*(6-x*(1-I
*23^(1/2)))*(I+23^(1/2))/(1-4*x-I*23^(1/2))/(7*I+23^(1/2)))/(1-(6-x*(1-I*23
^(1/2)))*((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/2)/(1-4*x-I*23^(1/2)))^2)^(1/2
)/(23+I*23^(1/2))/((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)/(2*x^2-x+3)^(1/2)/
(5*x^2+3*x+2)^(1/2)/(11-11*(6-x*(1-I*23^(1/2)))^2*(3*I-23^(1/2))/(1-4*x-I*2
3^(1/2))^2/(7*I+23^(1/2))-41*(6-x*(1-I*23^(1/2)))*(I+23^(1/2))/(1-4*x-I*23^
(1/2))/(7*I+23^(1/2)))^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1006, 949, 1117}

$$\frac{\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(-\sqrt{23}+11i\right)(5x^2+3x+2)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\left(1-\frac{\sqrt{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\left(6-\left(1-i\sqrt{23}\right)x\right)}{-4x-i\sqrt{23}+1}\right)\sqrt{\frac{-\frac{11\left(-\sqrt{23}+3i\right)\left(6-\left(1-i\sqrt{23}\right)x\right)^2}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}-\frac{41\left(\sqrt{23}+i\right)\left(6-\left(1-i\sqrt{23}\right)x\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)}+11}{\left(1-\frac{\sqrt{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\left(6-\left(1-i\sqrt{23}\right)x\right)}{-4x-i\sqrt{23}+1}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\sqrt{6-\left(1-i\sqrt{23}\right)x}}{\sqrt{4x+i\sqrt{23}-1}}\right)\middle|\frac{1}{88}\left(44-\frac{41\left(i+\sqrt{23}\right)}{\sqrt{11+i\sqrt{23}}}\right)\right)}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}\sqrt{-\frac{11\left(-\sqrt{23}+3i\right)\left(6-\left(1-i\sqrt{23}\right)x\right)^2}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}-\frac{41\left(\sqrt{23}+i\right)\left(6-\left(1-i\sqrt{23}\right)x\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)}+11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

```
[Out] (Sqrt[23/11]*(1 - I*Sqrt[23] - 4*x)*Sqrt[-1 + I*Sqrt[23] + 4*x]*Sqrt[6 - (1
 - I*Sqrt[23])*x]*Sqrt[((11*I - Sqrt[23])*(2 + 3*x + 5*x^2))/((7*I + Sqrt[2
3])*(1 - I*Sqrt[23] - 4*x)^2)]*(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23
]))]*(6 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x))*Sqrt[(11 - (41*(I +
Sqrt[23])*(6 - (1 - I*Sqrt[23])*x))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x
)) - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1
- I*Sqrt[23] - 4*x)^2))/(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23]))]*(6
 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x))^2]*EllipticF[2*ArcTan[((-((
3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[6 - (1 - I*Sqrt[23])*x])/Sqrt
[-1 + I*Sqrt[23] + 4*x]], (44 - (41*(I + Sqrt[23]))/Sqrt[11 + I*Sqrt[23]])/
88])/((23 + I*Sqrt[23])*(-((3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[3
 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]*Sqrt[11 - (41*(I + Sqrt[23])*(6 - (1 -
I*Sqrt[23])*x))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)) - (11*(3*I - Sqrt
[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^
2)])
```

Rule 949

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2])), Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*e
*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 - b
*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 1006

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b + r + 2
*c*x]*(Sqrt[2*a + (b + r)*x]/Sqrt[a + b*x + c*x^2]), Int[1/(Sqrt[b + r + 2*
c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \frac{\left(\sqrt{-1+i\sqrt{23}+4x}\sqrt{6+\left(-1+i\sqrt{23}\right)x}\right)\int \frac{}{\sqrt{-1+i\sqrt{23}}}}{\sqrt{3-x+2x^2}}$$

$$= -\frac{\left(2\left(-1+i\sqrt{23}+4x\right)^{3/2}\sqrt{6+\left(-1+i\sqrt{23}\right)x}\sqrt{\frac{}{\left(180-18\right.}}\right.}{}$$

$$= -\frac{\sqrt{\frac{23}{11}}\left(-1+i\sqrt{23}+4x\right)^{3/2}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(11i-\right.}{\left(7i+\sqrt{23}\right.}}}{}$$

Mathematica [A]
time = 2.25, size = 390, normalized size = 0.60

$$\frac{\left(1+i\sqrt{23}-4x\right)\left(3i+\sqrt{31}+10ix\right)\sqrt{\frac{6i-2\sqrt{31}+20ix}{\left(11i+5\sqrt{23}-2\sqrt{31}\right)\left(-i+\sqrt{23}+4ix\right)}}\sqrt{\frac{63-3i\sqrt{23}-i\sqrt{31}-\sqrt{713}+\left(-22-10i\sqrt{23}+4i\sqrt{31}\right)x}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(-i+\sqrt{23}+4ix\right)}}F\left(\sin^{-1}\left(\sqrt{2}\sqrt{-\frac{-63+3i\sqrt{23}+i\sqrt{31}+\sqrt{713}+2\left(11+5i\sqrt{23}-2i\sqrt{31}\right)x}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(-i+\sqrt{23}+4ix\right)}}\right)\middle|\frac{1}{484}\left(1197+41\sqrt{713}\right)\right)}{\left(-11i+5\sqrt{23}-2\sqrt{31}\right)\sqrt{\frac{3i+\sqrt{31}+10ix}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(-i+\sqrt{23}+4ix\right)}}\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

```
[Out] ((1 + I*Sqrt[23] - 4*x)*(3*I + Sqrt[31] + (10*I)*x)*Sqrt[(6*I - 2*Sqrt[31]
+ (20*I)*x)/((11*I + 5*Sqrt[23] - 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*S
```

```
qrt[(63 - (3*I)*Sqrt[23] - I*Sqrt[31] - Sqrt[713] + (-22 - (10*I)*Sqrt[23]
+ (4*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*
I)*x))]*EllipticF[ArcSin[Sqrt[2]*Sqrt[-((-63 + (3*I)*Sqrt[23] + I*Sqrt[31]
+ Sqrt[713] + 2*(11 + (5*I)*Sqrt[23] - (2*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[2
3] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x)))]], (1197 + 41*Sqrt[713])/484])
/((-11*I + 5*Sqrt[23] - 2*Sqrt[31])*Sqrt[(3*I + Sqrt[31] + (10*I)*x)/((11*I
 + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*Sqrt[3 - x + 2*x^2]
*Sqrt[2 + 3*x + 5*x^2])
```

Maple [A]
time = 0.39, size = 420, normalized size = 0.64

method	result
elliptic	$-\frac{i\sqrt{(2x^2-x+3)(5x^2+3x+2)}\left(\frac{11}{20}-\frac{i\sqrt{23}}{4}-\frac{i\sqrt{31}}{10}\right)\sqrt{\frac{\left(-\frac{11}{20}+\frac{i\sqrt{31}}{10}-\frac{i\sqrt{23}}{4}\right)\left(x-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)}{\left(-\frac{11}{20}+\frac{i\sqrt{31}}{10}+\frac{i\sqrt{23}}{4}\right)\left(x-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)}}\left(x-\right.}{115\sqrt{2x^2-x+3}\,\cdot}$
default	$\frac{4i\sqrt{5x^2+3x+2}\,\sqrt{2x^2-x+3}\left(2i\sqrt{31}+5i\sqrt{23}-11\right)\sqrt{-\frac{\left(2i\sqrt{31}-5i\sqrt{23}-11\right)\left(-1+4x+i\sqrt{23}\right)}{\left(2i\sqrt{31}+5i\sqrt{23}-11\right)\left(i\sqrt{23}-4x+1\right)}}\left(i\sqrt{\right.}{23\sqrt{10x^4+x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/23*I*(5*x^2+3*x+2)^(1/2)*(2*x^2-x+3)^(1/2)*(2*I*31^(1/2)+5*I*23^(1/2)-11)
*(-(2*I*31^(1/2)-5*I*23^(1/2)-11)*(-1+4*x+I*23^(1/2))/(2*I*31^(1/2)+5*I*23^
(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2)*(I*23^(1/2)-4*x+1)^2*(I*23^(1/2)*(I*31^
(1/2)+10*x+3)/(2*I*31^(1/2)-5*I*23^(1/2)+11)/(I*23^(1/2)-4*x+1))^(1/2)*(I*2
3^(1/2)*(I*31^(1/2)-10*x-3)/(2*I*31^(1/2)+5*I*23^(1/2)-11)/(I*23^(1/2)-4*x+
1))^(1/2)*23^(1/2)*10^(1/2)*EllipticF((-(2*I*31^(1/2)-5*I*23^(1/2)-11)*(-1+
4*x+I*23^(1/2))/(2*I*31^(1/2)+5*I*23^(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2),((
2*I*31^(1/2)+5*I*23^(1/2)+11)*(2*I*31^(1/2)+5*I*23^(1/2)-11)/(2*I*31^(1/2)-
5*I*23^(1/2)+11)/(2*I*31^(1/2)-5*I*23^(1/2)-11))^(1/2))/(10*x^4+x^3+16*x^2+
7*x+6)^(1/2)/(2*I*31^(1/2)-5*I*23^(1/2)-11)/((-1+4*x+I*23^(1/2))*(I*23^(1/2
)-4*x+1)*(I*31^(1/2)+10*x+3)*(I*31^(1/2)-10*x-3))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2 - x + 3}\,\sqrt{5x^2 + 3x + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+3*x+2)**(1/2)/(2*x**2-x+3)**(1/2),x)

[Out] Integral(1/(sqrt(2*x**2 - x + 3)*sqrt(5*x**2 + 3*x + 2)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2\,x^2 - x + 3}\,\sqrt{5\,x^2 + 3\,x + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^(1/2)),x)

[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^(1/2)), x)

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```